M9902: The Quarterly Projection Model

František Brázdik
Macroeconomic Forecasting Division
frantisek.brazdik@cnb.cz

Czech National Bank

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INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Outline

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Time series analysis

- Analysis of time series data is based on smoothing past data in order to separate the underlying pattern in the data series from randomness.
- The underlying pattern then can be projected into the future and used as the forecast.
- The underlying pattern can also be broken down into sub patterns to identify the component factors that influence each of the values in a series: decomposition
- Decomposition methods: identify separate components of the basic underlying pattern that tend to characterize economics and business series.

In search for trends

Decomposition Techniques

- Goal: separation of data into several unobservable components, generally in an additive or multiplicative form.
- Components: trend, seasonal pattern, cycle, and residual or irregular pattern
- Seasonal component: the periodic fluctuations of constant length
- Trend-cycle component: long term changes in the level of series

Detrending

- Trend Component: The tendency of a variable to grow over time, either positively or negatively.
- Basic forces in trend: population change, price change, technological change, productivity change, product life cycles
- The long term movements or trend in a series can be described by a straight line or a smooth curve.
- The long-term trend is estimated from the seasonally adjusted data for the variable of interest
- Interpretation:
  - Long run equilibrium: trends
  - Cyclical fluctuations: gaps

Trend analysis

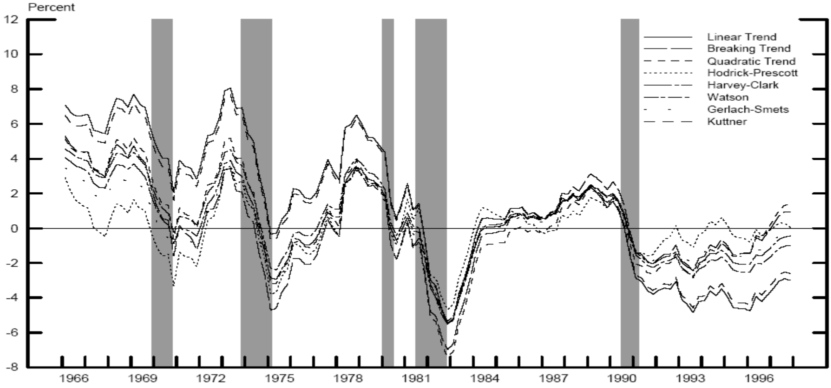
- Assume seasonally adjusted data
- Trend-Cycle decomposition: Series = Trend + Cycle + Noise
- No general-automatic techniques for detrending
- Simple techniques: Smoothing
  - Moving average: The average eliminate some higher frequency noise in the data, and leaves a smooth trend-cycle component. What order to use?
  - Simple centered moving average: can be defined for any odd order. A moving average of order k, is defined as the average consisting of an observation and the m = (k-1)/2 points on either side.
  - Centered moving average: take the simple centered moving average, assign weights and create weighted average
- Advanced techniques of detrending:
  - Fitting a polynomial
  - Using a structural model

Detrending techniques overview I

- Watson detrending: greater business cycle persistence; trend component follows a random walk with drift and cyclical component is a stationary finite order AR process.
- Harvey-Clark detrending: local linear trend model
- Hodrick-Prescott filter: univariate method
- Kalman filter: multivariate method, structural method
- Bandpass filter: not widely used, frequency domain analysis

Detrending techniques overview II

- Detrending comparison: US GDP gap



Outline

Motivation for QPM

- Separate econometric methods: Inconsistencies
- Short experience with FPAS: Forecasting and Policy Analysis System
- State:
  - Insufficient data and experience
  - Participation of other departments
  - Communication of results
- The further step on the way to complex structural models: DSGE
- Research tool

Features of QPM

- Reflects inflation targeting regime:
  - In December 1997: after an exchange rate crisis
  - CNB adopted a series of end-year inflation targets
  - Regime proved very effective in combating inflation and anchoring
  - Evolution toward a more transparent inflation targeting regime where monetary policy is anchored by a medium-term perspective
  - Change to point inflation target: Inflation target band
  - The character of the regime was further enhanced by publication of unconditional forecasts
- Linked to quarterly data
- Small open-economy gap model

Model of trends and cycle

- Two separate blocks:
  - Long run equilibrium trends
  - Cyclical fluctuations - gaps
  - These blocks are separable

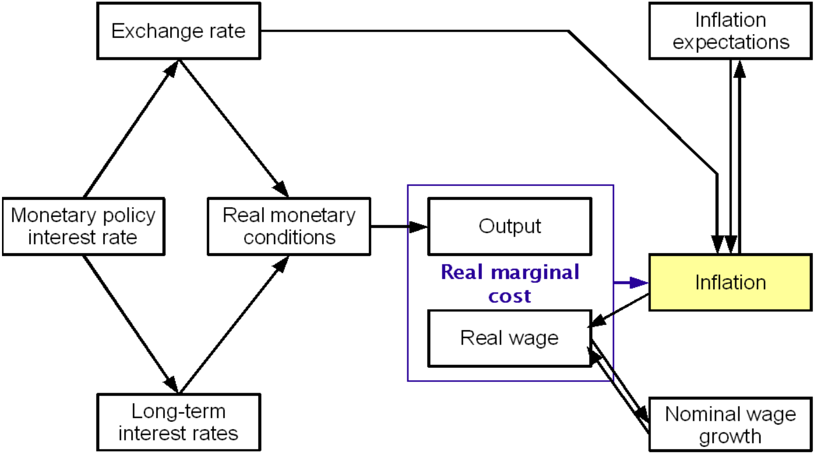
Long Run Trends

- First step: filter trend series
  - History - estimated by a simple statistical model (Kalman filter) and expert judgement
  - Forecast - exogenous (expert judgement), respecting steady state properties of QPM
- Important equilibrium values:
  - Real output growth
  - Real wage growth
  - Real exchange rate appreciation
  - Real interest rate
  - Stationarity is required: growth rates in focus
- Monetary decisions have small impact on long term real trends

Cyclical Part of QPM

- Description of the position of the Czech economy
- Monetary policy characteristics:
  - Inflation targeting regime
  - Forward looking policy
  - Focus on deviations from the target $\longrightarrow$ reaction to expected inflation a year ahead
  - Floating exchange rate - endogenous
- Description of behavior economic agents includes forward looking components
- Price frictions:
  - Wage stickiness
  - Final price stickiness
  - Expectation stickiness

Scheme of model



Real Economy I

IS curve (Aggregate demand):

- Output: function of lagged output, the real interest rate, the real exchange rate and foreign demand
- Includes impact of a change in interest rates with longer maturity on aggregate demand and take into account expectations about yield-curve on the dynamic properties of the model
- Real impact of monetary policy in a sticky-price model of a small open economy
- Marginal costs: cost of producing additional unit of a good

Real Economy II

Real Marginal Costs Gap:

- Approximation of inflationary pressures from the real economy.
- Marginal costs consist of the costs arising from the increasing volume of production (the "output gap") and wage costs (the "real wage gap").
- A positive real marginal cost gap implies an inflationary effect of the real economy

$$\widehat{\mathrm{mc}}_t = \lambda \widehat{\mathrm{y}}_t + \widehat{\mathrm{wr}}_t$$

Output Gap:

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Real Economy III

- Standard economic theory: higher real interest rate reduce aggregate demand by increasing the reward to saving
- Output gap: responds negatively to the difference between the real interest rate and its equilibrium value
- Open economy: the exchange rate matters
- Currency appreciation will, all else equal, make domestic goods more expensive in foreign markets and reduce demand for domestic goods abroad; cheaper imports may displace domestic goods

$$
\begin{aligned}
\widehat{\mathrm{y}}_t &= \alpha_1 \widehat{\mathrm{y}}_{t-1} - \widehat{\mathrm{rmci}}_{t-1} + \alpha_2 \widehat{\mathrm{y}}_t^f + \varepsilon_t^{\widehat{\mathrm{y}}} \\
\widehat{\mathrm{rmci}}_t &= \beta_1 \left(\beta_3 \widehat{\mathrm{rc}}_t + \beta_4 \widehat{\mathrm{r4}}_t + (1 - \beta_3 - \beta_4)\, \widehat{\mathrm{r4}}_t^f \right) + \beta_2 \widehat{\mathrm{z}}_t
\end{aligned}
$$

Real Wage Gap:

Real Economy IV

- Introduced in January 2007
- Wage costs are above their equilibrium level, they have an inflationary effect
- The effect of a deviation of the current level of the average real wage from its equilibrium level, which in the long run rises at the same rate as equilibrium real output (non-accelerating inflation real output)

$$\widehat{\mathrm{wr}}_t = \widehat{\mathrm{wr}}_{t-1} + \frac{\mathrm{w}_t}{4} - \frac{\pi_t}{4} - \frac{\triangle\overline{\mathrm{wr}}_t}{4} + \varepsilon_t^{\widehat{\mathrm{wr}}}$$

Phillips Curves I

Price Inflation:

- Phillips curve has been modified for a small open economy
- Blocks for various goods
- Import price effects
- Wage setters derive their nominal wage demand real consumer wage

- x for fuel, food, or adjusted excl. fuel inflation
- Administered prices are exogenous in baseline

Phillips Curves II

$$\begin{aligned}
\pi_t^x \;&=\; \gamma_1^x\left(\pi 4_t^{\mathrm{M}x} + \triangle_4 \overline{\mathrm{z}}_t^x\right) + \gamma_2^x\left(\mathrm{E}\pi 4_t + \triangle_4 \overline{\mathrm{z}}_t^x - \triangle_4 \overline{\mathrm{z}}_t\right) \\
&\quad + \left(1 - \gamma_1^x - \gamma_2^x\right)\pi_{t-1}^x + \gamma_3^x \widehat{\mathrm{mc}}_t + \varepsilon_t^{\pi^x}
\end{aligned}$$

Wage Inflation:

$$\mathrm{w}_t \;=\; \delta_1 \mathrm{Ew}4_t + (1-\delta_1)\mathrm{w}_{t-1} - \delta_2\left(\widehat{\mathrm{wr}}_t - \delta_3 \widehat{\mathrm{y}}_t\right) + \varepsilon_t^{\mathrm{w}}$$

Expectations I

Price Inflation Expectations:

- Expected inflation: a weighted combination of a backward-looking and a forward-looking component (the expected value of overall CPI inflation over the next four quarters)
- Overall CPI: an explicit link between changes in administered and energy prices and pressures on the rate of inflation for market prices

$$\mathrm{E}\pi 4_t = \lambda_1 \pi_{t+1} + (1-\lambda_1)\pi_{t-1} + \varepsilon_t^{\mathrm{E4}}$$

Wage Inflation Expectations:

$$\mathrm{Ew}4_t = \lambda_2 \mathrm{w}_{t+1} + (1-\lambda_2)\mathrm{w}_{t-1} + \varepsilon_t^{\mathrm{Ew4}}$$

Uncovered interest rate parity

Nominal Exchange Rate:

- UIP condition: arbitrage condition; international investors will equalize effective rates of return on investments in different currencies, allowing for any country-specific risk premiums
- foreign investor expecting a depreciation (appreciation) of the koruna will demand a higher (lower) return from Czech assets
- Moving average form

$$
\begin{aligned}
\mathrm{s}_t \quad = \quad & \phi \mathrm{s}_{t+1} + (1-\phi)\left(\mathrm{s}_{t-1} + 2\left(\frac{\mathrm{E}_t\pi}{4} - \frac{\mathrm{E}_t\pi^f}{4}\right) + 2\bigtriangleup \overline{\mathrm{z}}_t\right) \\
& + \frac{\mathrm{i}_t}{4} - \frac{\mathrm{i}_t^f}{4} - \mathrm{prem}_t + \varepsilon_t^{\mathrm{s}}
\end{aligned}
$$

Reaction Function

Nominal Interest Rate:

- Forward-looking reaction function
- CPI inflation expecte to be above the target rate: central bank push up the short-term
- Excess demand: the central bank increases short-term interest rate
- Long-term level for rates and some additional dynamic structure
- Interest rate inertia: interest rate smoothing

$$
\begin{aligned}
\mathrm{i}_t &= \psi \mathrm{i}_{t-1} + (1-\psi)\left(\mathrm{i}_t^{neutral} + \mathit{\Pi}_t\right) + \varepsilon_t^{\mathrm{i}} \\
\mathrm{i}_t^{neutral} &= \bar{\mathrm{r}}_t + \pi 4_{t+4} + \varepsilon_t^{\mathrm{i}} \\
\mathit{\Pi}_t &= \kappa_1\left(\pi 4_{t+4} - \pi 4_{t+4}^{target}\right) + \kappa_2 \widehat{\mathrm{y}}_t
\end{aligned}
$$

Outline

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Calibration vs. Estimation

- QPM is calibrated, partially estimated
- Problems in estimation:
  - Short data sample
  - Structural changes in economy
  - Changes of monetary policy regime
  - It is impossible to estimate some parameters: identification problems

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Calibration of QPM

Parameters setup:

- Restrictions on parameters originating from economic theory
- Parameters are set to mach the properties of data
- Responses to structural shocks

Parameters checks:

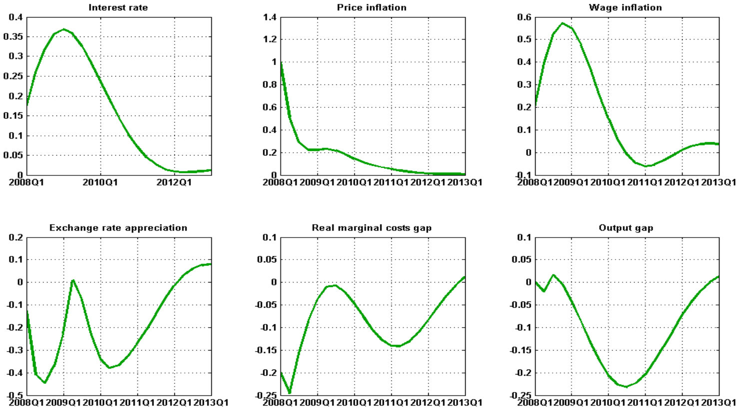
- Reactions to shocks
- Residuals
- In-sample simulations
- Curve-fitting estimates

Outline

Price shock I

- Positive shock to the output gap
- Upward pressure on inflation
- Currency depreciation
- Central bank increases interest rate
- Cumulative effect on output is very close to zero: feature of linear models;
- Offsetting of excess supply to counteract the effects of shocks that create excess demand

Price shock II



Aggregate demand shock I

- Positive shock to the output gap
- Upward pressure on inflation
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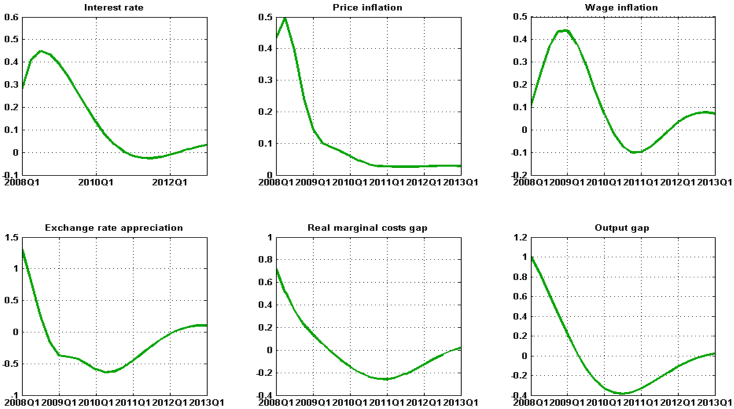
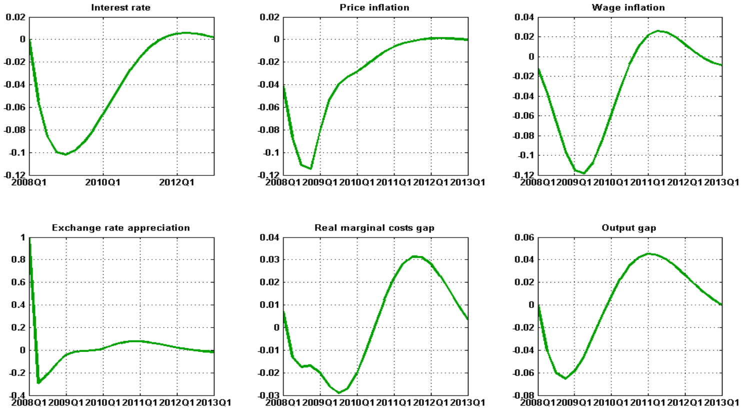
Aggregate demand shock II



Exchange rate shock I

- Depreciation acts to increase aggregate demand, opening a positive output gap

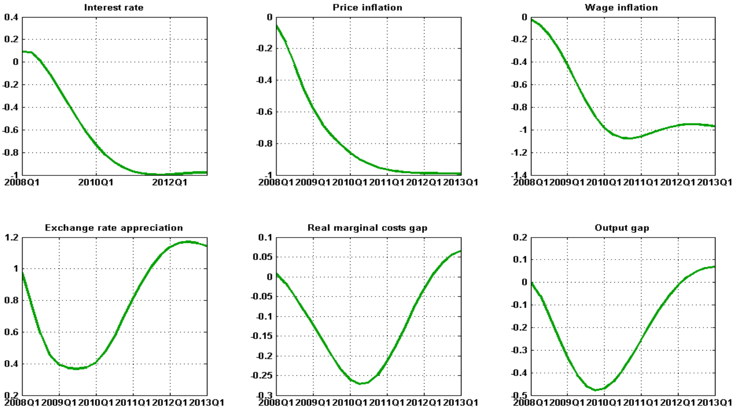
Exchange rate shock II



Inflation target change I

- Lower the target rate of inflation by one percentage point
- To achieve disinflation: raise the short rate
- Appreciation: Import prices fall
- The combined effect of the import price decline and the excess supply gap works to gradually pull down the rate of inflation
- Note: purely nominal shock, and since the model is super-neutral, there is no change to any real equilibrium in this shock, including the real exchange rate. The nominal exchange rate changes, of course, with the cumulative
- Cumulative effects on output and employment
- Sacrifice ratio: a cumulative loss of output vs. lower inflation by a percentage point

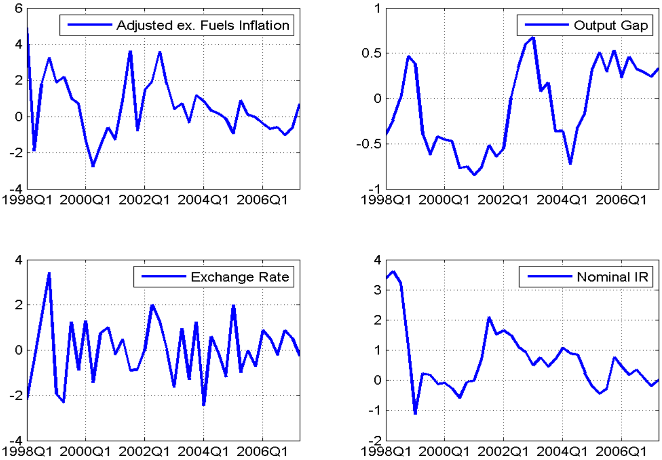
Inflation target change II



Residuals I

- Conflict between estimated parameters and calibrated
- The parameters have to be chosen so as to give reasonable model behavior
- Examined how well the model performs over the historical sample
- Identify systematic biases

Residuals II



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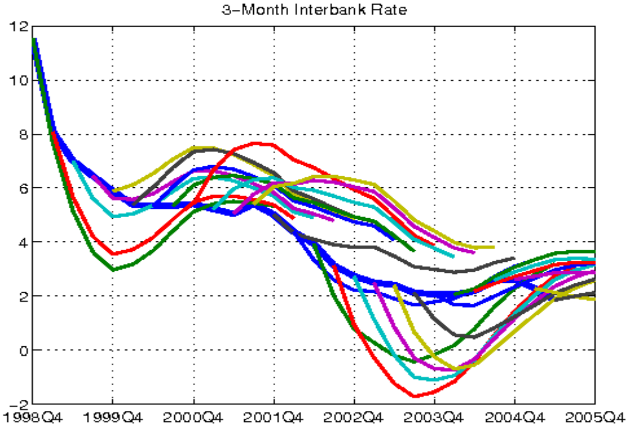
In-Sample Simulations



Modeling tools

- Implementation in Matlab
- IRIS by Jaromír Beneš

Univariate filtering I

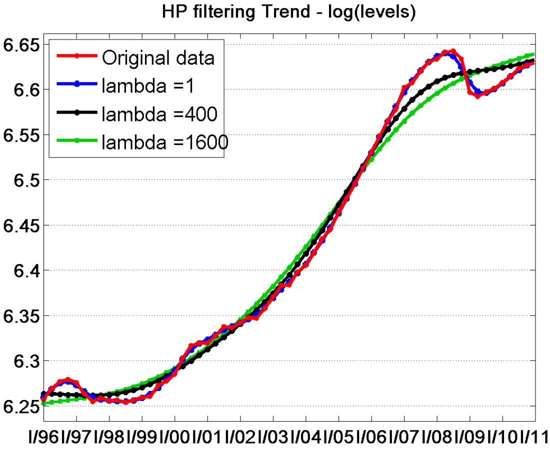
- Hodrick-Prescott filter: optimally extracts a trend which is stochastic but moves smoothly over time and is uncorrelated with the cyclical component
- Mathematics of HP filter:
  - Decomposition: $y_t = \tau_t + c_t$
  - Solve:
    $\min \sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda * \sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2$
  - $\lambda = 100 * (\textit{number of periods in a year})^2$
- Assumption that the trend is smooth is imposed by assuming that the sum of squares of the second differences of τ_t is small
- Sensitivity of the trend to short-term fluctuations is achieved by modifying a multiplier λ

Univariate filtering II

- Drawbacks:
  - One-time permanent shock, split growth rates present: Filter identifies non-existing shifts in the trend
  - It pushes noise in data to Normal distribution
  - Misleading predictive outcome: Analysis is purely historical and static

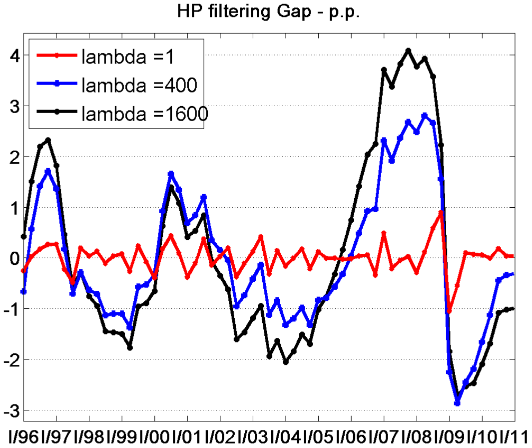
Univariate filtering III

- Trend:



Univariate filtering IV

- Gap:



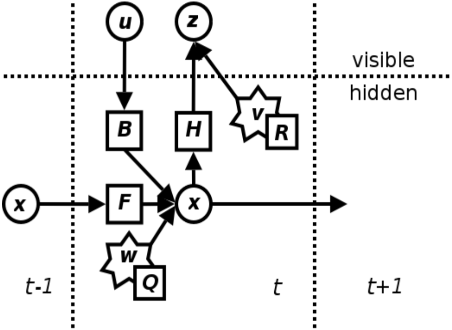
Kalman filter I

- Separate the cyclical component of a time series from raw data
- Can handle more series and exploit relations between them
- Kalman filter is a powerful tool for:
    - Estimation
    - Prediction
    - Smoothing

  Kalman filter:
    - Online estimation procedure
    - States are estimated, when the new observations are coming in

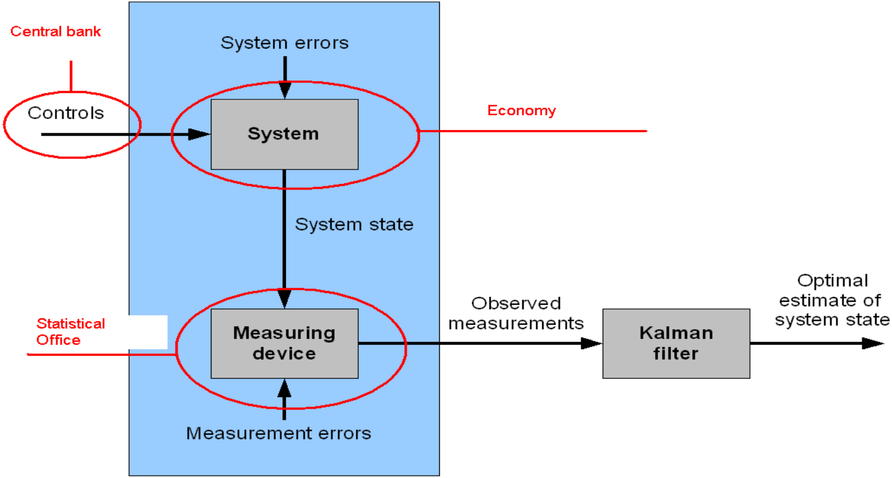
  Kalman smoother:
    - Off-line estimation procedure
    - The state estimation of is not only based on all previous observations, but also on all later observations

Kalman filter II



- F is the state transition model
- B is the control-input model
- H is the observation model
- w is the process noise
- z is the measurement
- v is the measurement error
- u is the exogenous control

Kalman filter structure



Description of variables

- Measurement variables: ΔEU_LGDP, $EU\_LGDPGAP\_EXPERT$
- State variables: ΔEU_LGDP_EQ, MU, $EU\_LGDPGAP$
- Exogenous-variables: $EU\_RMCIGAP$
- Shocks: ν's
- Coefficients: a_1, a_2, a_3 and μ_{SS}
- Variance: $\sigma_1, \sigma_2, \sigma_3, \sigma_4$
- Remark: In the following slides the filtering is actually smoothing

Description of model

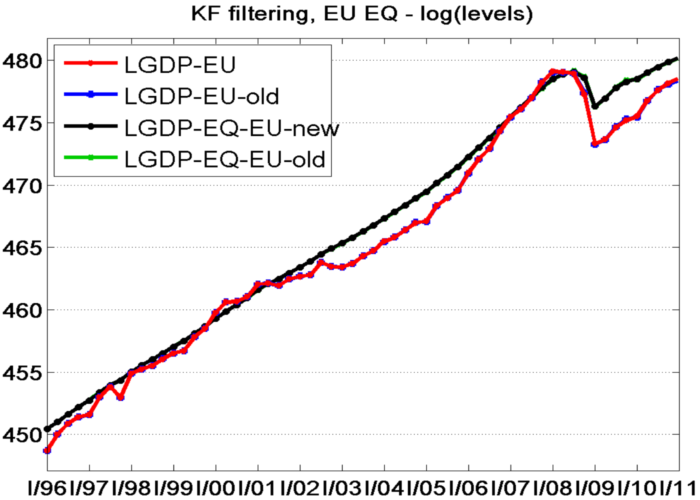
- Measurement equations:

$$
\begin{aligned}
\Delta EU\_LGDP &= \Delta EU\_LGDP\_EQ + \\
&+ 4 * (EU\_LGDPGAP - EU\_LGDPGAP\{-1\}) \\
EU\_LGDPGAP &= EU\_LGDPGAP\_EXPERT + \sigma_4 * \nu_4
\end{aligned}
$$

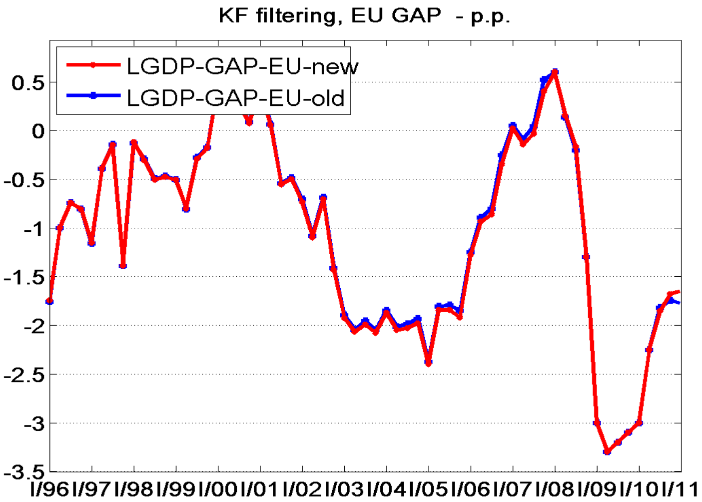
- State equations:

$$
\begin{aligned}
\Delta EU\_LGDP\_EQ &= \mu + \sigma_1 * \nu_1 \\
\mu &= (1 - a_3) * \mu_{SS} + a_3 * \mu\{-1\} + \sigma_3 * \nu_3 \\
EU\_LGDPGAP &= a_1 * EU\_LGDPGAP\{-1\} + \\
&+ a_2 * EU\_RMCIGAP\{-1\} + \sigma_2 * \nu_2
\end{aligned}
$$

Filtering results: EU Eq. trajectories



Filtering results: EU Gap estimate



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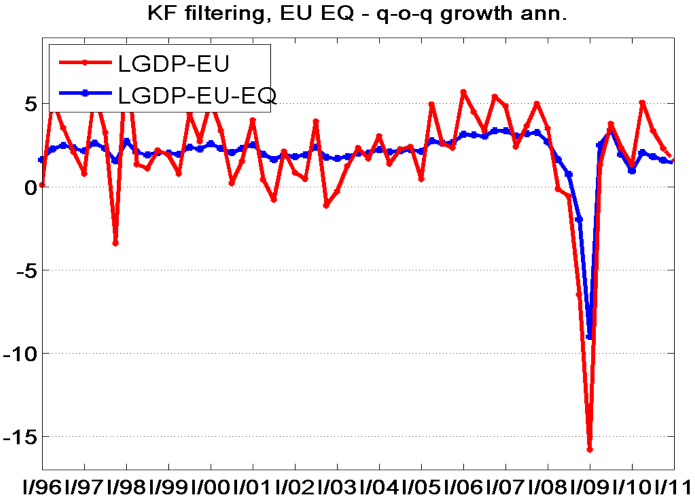
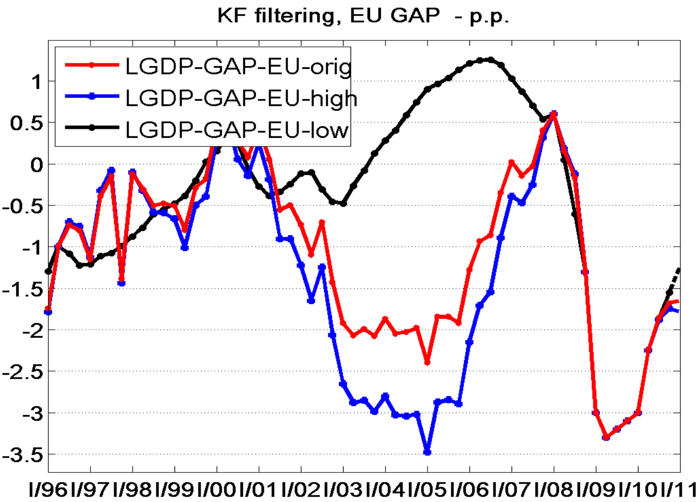
Filtering results: Removing volatility



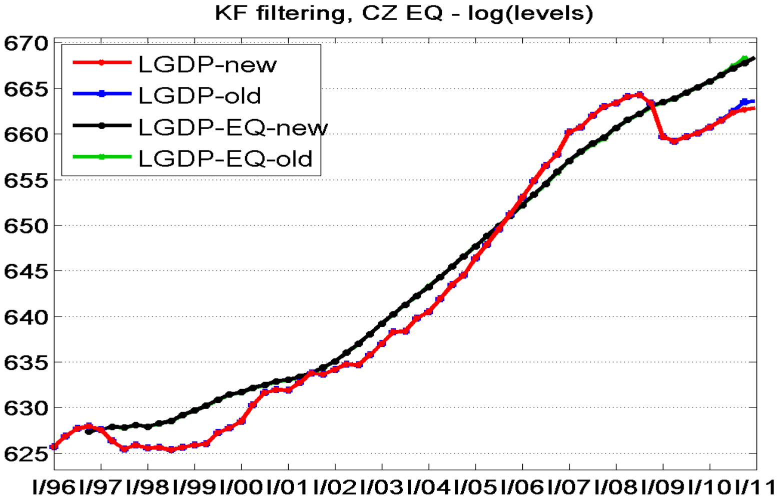
Model setting: Changes in volatility of gap σ_2



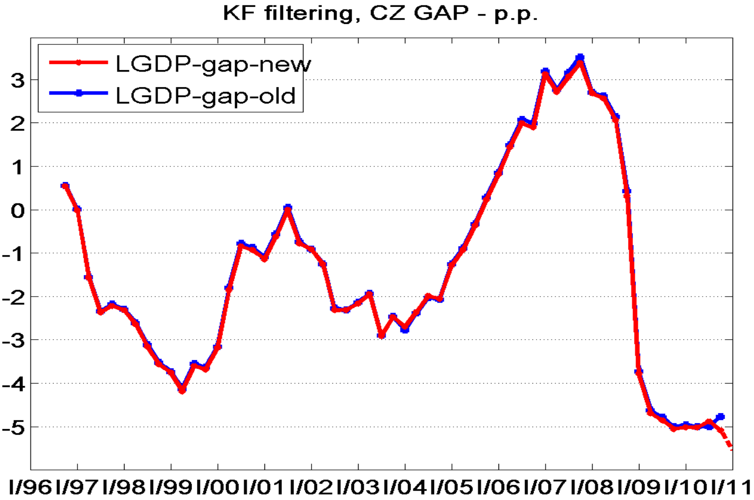
Filtering domestic variables

- First step:
  - Decompose real variables: trend and cycle
  - Simple model for: Real interest rate, Real exchange rate, Exchange risk premium
- Second step:
  - Utilize measurement of inflation and wage growth
  - Fit simple backward-looking Phillips curves: relation between inflation and output gap
  - Fit IS curve: relation between output gap and gaps in real interest and exchange rate
  - Decompose: domestic output, real wage, unemployment

Filtering results: Domestic Eq. trajectory



Filtering results: Domestic output gap



Description: Second step model

- Measurement variables: *DOT_LGDP*, *DOT_UNR*, *PIE_CORE*, *PIE_W*, *DOT_LWR*, *LWR_GAP_EXPERT*, *LGDP_GAP_EXPERT*, *UNR_GAP_EXPERT*
- State variables: *DOT_LGDP_EQ*, *MU*, *LGDP_GAP*, *DOT_UNR_EQ*, *UNR_GAP*, *PIE_CORE_S*, *PIE_W_S*, *DOT_LWR_EQ*, *LWR_GAP*
- Exogenous-variables: *RRC_GAP*, *RR4_GAP*, *EU_RR4_GAP*, *LZ_GAP*, *EU_LGDP_GAP*, *PIE_M_XENERGY4*, *DOT_LZ_CORE_EQ4*, *DOT_LZ_EQ4*, *E0_CORE4*, *E0_PIE_W4*, *DOT_LWR_PRIOR*, *E0_PIE4*
- Shocks: νs
- Variance: σs

Model I

- Measurement equations:

$$
\begin{aligned}
DOT\_LGDP &= DOT\_LGDP\_EQ + 4 * (LGDP\_GAP - LGDP\_GAP\{-1\}) \\
DOT\_UNR &= DOT\_UNR\_EQ - 4 * (UNR\_GAP - UNR\_GAP\{-1\}) \\
PIE\_CORE &= PIE\_CORE\_S \\
PIE\_W &= PIE\_W\_S \\
DOT\_LWR &= DOT\_LWR\_EQ + 4 * (LWR\_GAP - LWR\_GAP\{-1\}) \\
LWR\_GAP &= LWR\_GAP\_EXPERT + std\_w3 * \nu\_LWR\_GAP\_EXPERT \\
LGDP\_GAP &= LGDP\_GAP\_EXPERT + std\_w1 * \nu\_LGDP\_GAP\_EXPERT \\
UNR\_GAP &= UNR\_GAP\_EXPERT + std\_w2 * \nu\_UNR\_GAP\_EXPERT
\end{aligned}
$$

Model II

- State equations:

$$
\begin{aligned}
DOT\_LGDP\_EQ &= MU\{-1\} + a1 * DOT\_UNR\_EQ + std\_v1 * \nu\_DOT\_LGDP\_EQ \\
LGDP\_GAP &= LGDP\_GAP\_C01 * LGDP\_GAP\{-1\} - RMCI\_GAP\_C02 * (b2 * RRC\_GAP\{-1\} \\
&+ b3 * RR4\_GAP\{-1\} + b4 * EU\_RR4\_GAP\{-1\}) \\
&\quad -RMCI\_GAP\_C01 * LZ\_GAP\{-1\} + \\
&\quad LGDP\_GAP\_C02 * EU\_LGDP\_GAP + std\_v2 * \nu\_LGDP\_GAP \\
MU &= (1 - a3) * MU\_SS + a3 * MU\{-1\} + std\_v3 * \nu\_MU \\
DOT\_UNR\_EQ &= std\_v4 * \nu\_DOT\_UNR\_EQ \\
UNR\_GAP &= UNR\_GAP\_C01 * UNR\_GAP\{-1\} \\
&+ UNR\_GAP\_C02 * LGDP\_GAP + std\_v5 * \nu\_UNR\_GAP \\
PIE\_CORE\_S &= PIE\_CORE\_C01 * (PIE\_M\_XENERGY4 + DOT\_LZ\_CORE\_EQ4) \\
&+ PIE\_CORE\_C02 * (PIE\_CORE\_C05 * E0\_CORE4 \\
&+ (1 - PIE\_CORE\_C05) * E0\_PIE4) \\
&+ (1 - PIE\_CORE\_C01 - PIE\_CORE\_C02) * PIE\_CORE\_S\{-1\} \\
&+ RMC\_GAP\_C01 * PIE\_CORE\_C03 * LGDP\_GAP \\
&+ PIE\_CORE\_C03 * LWR\_GAP \\
&+ std\_v6 * \nu\_PIE\_CORE \\
PIE\_W\_S &= PIE\_W\_C01 * E0\_PIE\_W4 + (1 - PIE\_W\_C01) * PIE\_W\_S\{-1\} \\
&+ PIE\_W\_C02 * (LWR\_GAP - PIE\_W\_C03 * LGDP\_GAP) + std\_v7 * \nu\_PIE\_W \\
DOT\_LWR\_EQ &= DOT\_LGDP\_EQ + DOT\_LWR\_PRIOR + std\_v8 * \nu\_DOT\_LWR\_EQ \\
LWR\_GAP &= f1 * LWR\_GAP\{-1\} + std\_v9 * \nu\_LWR\_GAP
\end{aligned}
$$

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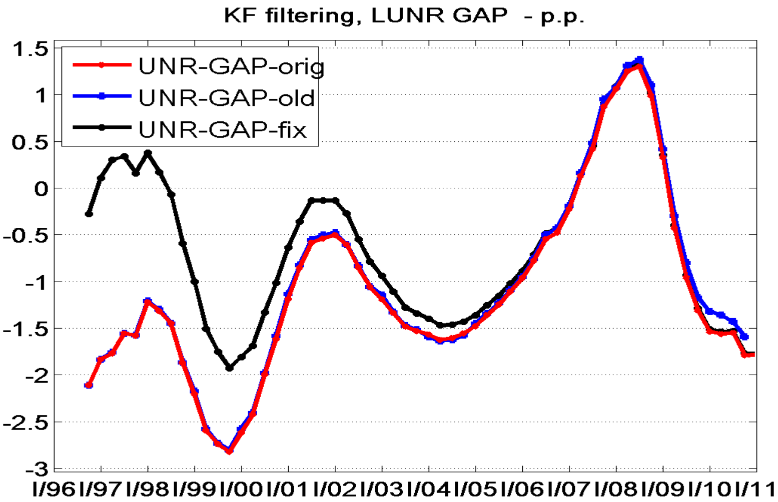
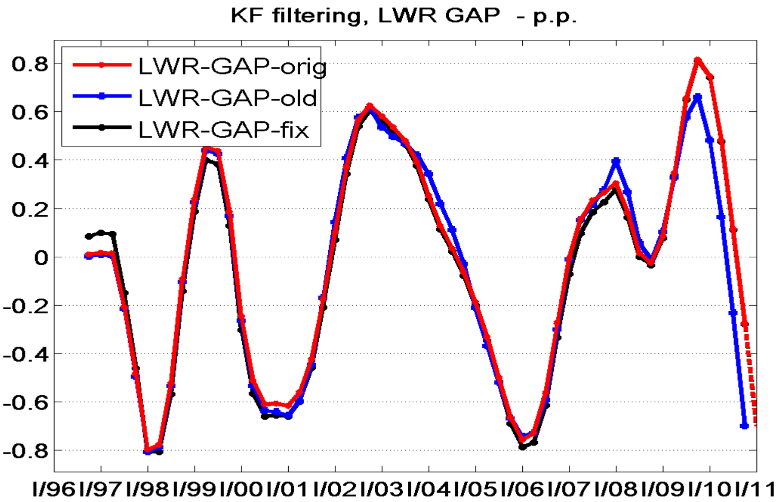
Filtering results: Expert judgement



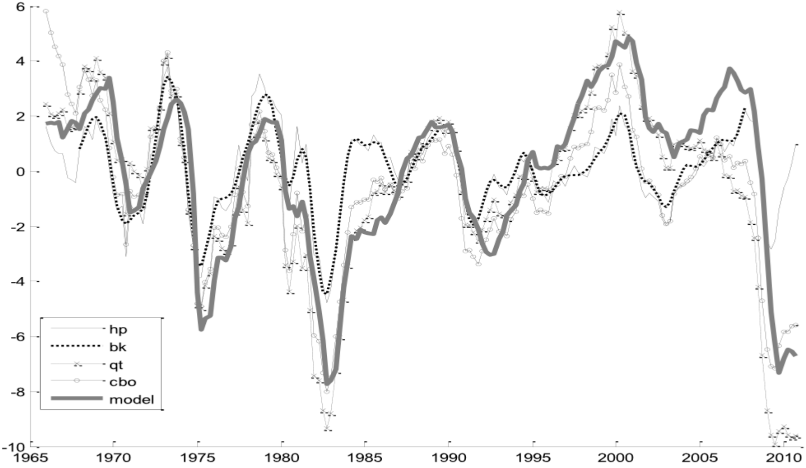
Filtering results: Expert judgement



Advanced filtering

- Criticism of simple models: lack of reference to unemployment
- J. Galí,F. Smets and R. Wouters (2011):
    - Address this issue in an extended model
    - Conclusion: Model-based output gap resembles conventional measures of the cyclical component of log GDP.
    - Comparison of a variety of statistical detrending methods
    - HP filter, band-pass filter, quadratic detrending, and the Congressional Budget Office's measure

Advanced filtering



In search for future trends

List of Variables I

$\widehat{a}$	gap of the variable a
$\bar{a}$	trend (equilibrium) value of the variable a
a^f	variable a for the foreign country
ε^a	residual in the equation for the variable a

mc	real marginal costs
y	real output
rw	real wage
rmci	real monetary condition index
r4	real 1Y interbank rate
r	real 3M interbank rate
rc	real rate of newly-issued bank loans
z	real exchange rate

List of Variables II

$\pi4^{target}$	inflation target (y-o-y)
π	price inflation (q-o-q)
$\pi4$	price inflation (y-o-y)
w	wage inflation (q-o-q)
$\mathrm{w4}$	wage inflation (y-o-y)
$\pi4^{\mathrm{M}}$	imported inflation (y-o-y)
s	nominal exchange rate
prem	risk premium
i	nominal short-term interest rate
$\mathrm{i}^{neutral}$	policy neutral short-term interest rate
$\alpha,\ \beta,\ \gamma,\ \delta,\ \phi,\ \psi,\ \kappa,\ \lambda$	parameters

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Additional one ...