Homework number 4

Do one of the exercises 10 and 11 and the exercise 12.

Exercise 10. Compute $H_n(\mathbb{R}P^{10}/\mathbb{R}P^5)$ using the CW-structure of this space.

Exercise 11. Let $X = D^{n+1} \cup_f S^n$, where $f : S^n \to S^n$ is a map of degree k. Compute $H_i(X)$ and determine all nontrivial homomorphisms in the long exact sequence for the couple (X, S^n).

Exercise 12. Let x be a point in a space X. The local homology groups of X in the point x are the groups $H_n(X, X - \{x\})$. Using excission theorem they are isomorphic to $H_n(U, U - \{x\}$ where U is an open neighbourhood of the point x. These local homology groups are preserved by homeomorphisms and so they are important if you want to decide if two spaces are homeomorphic.
(**a**) Consider X as a one-skeleton of tetrahetron $A_0A_1A_2A_3$ and compute the local homology groups of this space with respect to a vertex of the tetrahedron and with respect to a center of an edge.
(**b**) Now add the center T of the tetrahedron and let Y be the union of all six triangles TA_iA_j, $i \neq j$. Compute the local homology groups of Y with respect to T.