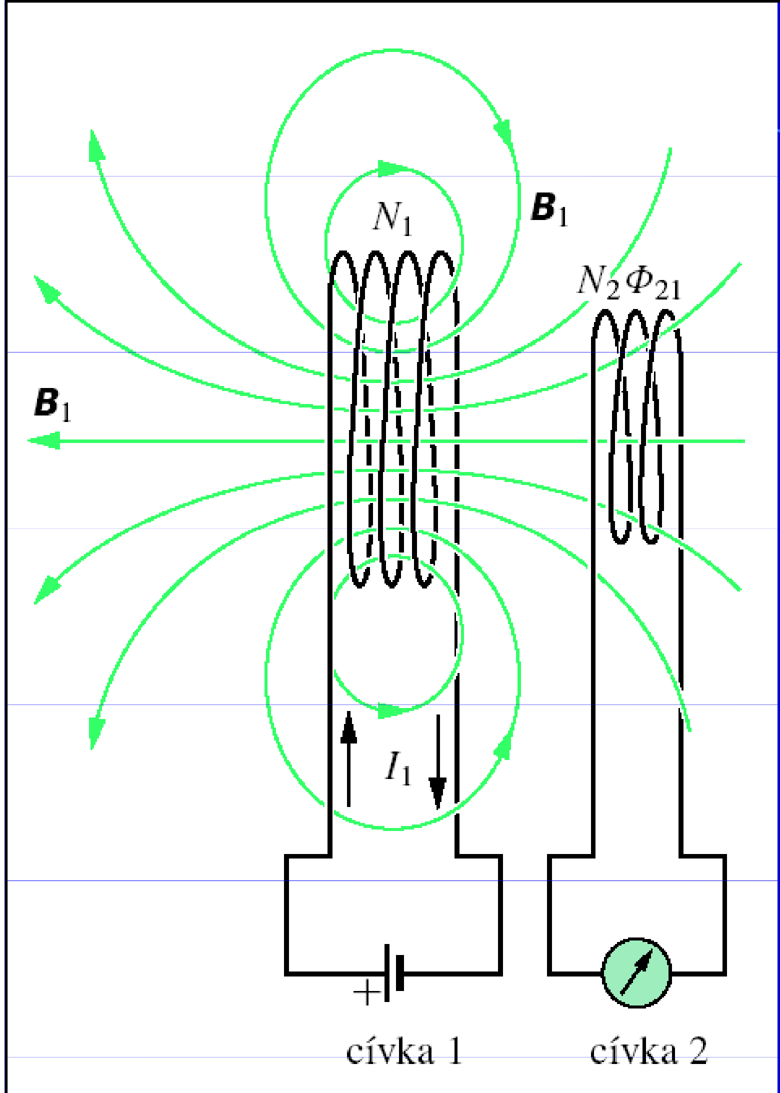
Indukce

a

indukčnost



$$\oint_C \vec{B}_1 \cdot d\vec{s} = \mu_0 N_1 I_1(t)$$

$$B_1 \propto I_1$$

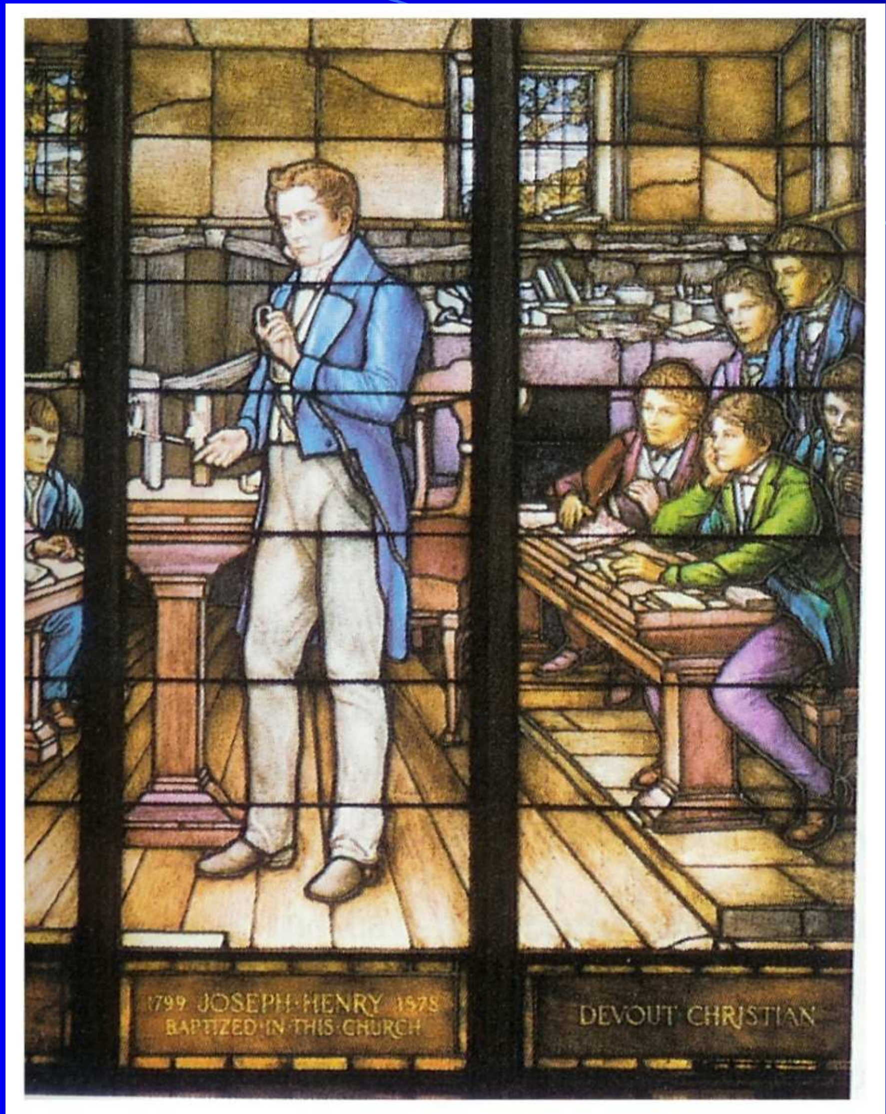
$$N_2\Phi_{21} = N_2 \iint_{S_2} \vec{B}_1(2)\cdot d\vec{S} = M_{21} I_1$$

vzájemná indukčnost

$$\mathcal{E}_2^{ind} = -\frac{d\left(N_2\Phi_{21}\right)}{dt}$$

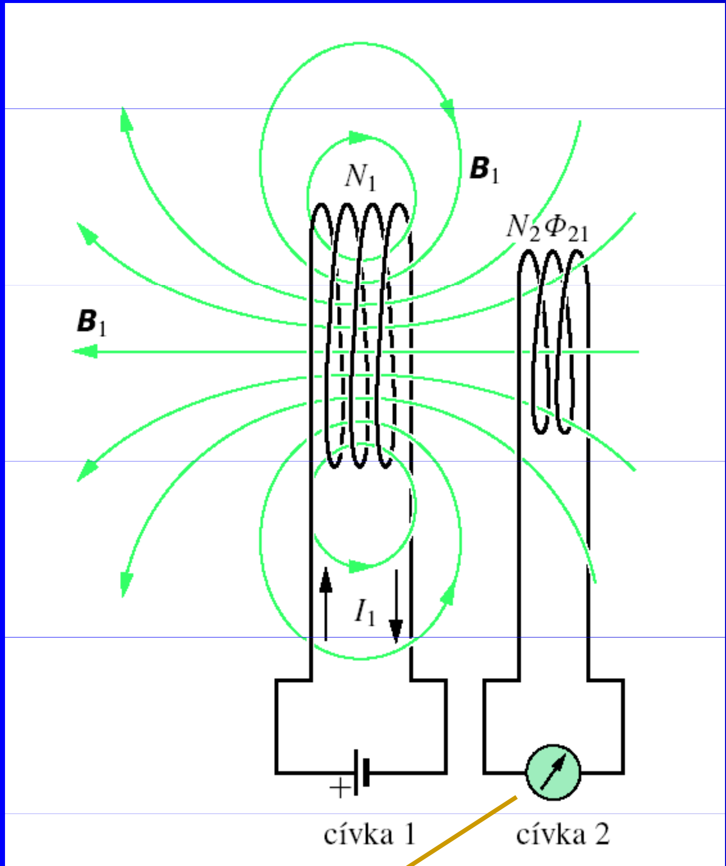
1 H (henry) =1 T·m^2·A^{-1}

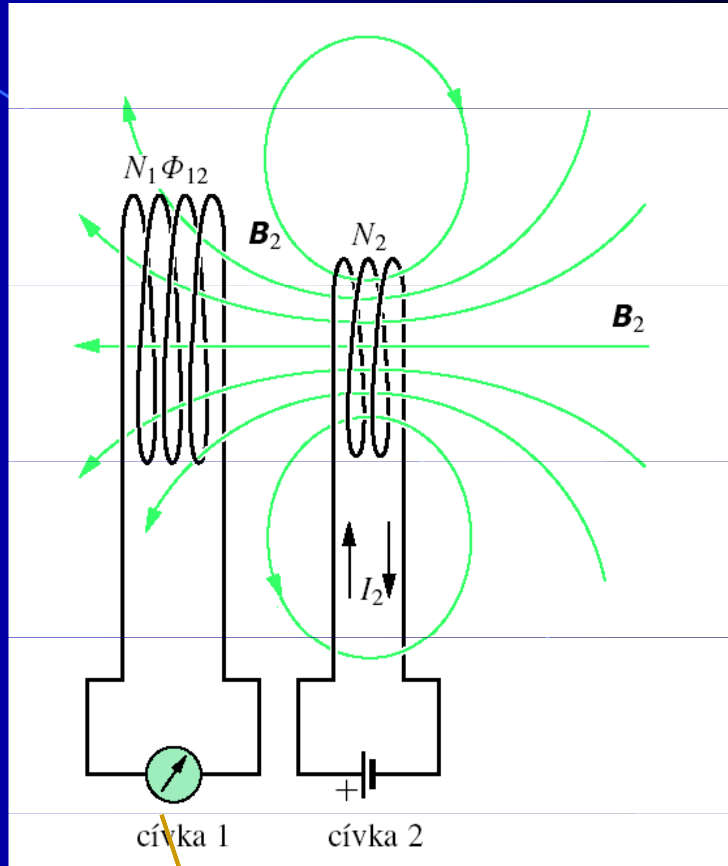
$$\mathcal{E}_2^{ind} = -M_{21}\frac{dI_1}{dt}$$



Joseph Henry
(vitráž v Prvním presbyteriánském kostele v Albany (NY), kde byl pokřtěn)

Vzájemná indukce





$$\mathscr{E}_2 = -M_{21}\frac{\mathrm{d}I_1}{\mathrm{d}t}$$

$$\mathscr{E}_1 = -M_{12}\frac{\mathrm{d}I_2}{\mathrm{d}t}$$

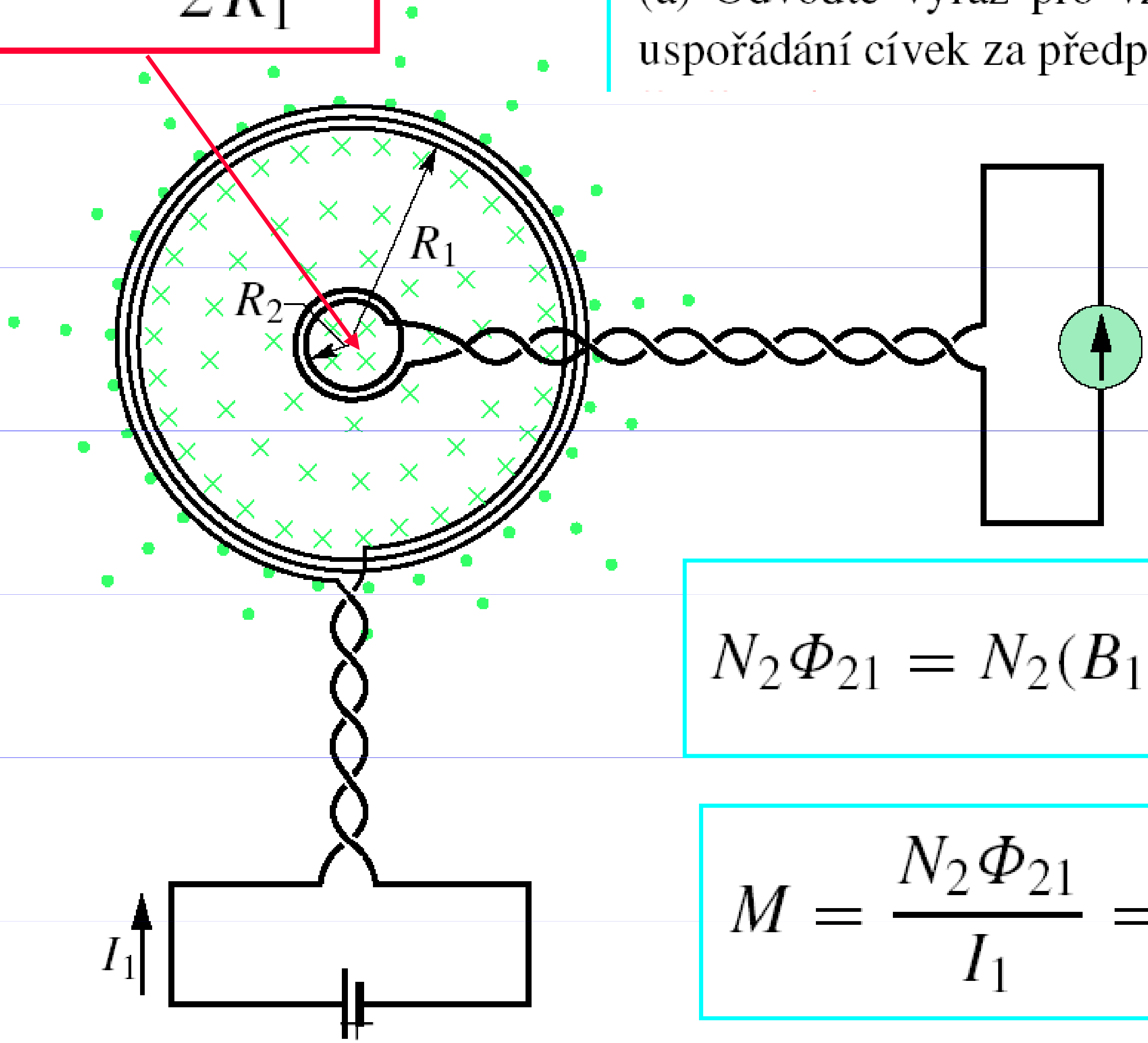
$$M_{21} = M_{12} = M$$

PŘÍKLAD 31.12

Na obr. 31.25 jsou dvě kruhové hustě vinuté souosé cívky ležící ve stejné rovině. Menší má poloměr R_2 a počet závitů N_2, větší má poloměr R_1 a počet závitů N_1.

(a) Odvoďte výraz pro vzájemnou indukčnost M pro toto uspořádání cívek za předpokladu, že $R_1 \gg R_2$.

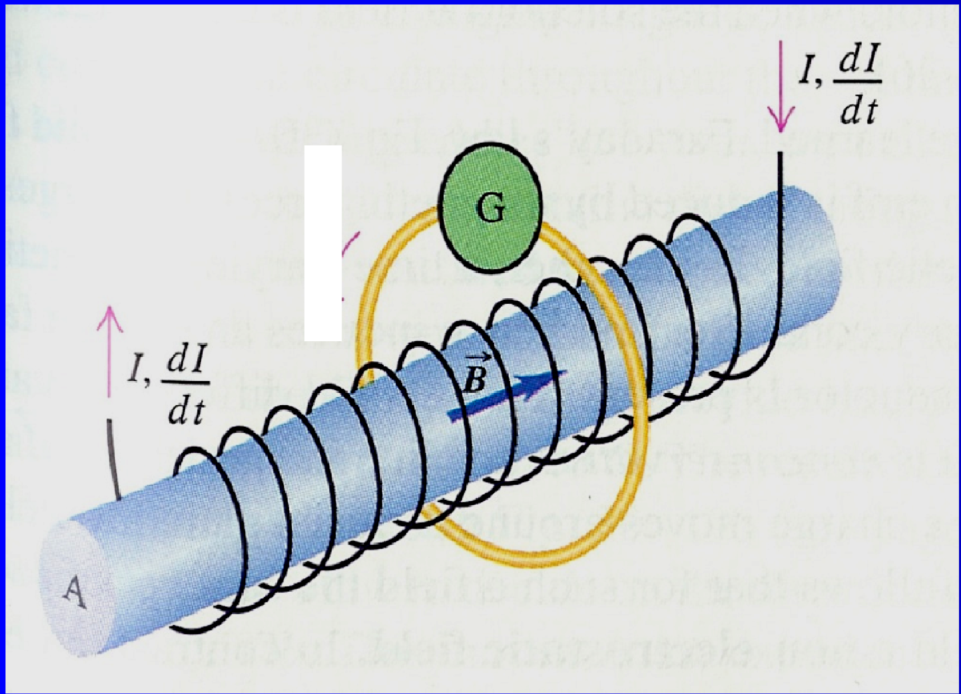
$$B_1 = \frac{\mu_0 I_1 N_1}{2R_1}$$



$$N_2 \Phi_{21} = N_2 (B_1)(\pi R_2^2) = \frac{\pi \mu_0 N_1 N_2 R_2^2 I_1}{2R_1}$$

$$M = \frac{N_2 \Phi_{21}}{I_1} = \frac{\pi \mu_0 N_1 N_2 R_2^2}{2R_1}$$

Vlastní indukce



$$\oint_C \vec{B} \cdot \mathrm{d}\vec{s} = \mu_0 N I(t)$$

$B \propto I$

$$N\Phi = N \iint_S \vec{B} \cdot \mathrm{d}\vec{S} = L I$$

vlastní indukčnost

1 H

$$\mathcal{E}_{ind} = -\frac{\mathrm{d}(N\Phi)}{\mathrm{d}t}$$

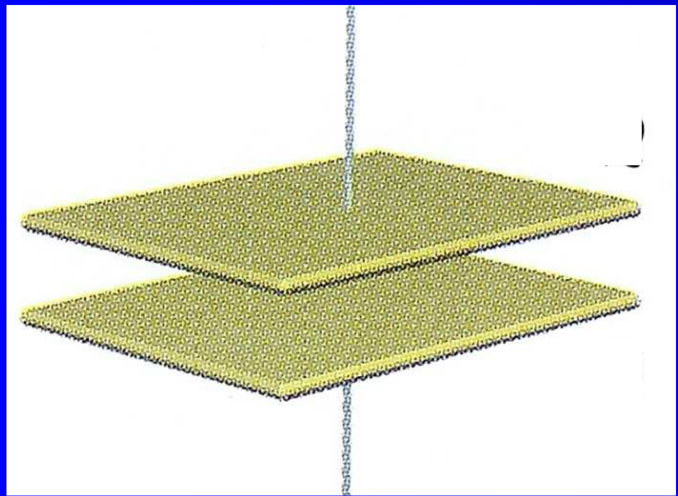
$$\mathcal{E}_L = -L\frac{\mathrm{d}I}{\mathrm{d}t}$$

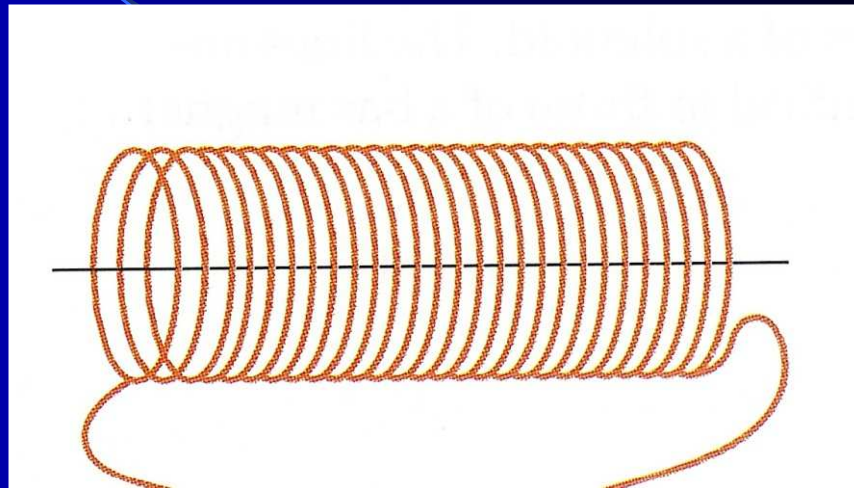
Indukčnost solenoidu

$$B = \mu_0 I n$$

$$N\Phi_B = (nl)(BS)$$

$$L = \frac{N\Phi_B}{I} = \mu_0 n^2 l S$$

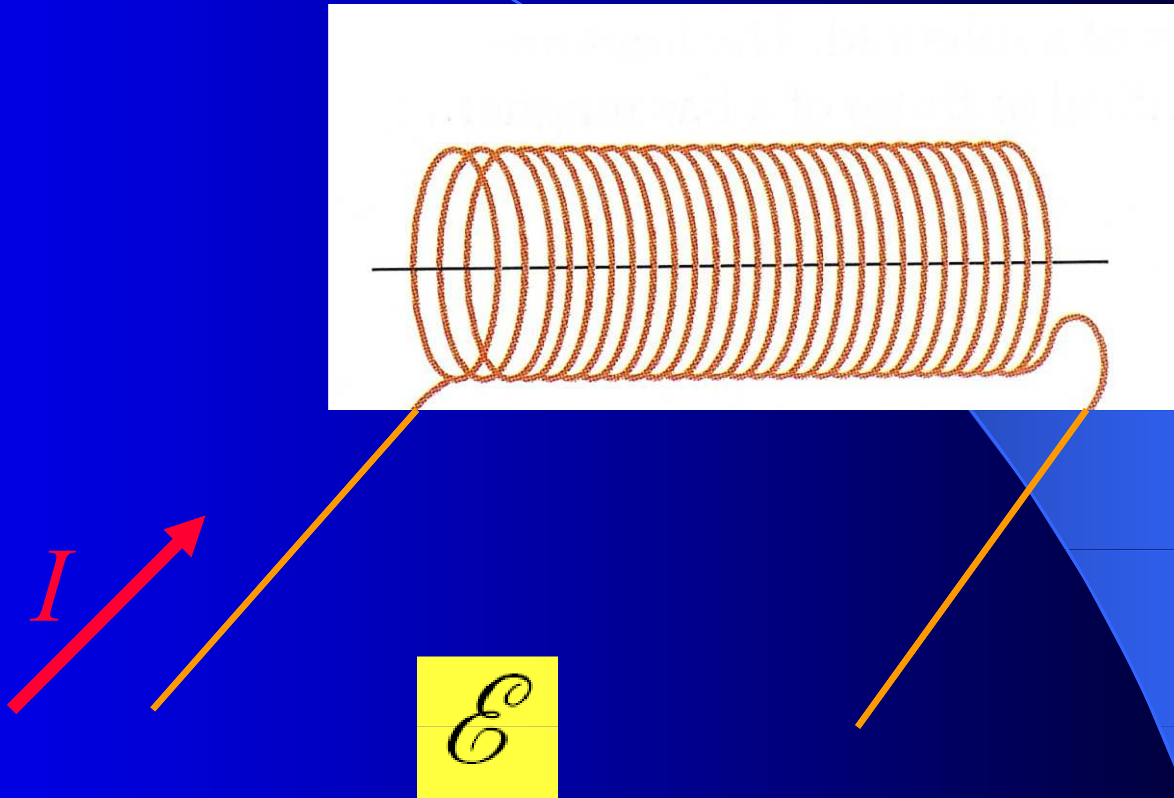




$$C = \frac{\varepsilon_r \varepsilon_0 S}{d}$$

$$L = \frac{\mu_r \mu_0 N^2 S}{l}$$

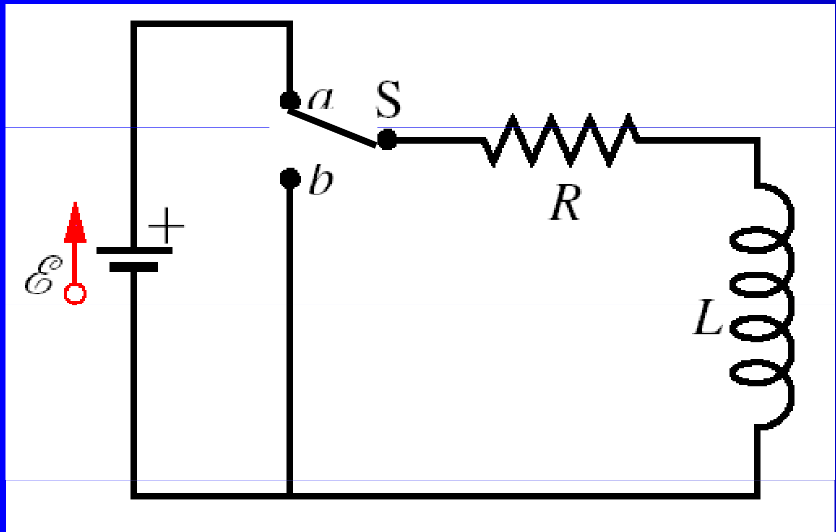
Směr indukovaného napětí



proud roste: + ← −

proud klesá: − → +

OBVODY *RL*



$\mathcal{E}_L$

$$-IR - L\frac{\mathrm{d}I}{\mathrm{d}t} + \mathcal{E} = 0$$

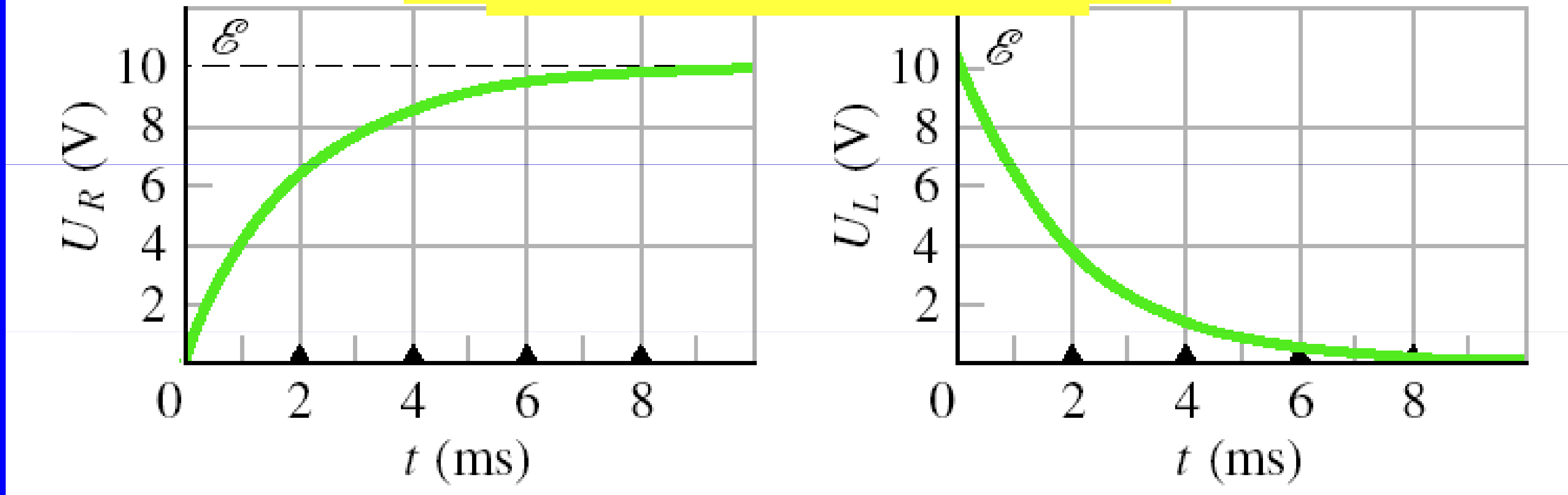
$$L\frac{\mathrm{d}I}{\mathrm{d}t} + RI = \mathcal{E}$$

L/R

$U_R = RI$

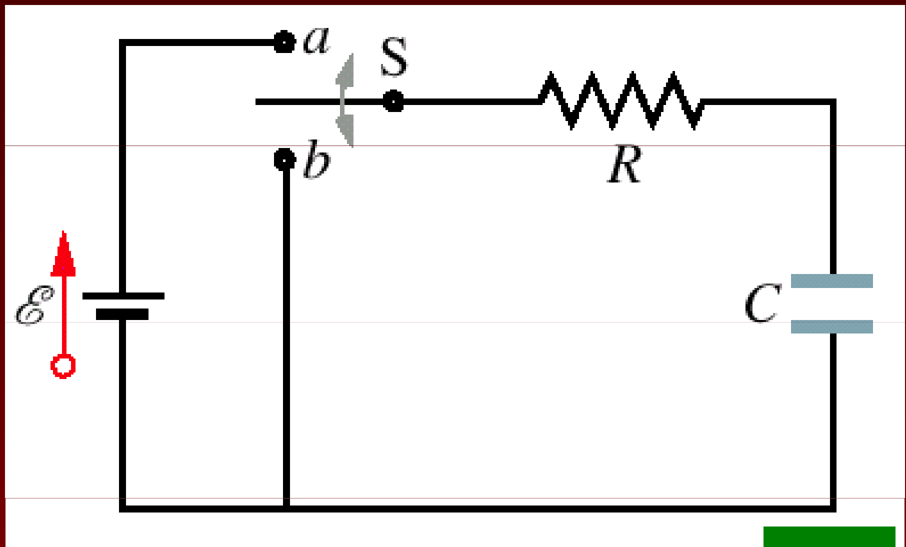
$$Q = C\mathcal{E}\left(1 - \mathrm{e}^{-t/(RC)}\right)$$

$$U_L = L\frac{\mathrm{d}I}{\mathrm{d}t}$$



OBVODY *RC*

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}$$



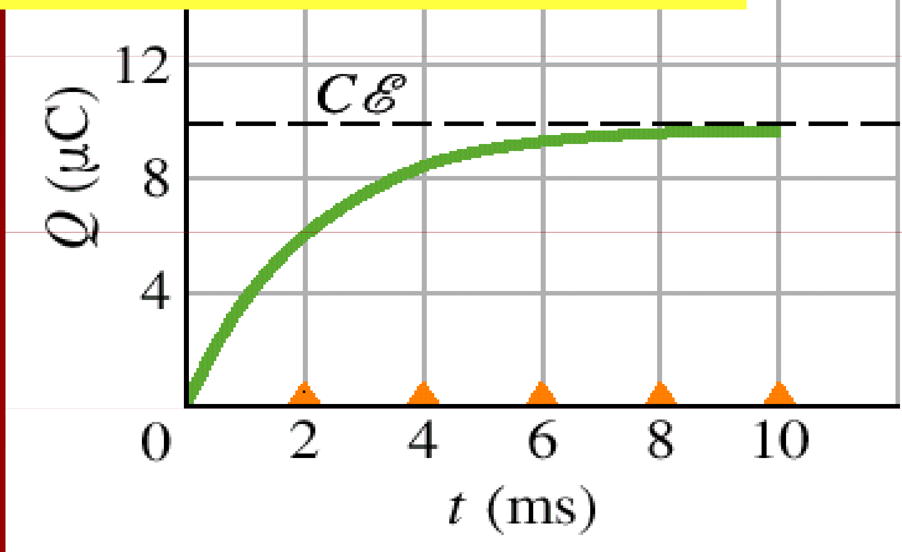
$$\mathcal{E} - IR - \frac{Q}{C} = 0.$$

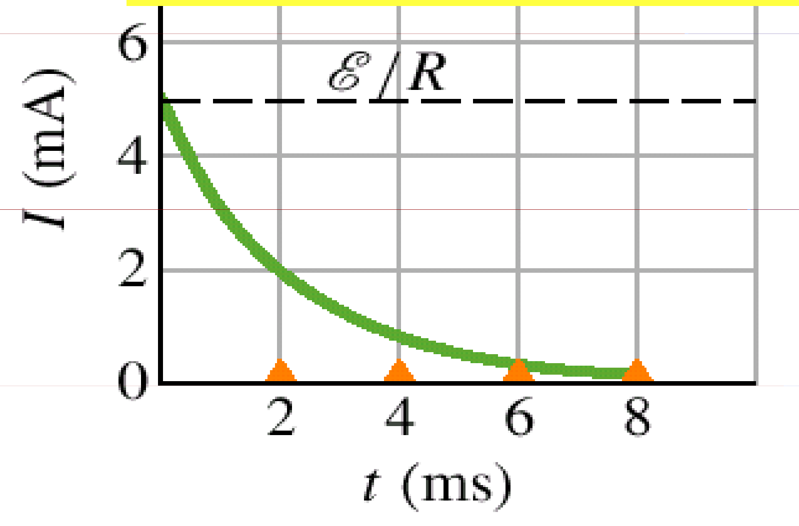
$$R\frac{\mathrm{d}Q}{\mathrm{d}t} + \frac{Q}{C} = \mathcal{E}$$

τ_C

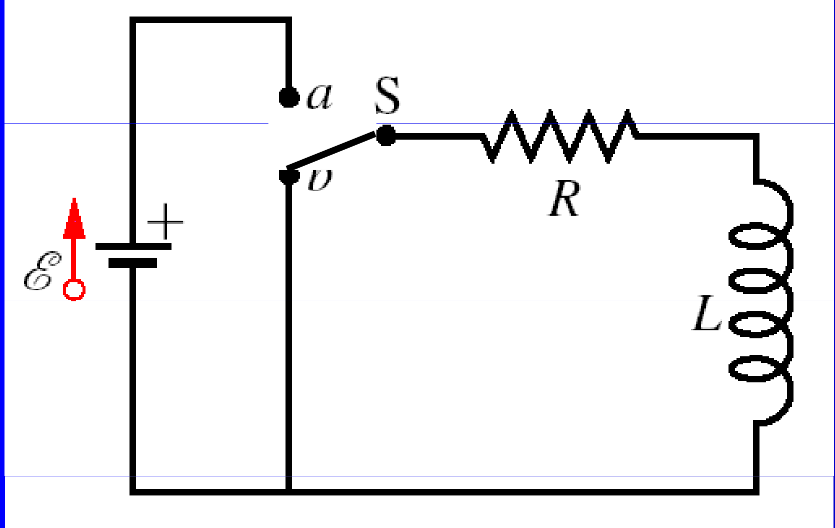
$$Q = C\mathcal{E}\left(1 - \mathrm{e}^{-t/(RC)}\right)$$

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{\mathcal{E}}{R}\mathrm{e}^{-t/(RC)}$$





OBVODY *RL*



ε_L

$$-IR - L\frac{dI}{dt} = 0.$$

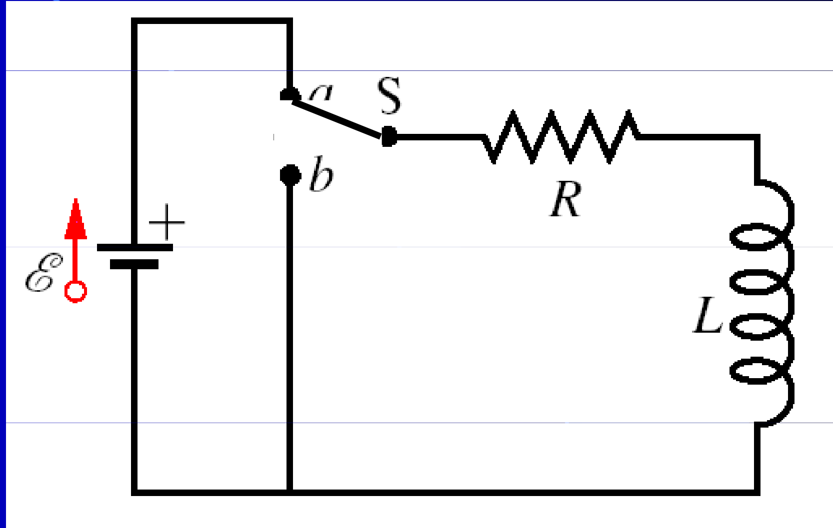
$$L\frac{dI}{dt} + RI = 0$$

$$I = \frac{\mathcal{E}}{R}e^{-t/\tau_L} = I_0 e^{-t/\tau_L}$$

?

ENERGIE MAGNETICKÉHO POLE

$$\mathscr{E} = L\frac{\mathrm{d}I}{\mathrm{d}t} + IR$$



$$\mathscr{E}I = LI\frac{\mathrm{d}I}{\mathrm{d}t} + I^2R,$$

$$\mathrm{d}E_{\mathrm{mg}} = LI\,\mathrm{d}I$$

$$\int_0^{E_{\mathrm{mg}}} \mathrm{d}E'_{\mathrm{mg}} = \int_0^I LI'\,\mathrm{d}I'$$

$$E_{\mathrm{mg}} = \tfrac{1}{2}LI^2$$

Kde je energie?

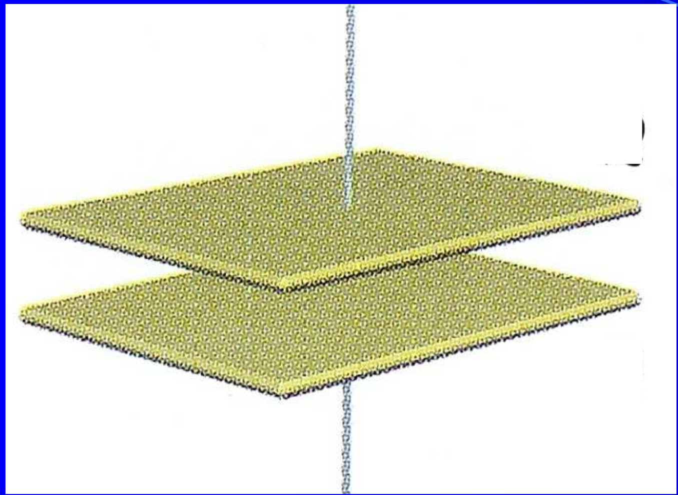
V magnetickém poli

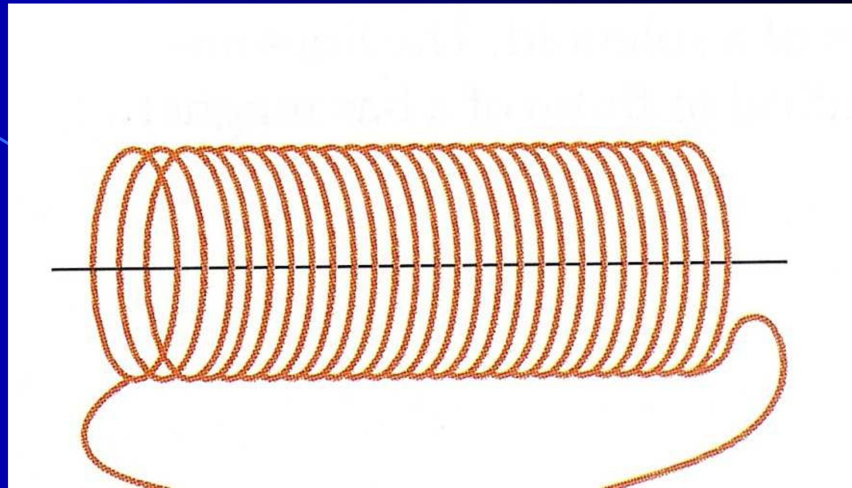
BS — solenoid — $\dfrac{Bl}{\mu_0 N}$ — V

$$E_{\mathrm{mg}} = \frac{1}{2}(LI)I = \frac{1}{2}N\Phi I = \frac{1}{2}(NBS)\left(\frac{Bl}{\mu_0 N}\right) = \left(\frac{B^2}{2\mu_0}\right)(Sl)$$

s hustotou

$$w_{\mathrm{mg}} = \frac{B^2}{2\mu_0}$$





$$C = \frac{\varepsilon_r \varepsilon_0 S}{d}$$

$$L = \frac{\mu_r \mu_0 N^2 S}{l}$$

$$E_C = \frac{1}{2} C U^2$$

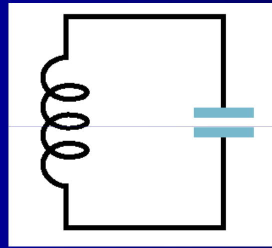
$$E_L = \frac{1}{2} L I^2$$

$$w_{el} = \frac{1}{2} \varepsilon_0 E^2$$

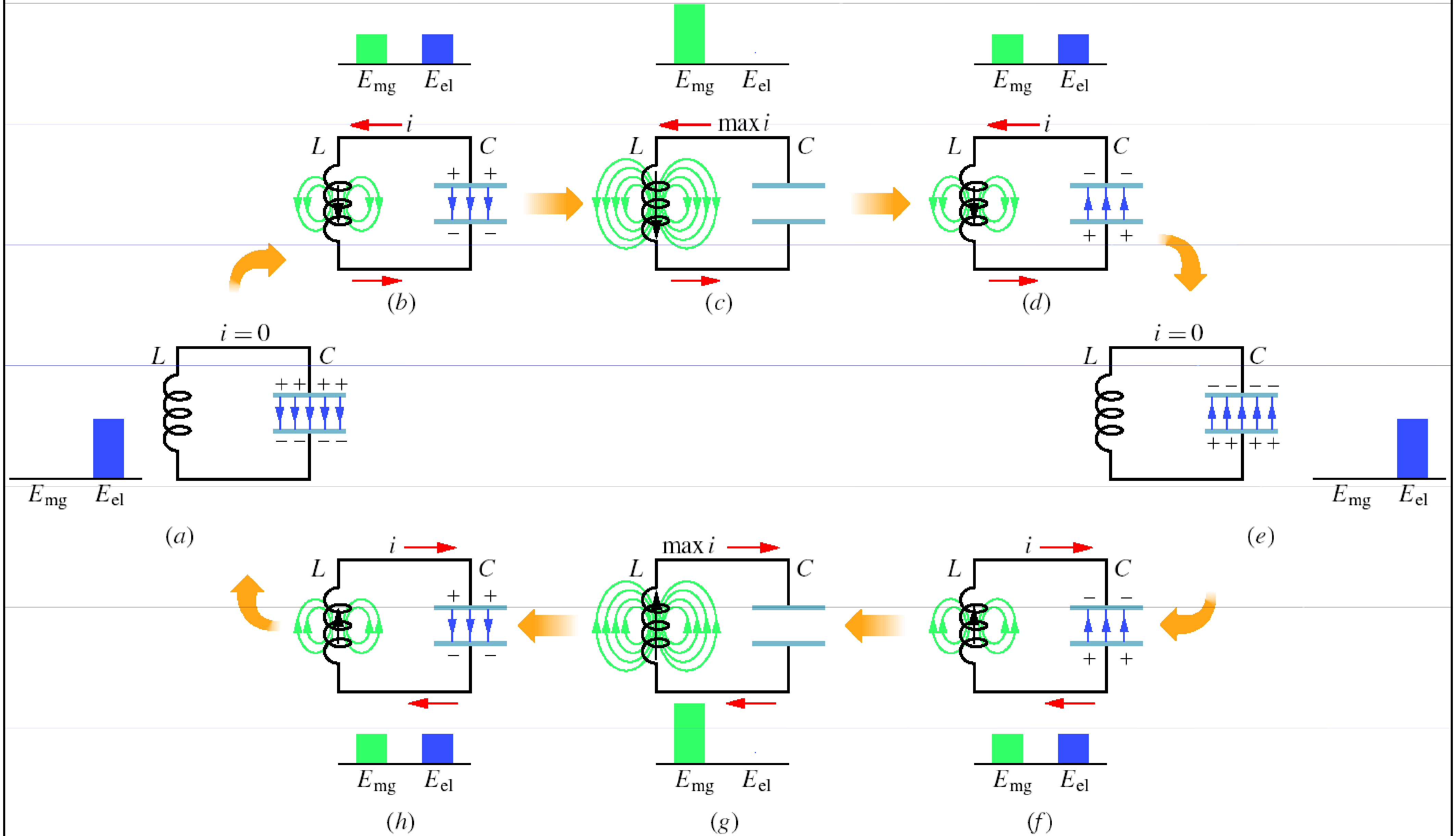
$$w_{mg} = \frac{B^2}{2\mu_0}$$

Obvod LC

$$E_{mg} = \frac{Li^2(t)}{2}$$



$$E_{el} = \frac{q^2(t)}{2C}$$



$$E = E_{mg} + E_{el} =$$

$$= \frac{Li^2}{2} + \frac{q^2}{2C}$$

$$\frac{dE}{dt} = 0 =$$

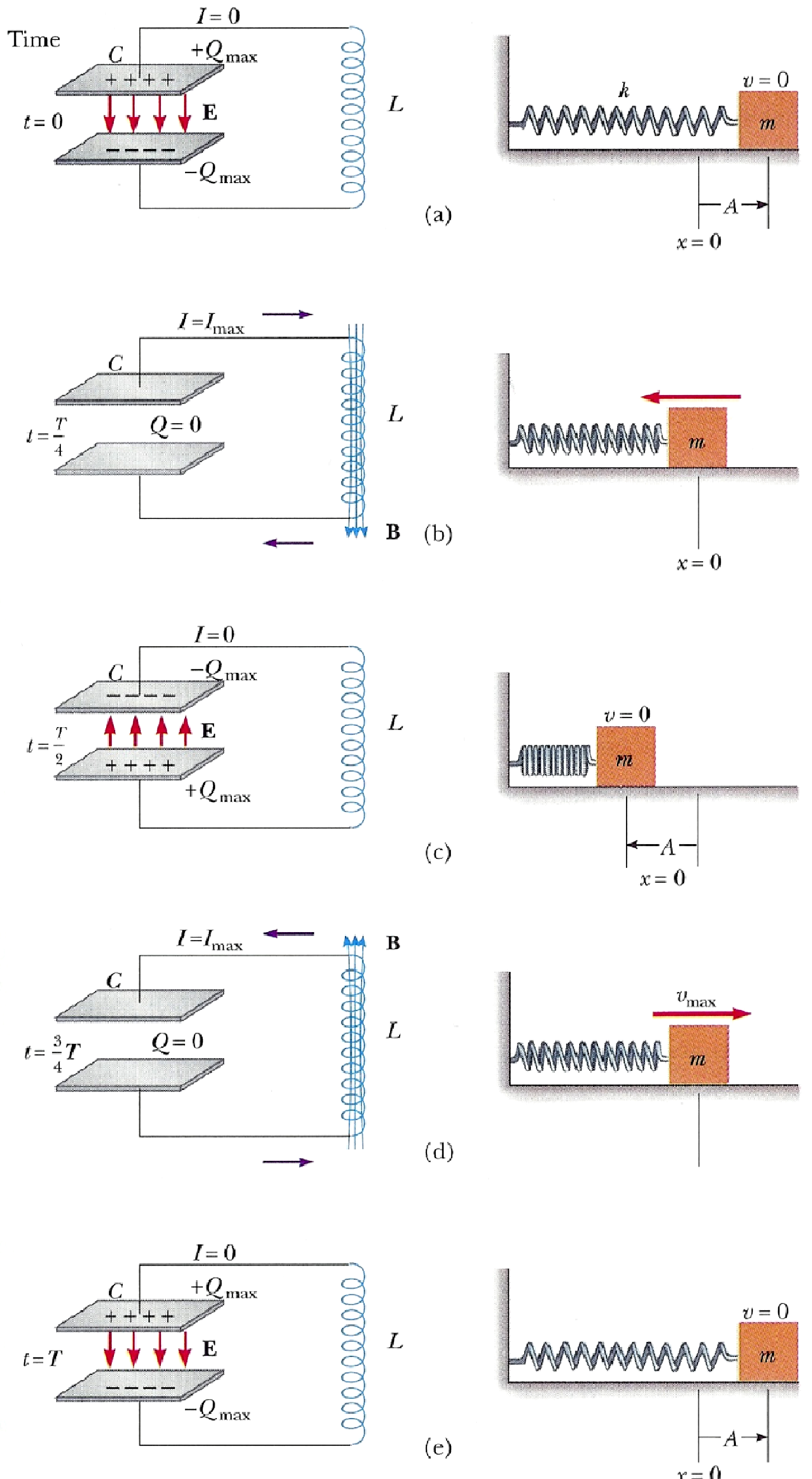
$$= Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt}$$

$\ddot{q}$ $\quad i$

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

ω^2

$$q(t) = Q\cos(\omega t + \varphi)$$



$$E = E_k + E_p =$$

$$= \frac{mv^2}{2} + \frac{kx^2}{2}$$

$$\frac{dE}{dt} = 0 =$$

$$= mv\frac{dv}{dt} + kx\frac{dx}{dt}$$

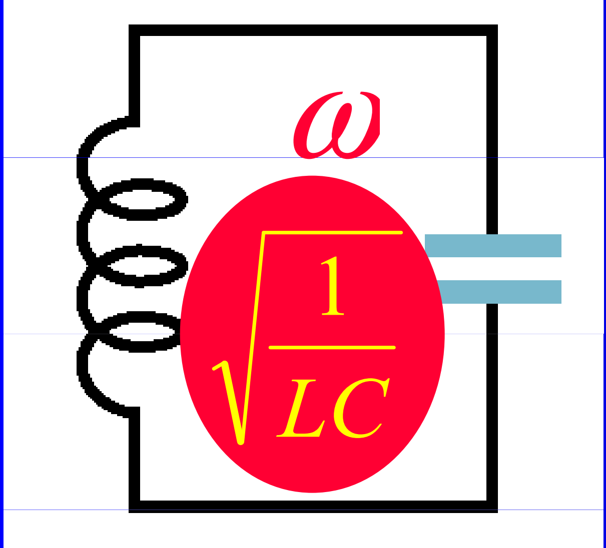
$\ddot{x}$ $\quad v$

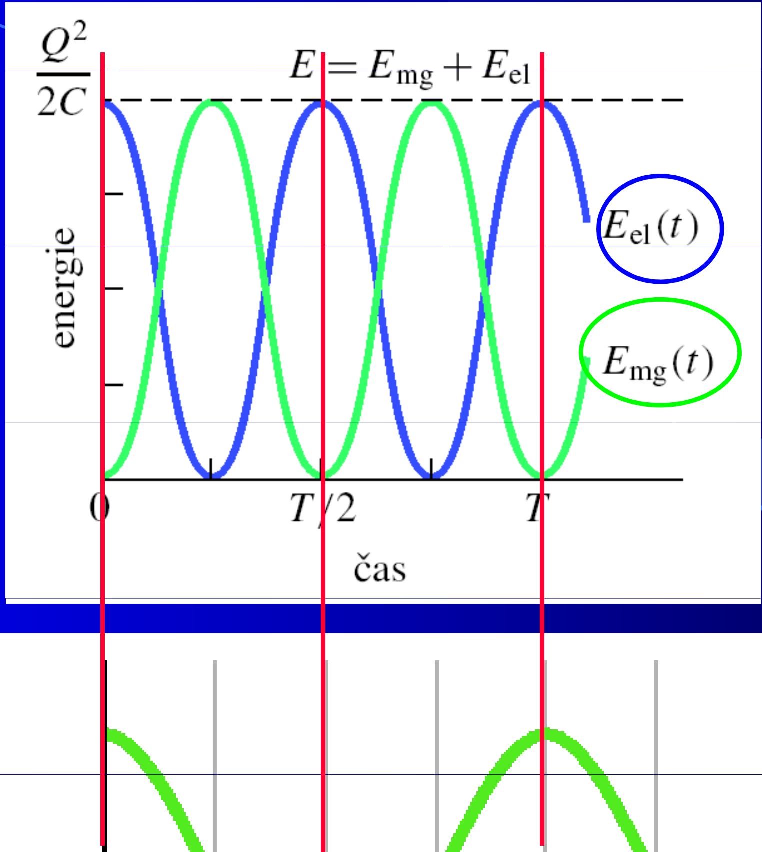
$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

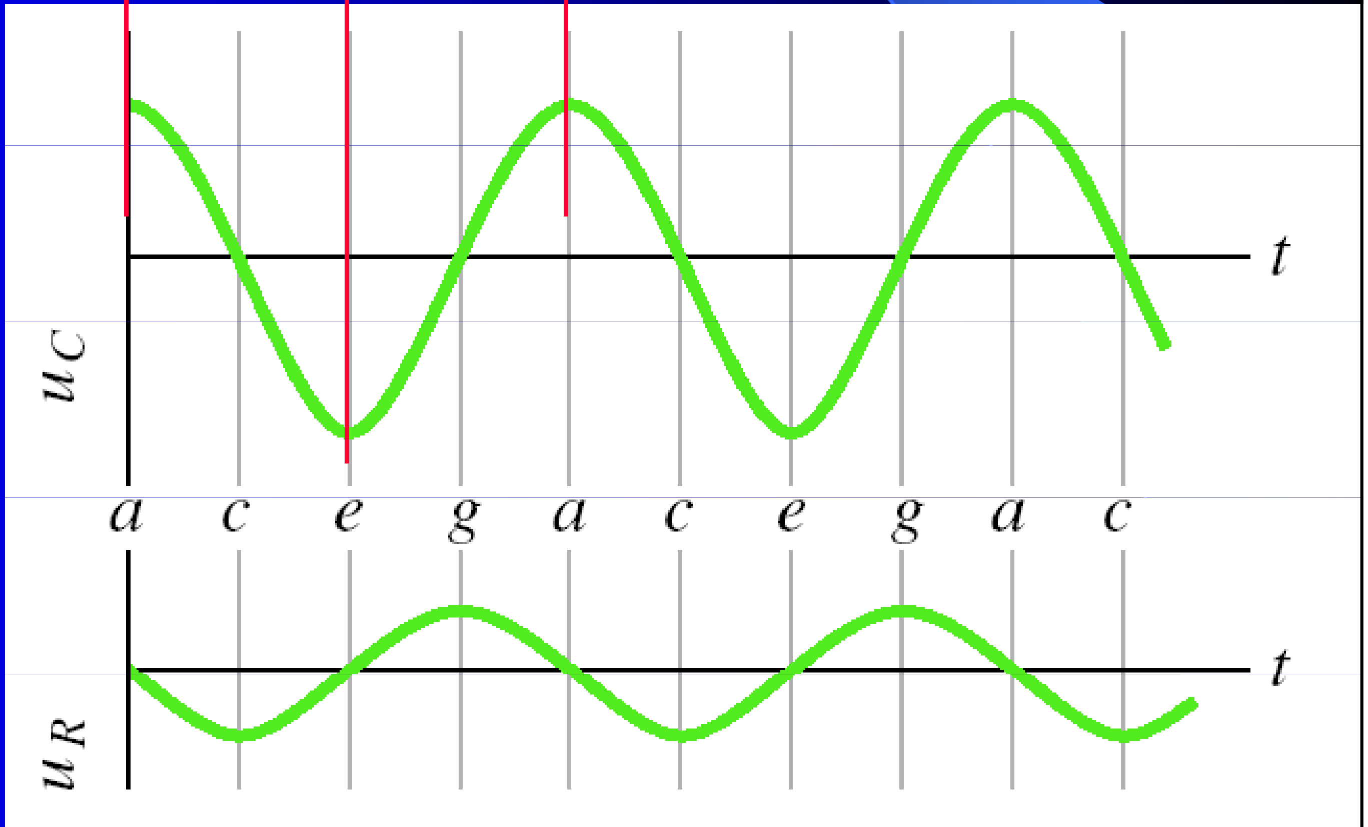
ω^2

$$x(t) = X\cos(\omega t + \varphi)$$

Oscilátor LC







$$q(t) = Q\cos\omega t$$

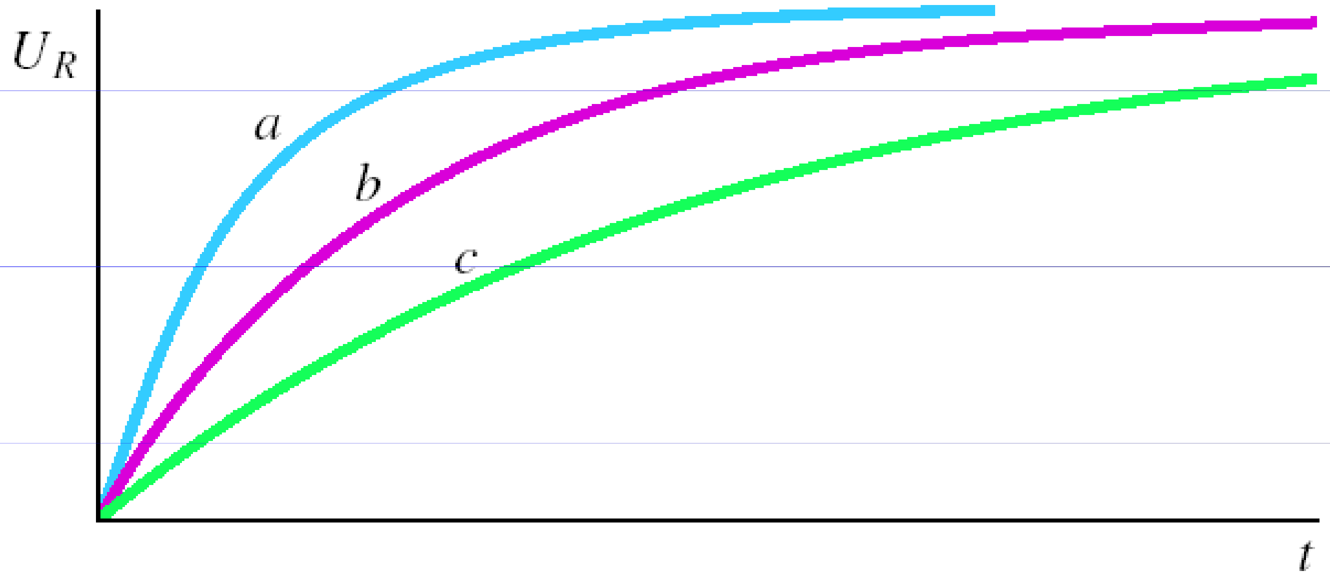
$$i(t) = -I\sin\omega t$$

OTÁZKY

11. V následující tabulce jsou dány počty závitů na jednotku délky, proudy a obsahy průřezů pro tři stejně dlouhé solenoidy. Seřaďte tyto solenoidy v sestupném pořadí podle (a) jejich indukčnosti, (b) magnetického toku jedním závitem.

Solenoid	Závity na jednotku délky	Proud	Obsah průřezu
1	$2n_1$	I_1	$2S_1$
2	n_1	$2I_1$	S_1
3	n_1	I_1	$4S_1$

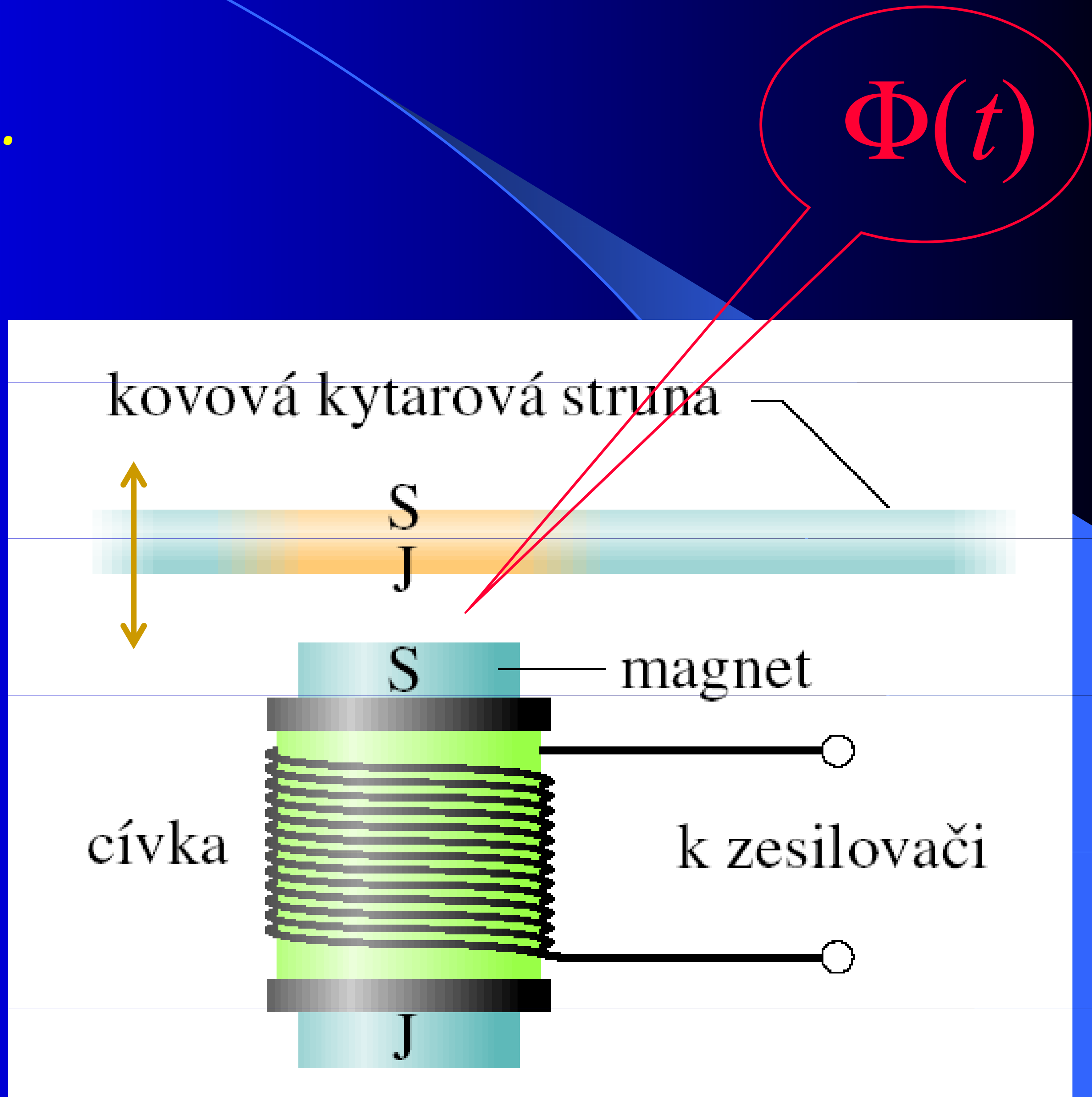
12. Obr. 31.35 ukazuje časový průběh napětí na rezistoru ve třech obvodech stejného typu jako na obr. 31.20. Obvody mají stejné odpory R a emn $\mathscr{E}$, ale liší se indukčností L. Seřaďte je sestupně podle velikosti L.



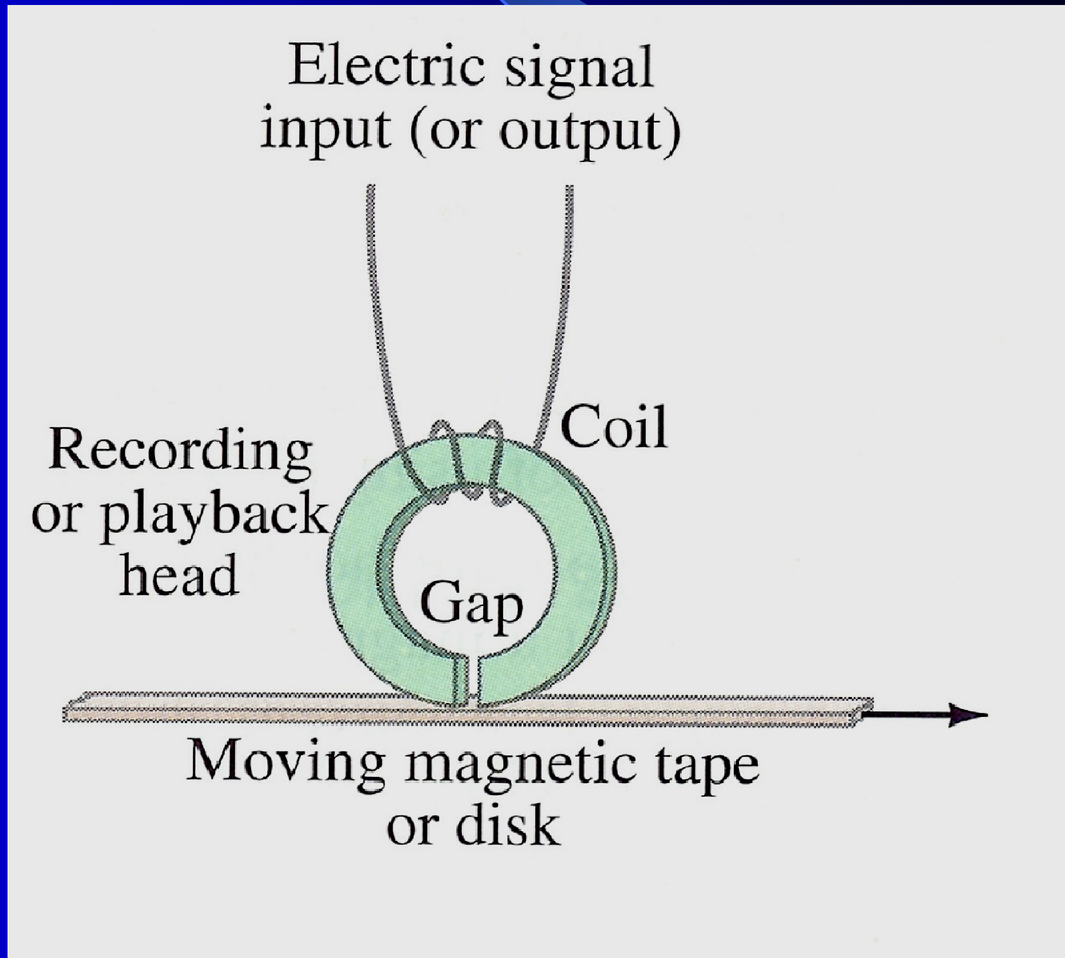
Obr. 31.35 Otázka 12

Aplikace

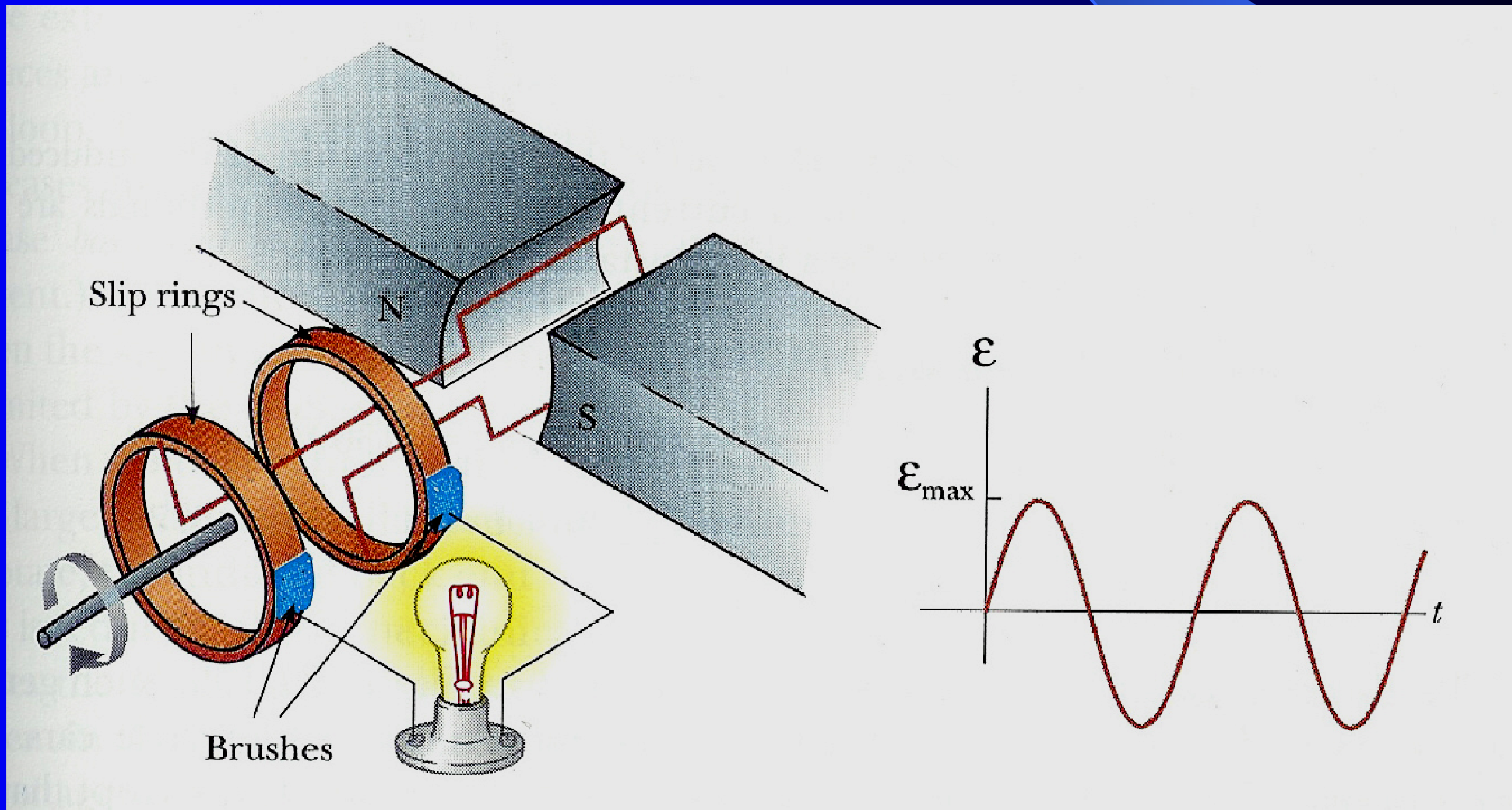
Čidla, telefon …



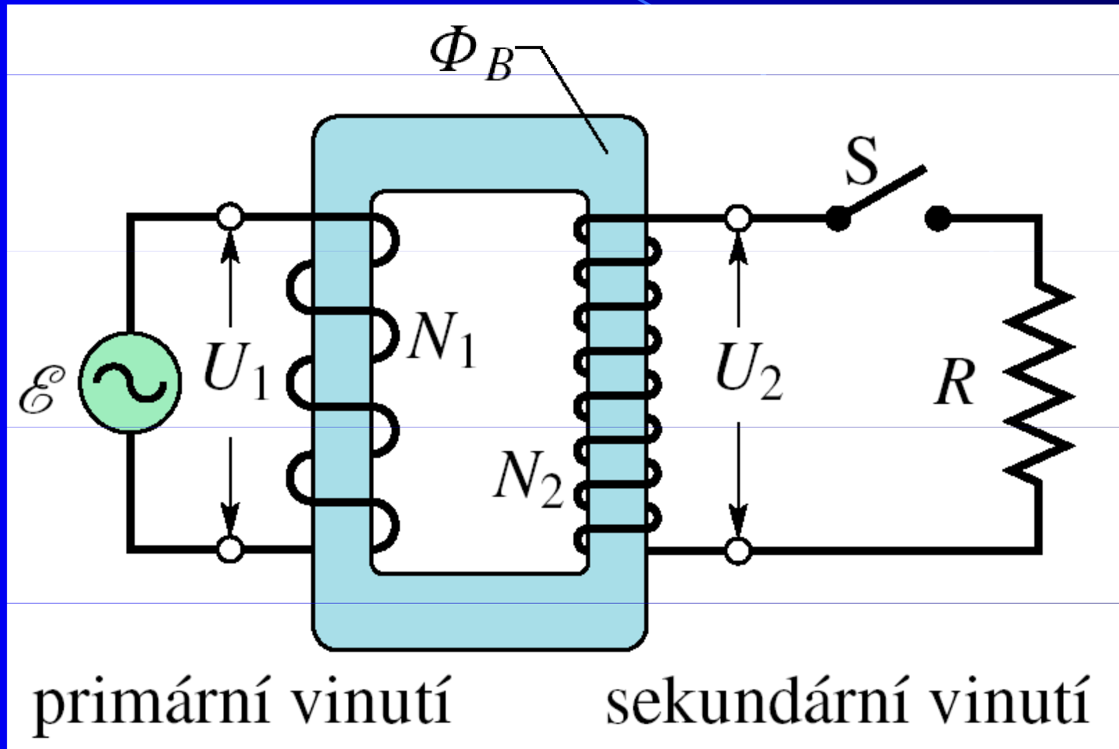
Záznam signálu na magnetické medium



Výroba elektrického proudu



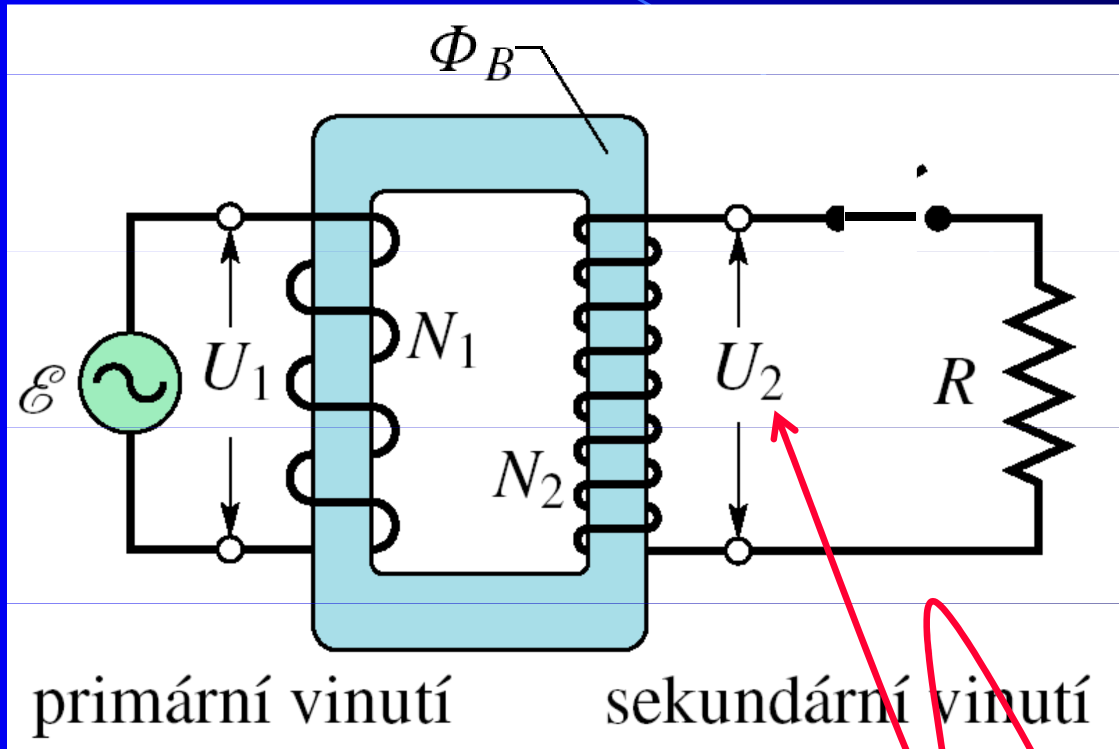
Transformátor



$$\mathcal{E}(t) - \frac{\mathrm{d}(N_1\Phi)}{\mathrm{d}t} = 0$$

$$U_2 = -\frac{\mathrm{d}(N_2\Phi)}{\mathrm{d}t}$$

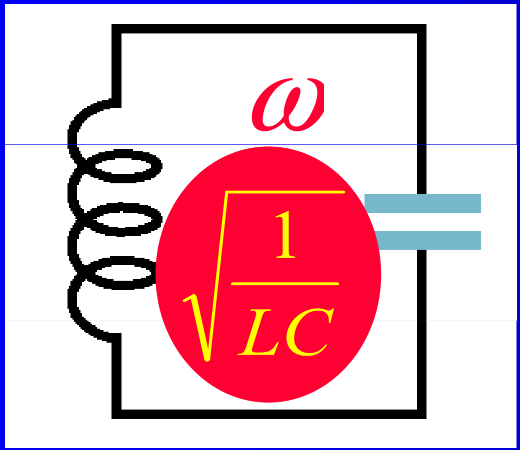
$$\boxed{\frac{U_2}{U_1} = \frac{N_2}{N_1}}$$

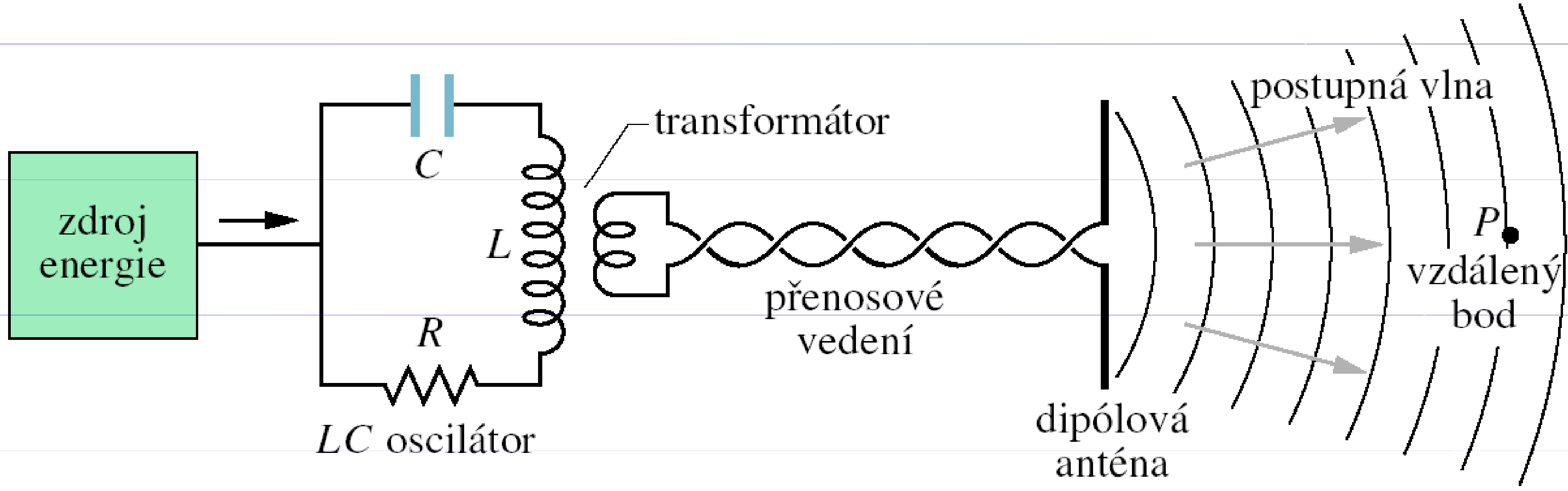


$$\mathcal{E}(t) - L_1 \dot{I}_1 - M \dot{I}_2 = R_1 I_1$$

$$-M \dot{I}_1 - L_2 \dot{I}_2 = R_2 I_2$$

Generátor oscilací





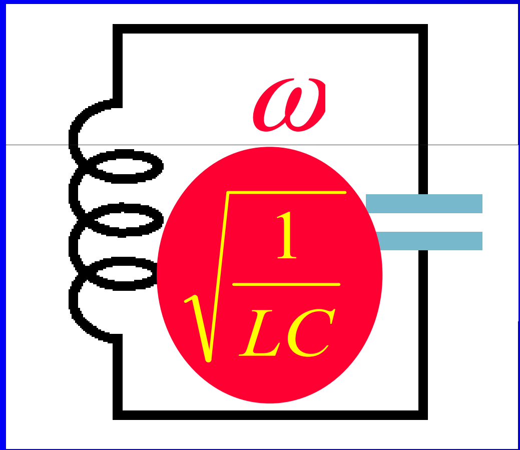
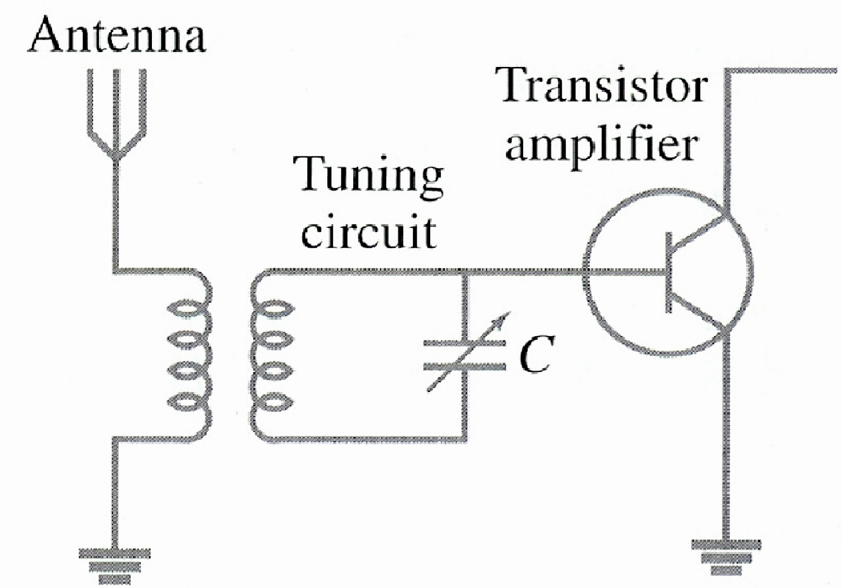
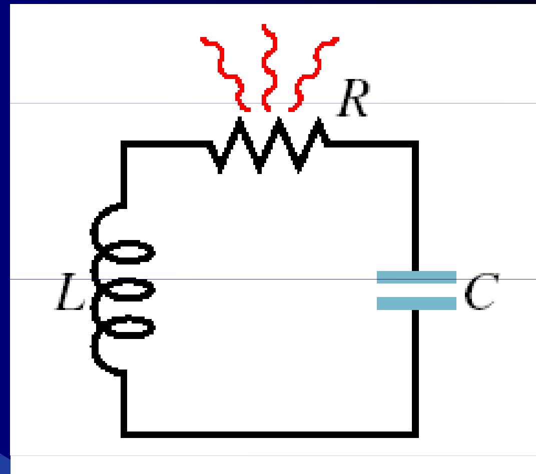


FIGURE 32–20 Simple tuning stage of a radio.



Tlumené kmity v obvodu RLC



energie v obvodu:

$$E = E_{\mathrm{mg}} + E_{\mathrm{el}} = \frac{Li^2}{2} + \frac{q^2}{2C}.$$

její úbytek:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -Ri^2$$

po úpravě:

$$L\frac{\mathrm{d}^2 q}{\mathrm{d}t^2} + R\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{1}{C}q = 0$$

řešení:

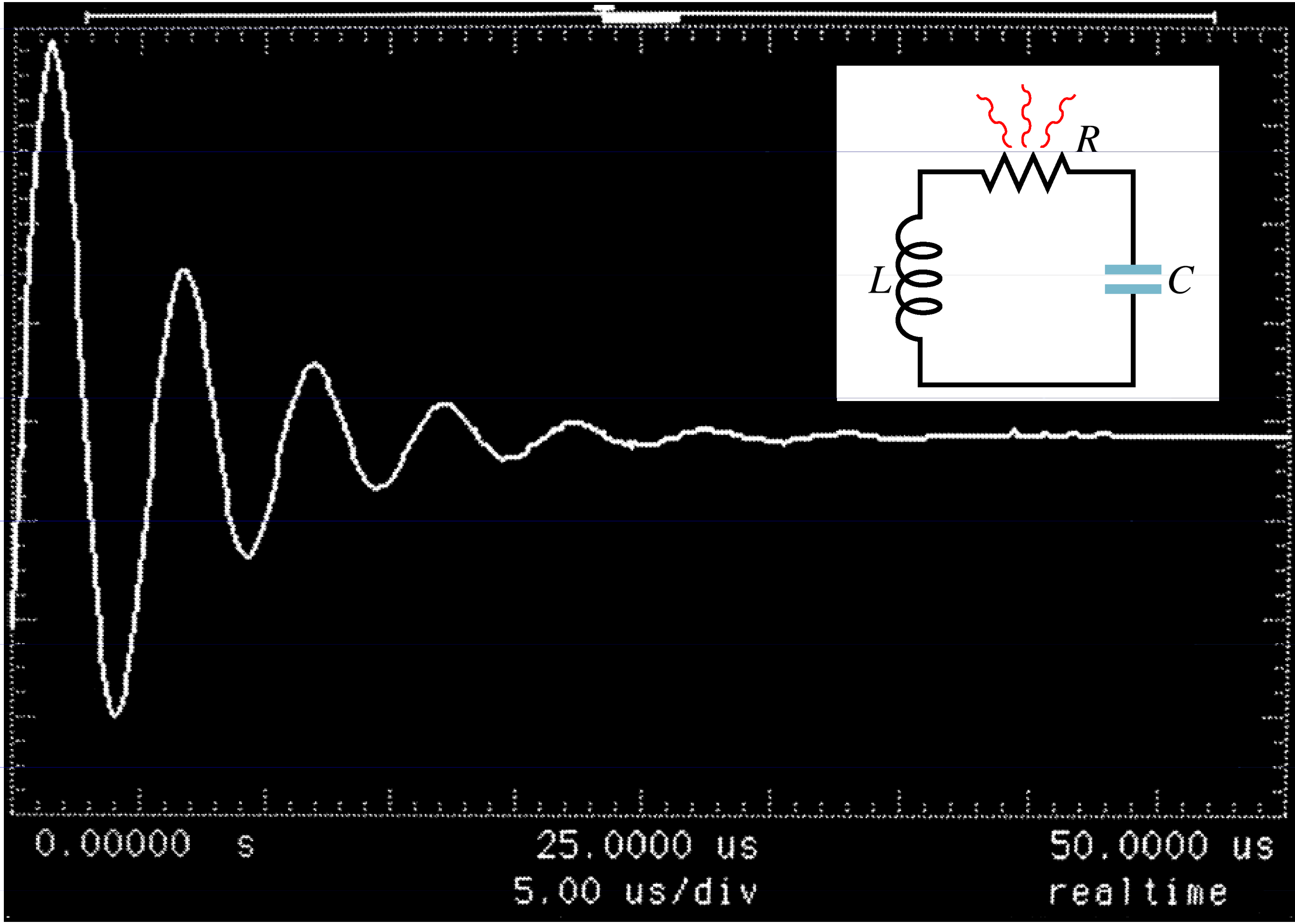
$$q = Q\mathrm{e}^{-Rt/(2L)}\cos(\omega' t + \varphi)$$

$$\omega' = \sqrt{\omega^2 - (R/2L)^2}$$

$$\omega = 1/\sqrt{LC}$$

střední hodnota energie v obvodu:

$$E = \frac{Q^2}{2C}\mathrm{e}^{-Rt/L}.$$

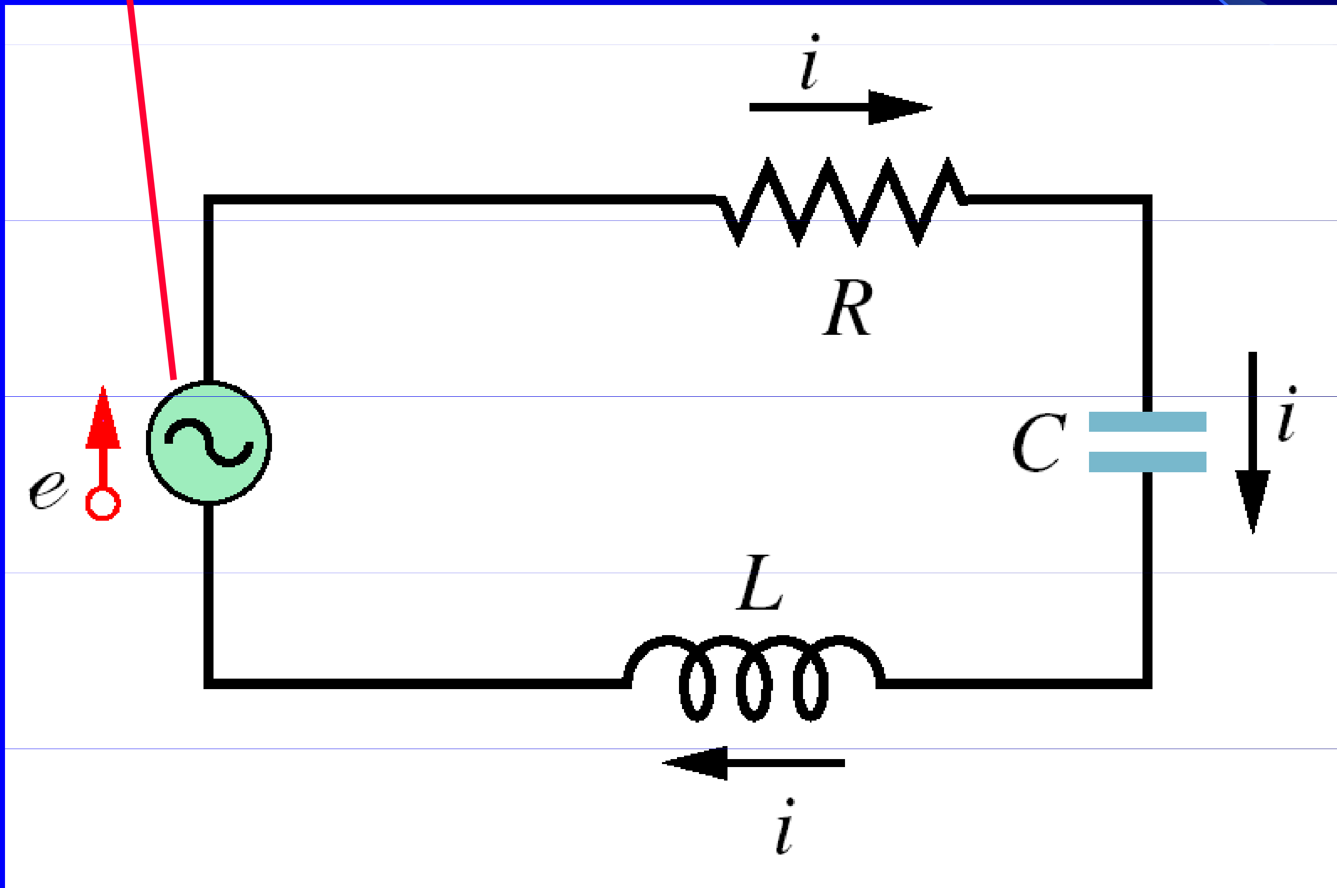


R
L
C
0.00000 s
25.0000 us
50.0000 us
5.00 us/div
realtime

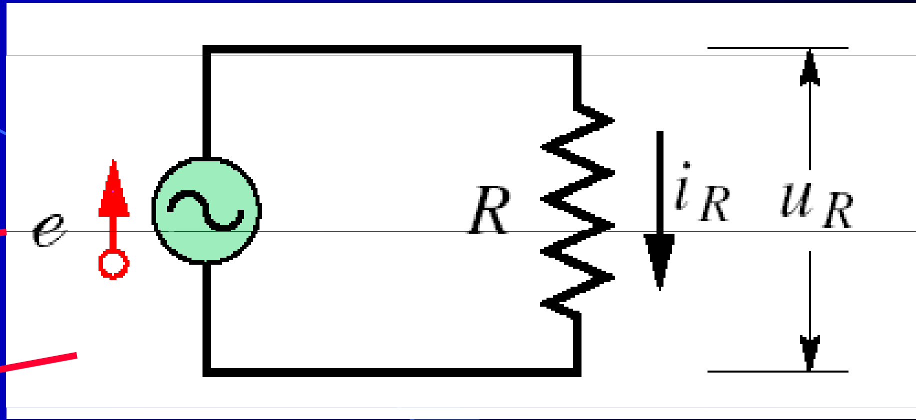
Nucené kmity v obvodech RLC

$$e = \mathscr{E} \sin \omega_b t .$$

$$i = I \sin (\omega_b t - \varphi),$$



Odporová zátěž



$$e = \mathscr{E} \sin \omega_b t.$$

$$e - u_R = 0.$$

$$u_R = \mathscr{E} \sin \omega_b t.$$

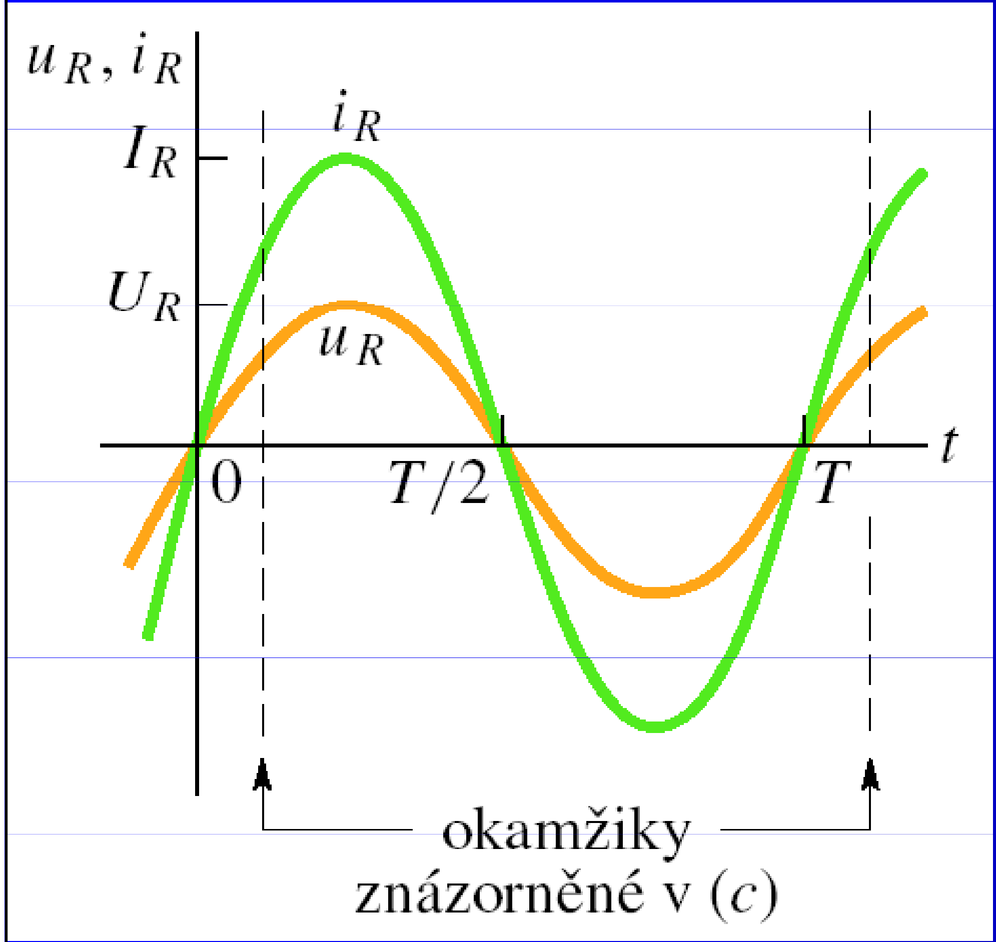
$$u_R = U_R \sin \omega_b t.$$

$$i_R = \frac{u_R}{R} = \frac{U_R}{R} \sin \omega_b t.$$

$$\varphi = 0^\circ$$

$$U_R = I_R R \qquad \text{(rezistor).}$$

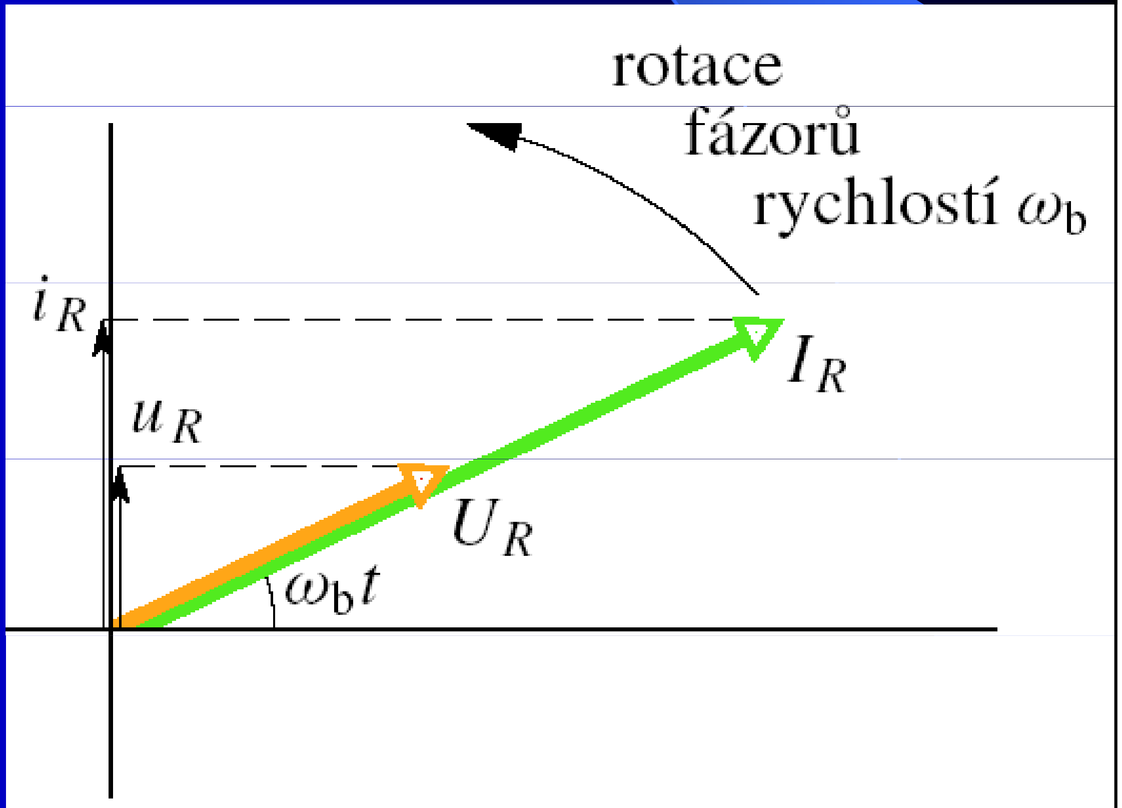
Odporová zátěž



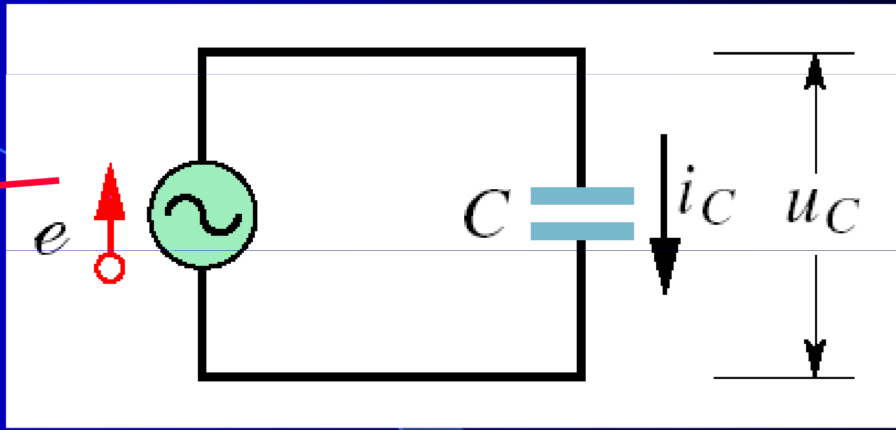
$$i_R = \frac{u_R}{R} = \frac{U_R}{R} \sin \omega_b t.$$

$$u_R = U_R \sin \omega_b t.$$

$$\varphi = 0^\circ$$



Kapacitní zátěž



$$u_C = U_C \sin \omega_b t$$

$$q_C = C u_C = C U_C \sin \omega_b t$$

$$i_C = \frac{\mathrm{d}q_C}{\mathrm{d}t} = \omega_b C U_C \cos \omega_b t$$

$$X_C = \frac{1}{\omega_b C} \quad \text{(kapacitní reaktance).}$$

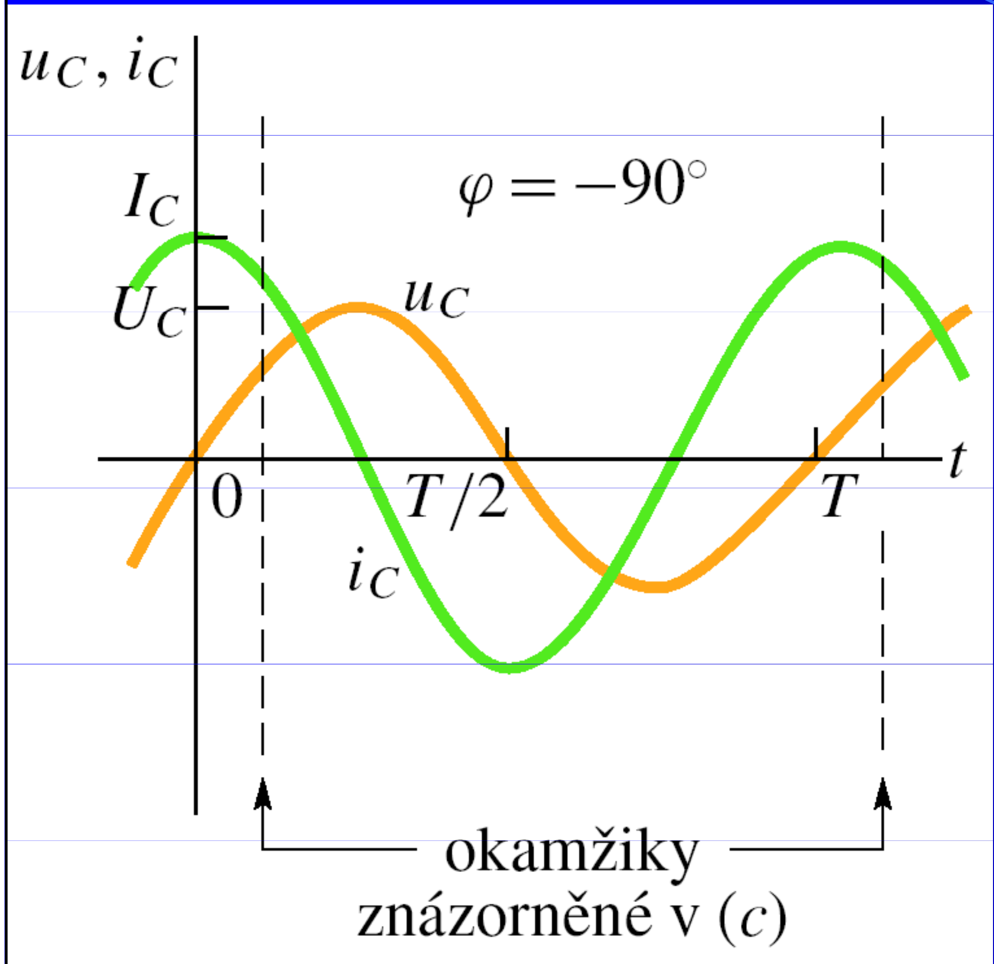
$$\cos \omega_b t = \sin(\omega_b t + 90^\circ)$$

$$i_C = \left(\frac{U_C}{X_C}\right) \sin(\omega_b t + 90^\circ)$$

$$U_C = I_C X_C$$

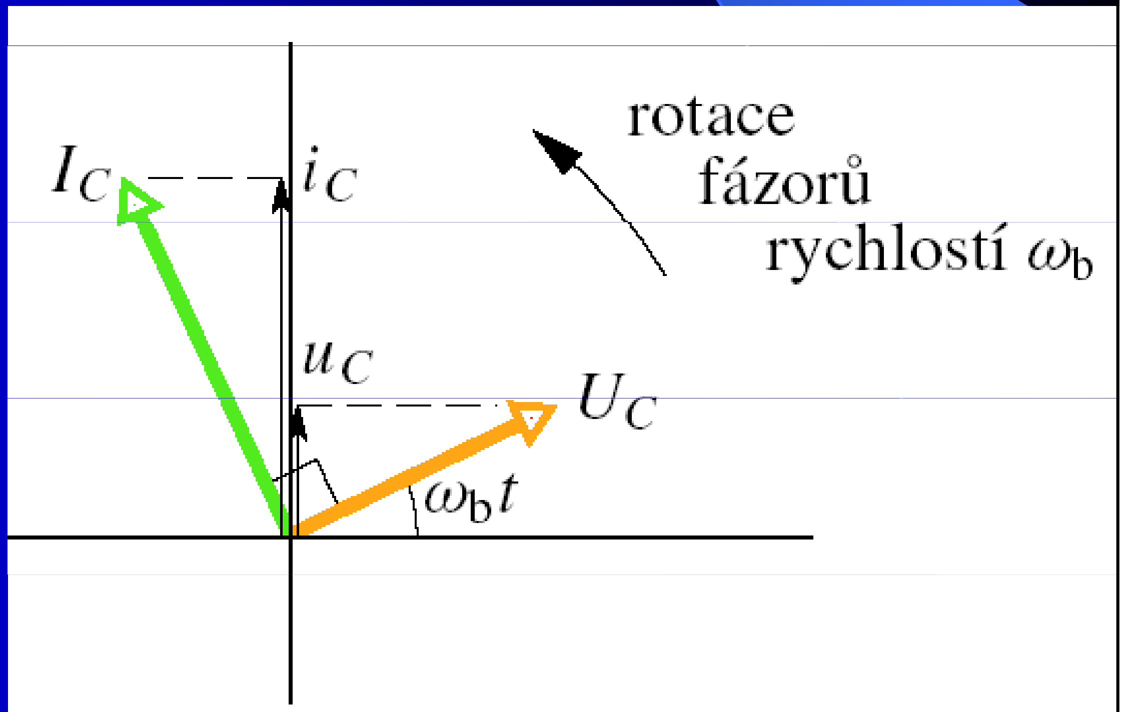
$$\varphi = -90^\circ$$

Kapacitní zátěž



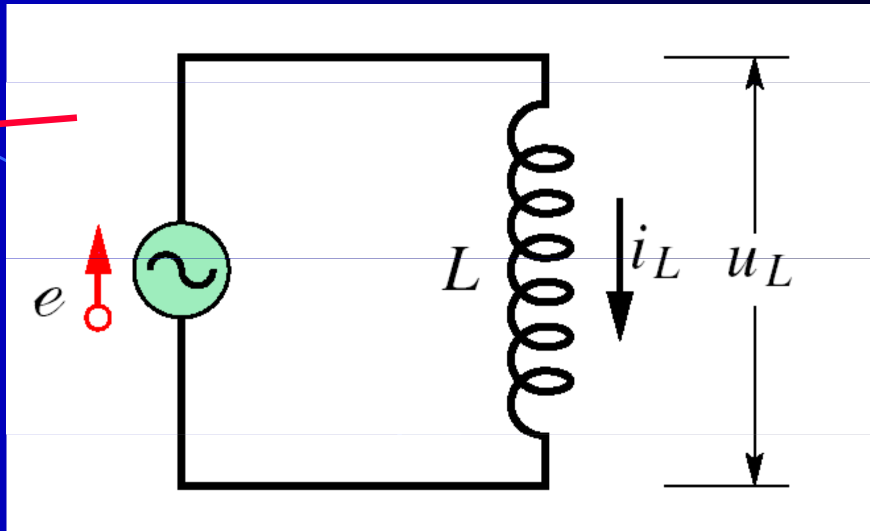
$$u_C = U_C \sin \omega_b t$$

$$i_C = \left(\frac{U_C}{X_C}\right) \sin(\omega_b t + 90^\circ)$$



$$\varphi = -90^\circ$$

Induktivní zátěž



$$u_L = U_L \sin \omega_b t$$

$$u_L = L \frac{di_L}{dt}.$$

$$\frac{di_L}{dt} = \frac{U_L}{L} \sin \omega_b t.$$

$$i_L = -\left(\frac{U_L}{\omega_b L}\right) \cos \omega_b t$$

$$X_L = \omega_b L \qquad \text{(induktivní reaktance).}$$

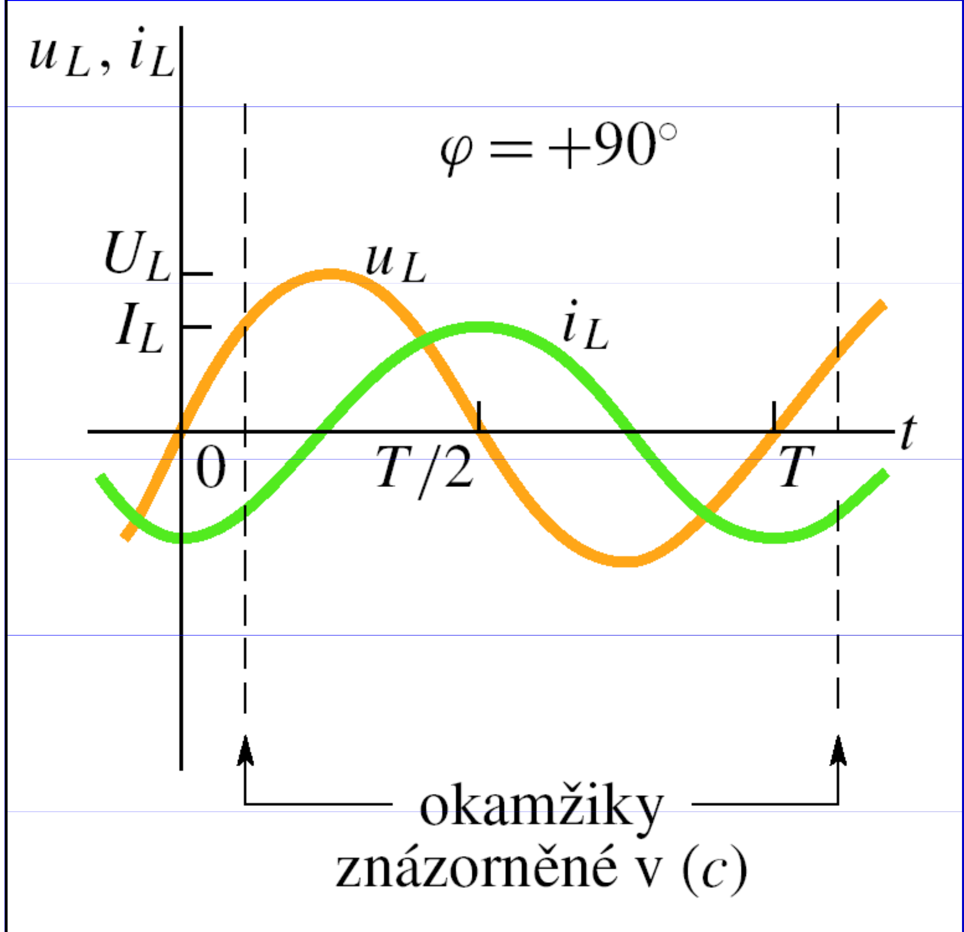
$$-\cos \omega_b t = \sin(\omega_b t - 90^\circ).$$

$$i_L = \frac{U_L}{X_L} \sin(\omega_b t - 90^\circ).$$

$$U_L = I_L X_L$$

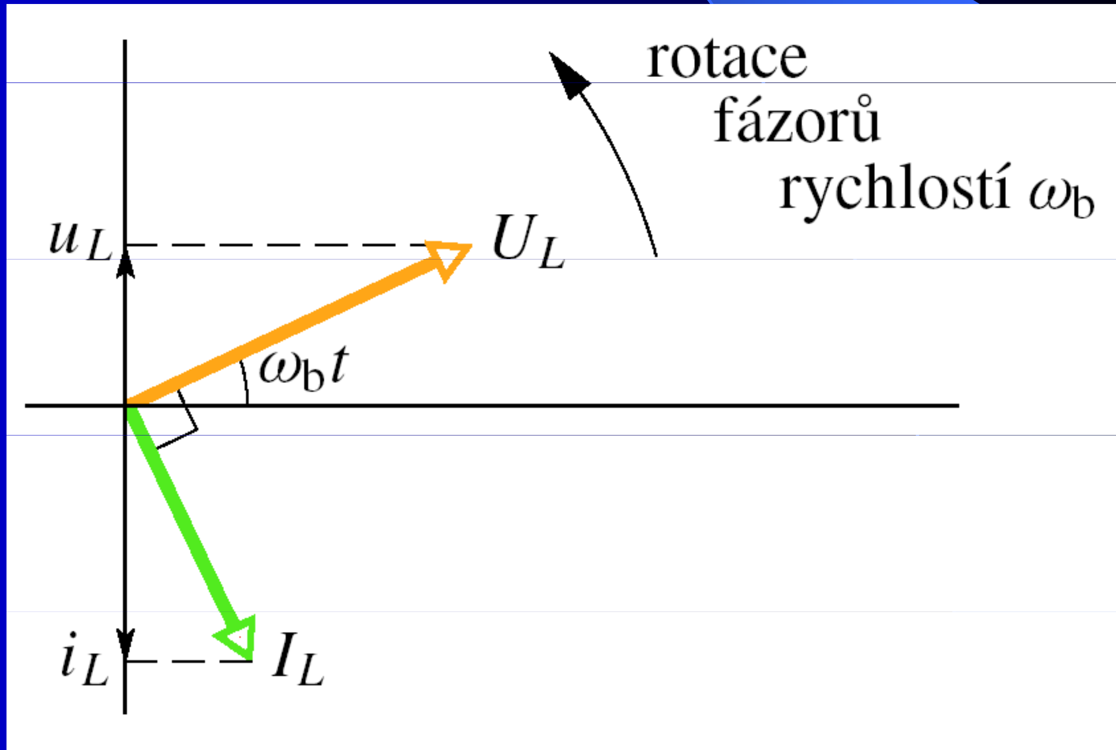
$$\varphi = +90^\circ$$

Induktivní zátěž



$$u_L = U_L \sin \omega_b t$$

$$i_L = \frac{U_L}{X_L} \sin(\omega_b t - 90^\circ).$$



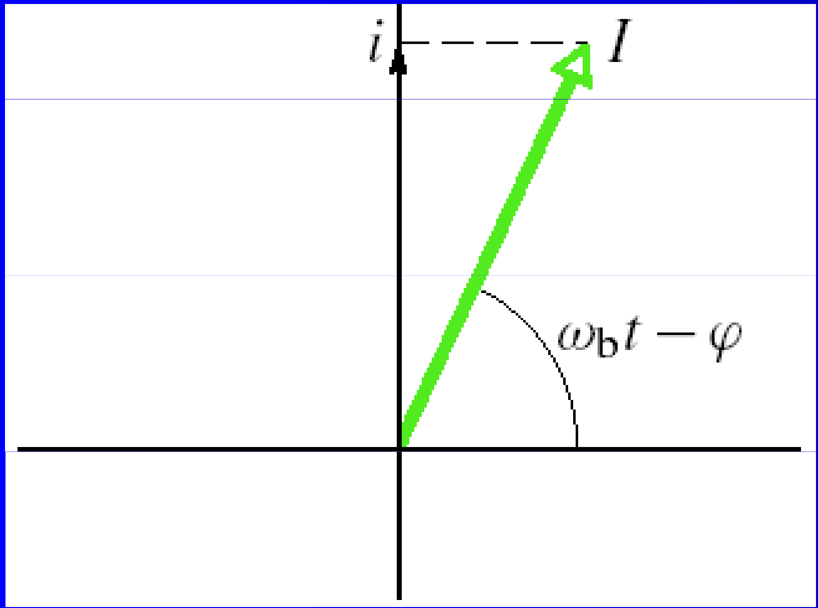
$$\varphi = +90^\circ$$

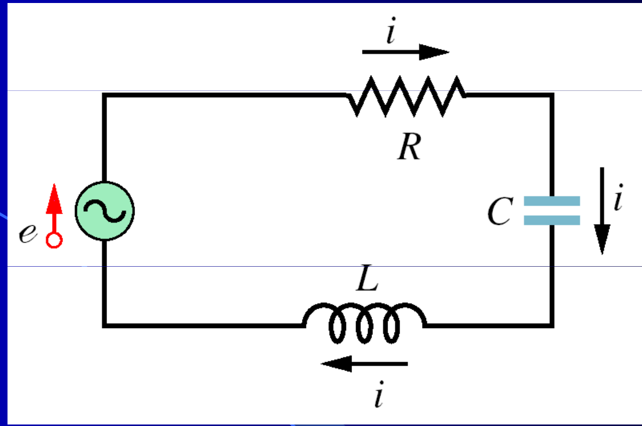
Tabulka 33.2 Vztahy mezi amplitudou a fází pro střídavé proudy a napětí

OBVODOVÝ PRVEK	SYMBOL	REZISTANCE NEBO REAKTANCE*	FÁZE PROUDU	FÁZOVÝ POSUN φ	VZTAH MEZI AMPLITUDAMI
Rezistor	R	R	ve fázi s u_R	0°	$U_R = I_R R$
Kondenzátor	C	$X_C = 1/(\omega_b C)$	předbíhá u_C o 90°	-90°	$U_C = I_C X_C$
Cívka	L	$X_L = \omega_b L$	zpožděna za u_L o 90°	$+90^\circ$	$U_L = I_L X_L$

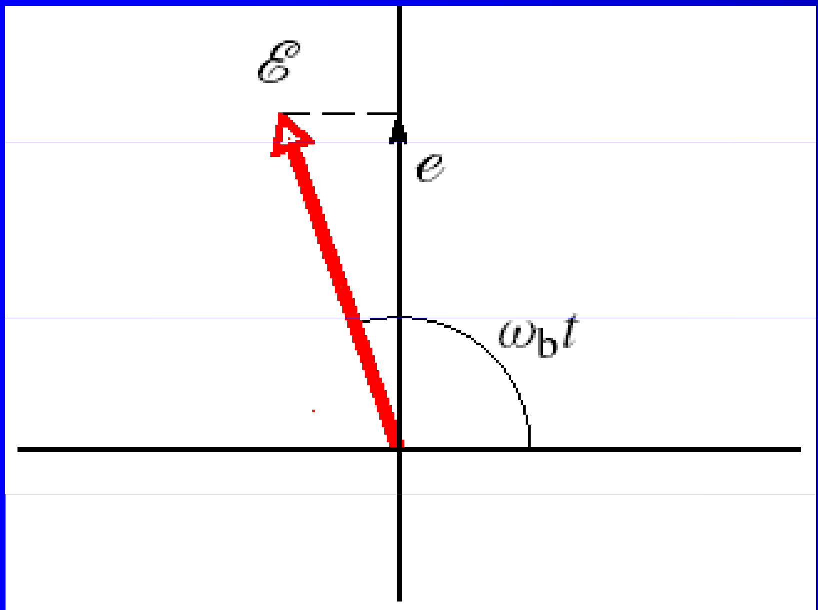
* Někdy se pro kapacitní reaktanci užívá název **kapacitance** a pro induktivní reaktanci název **induktance**.

Sériový obvod RLC

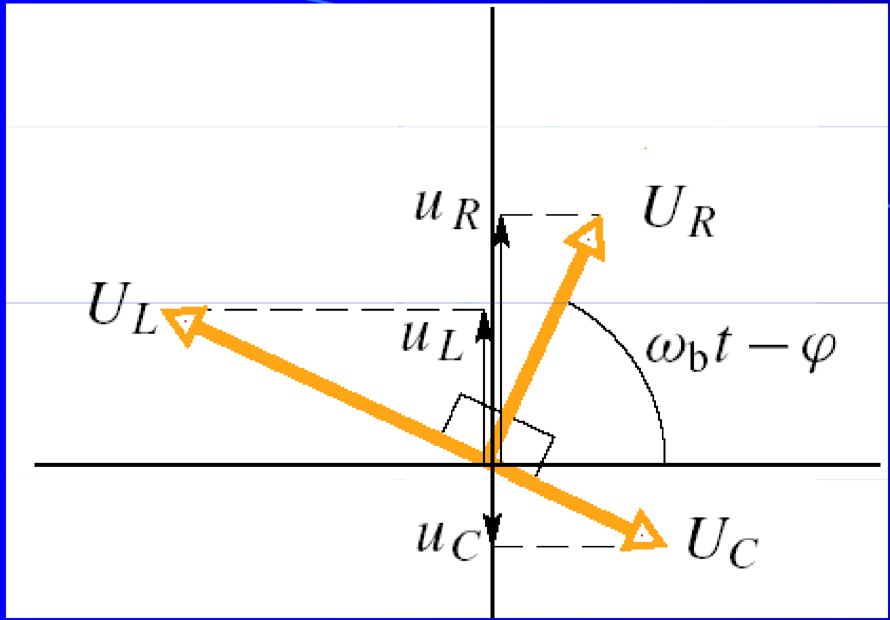




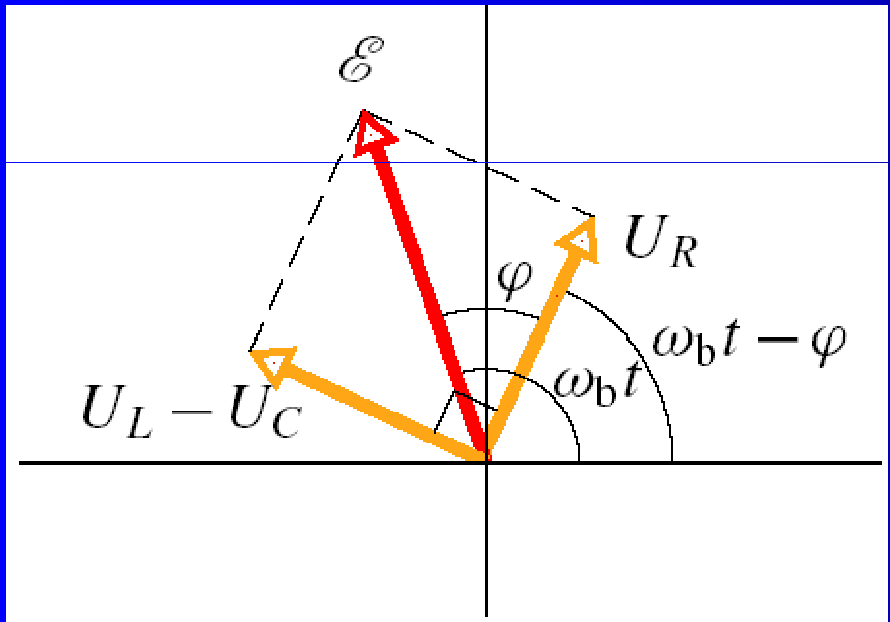
$$i = I \sin(\omega_b t - \varphi)$$



$$e = \mathcal{E} \sin \omega_b t$$



$$e = u_R + u_C + u_L .$$



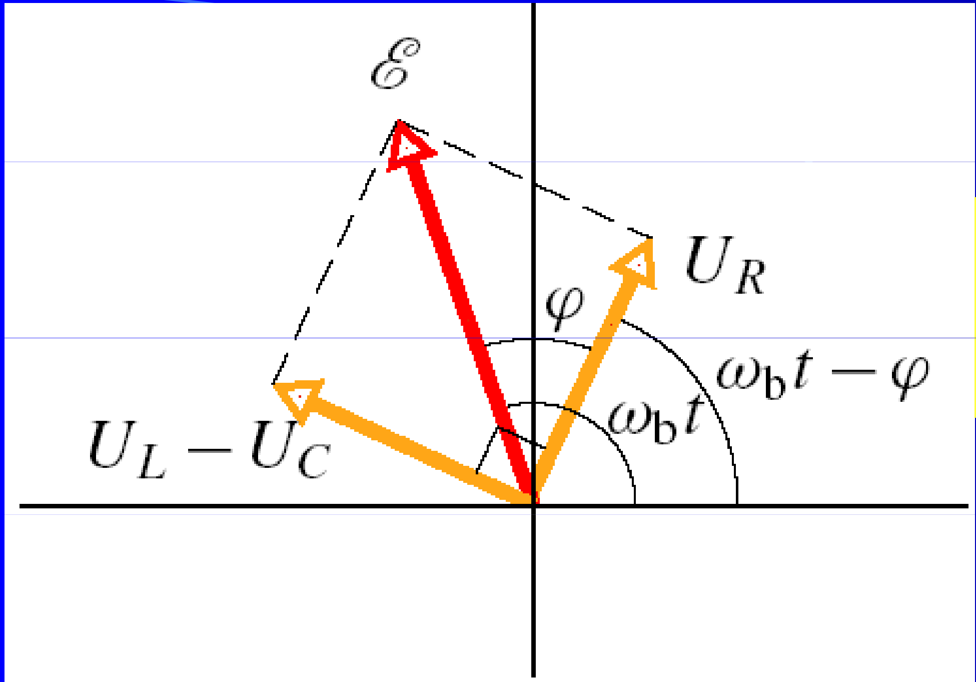
$$\mathscr{E}^2 = U_R^2 + (U_L - U_C)^2 .$$

$$\mathscr{E}^2 = (IR)^2 + (IX_L - IX_C)^2$$

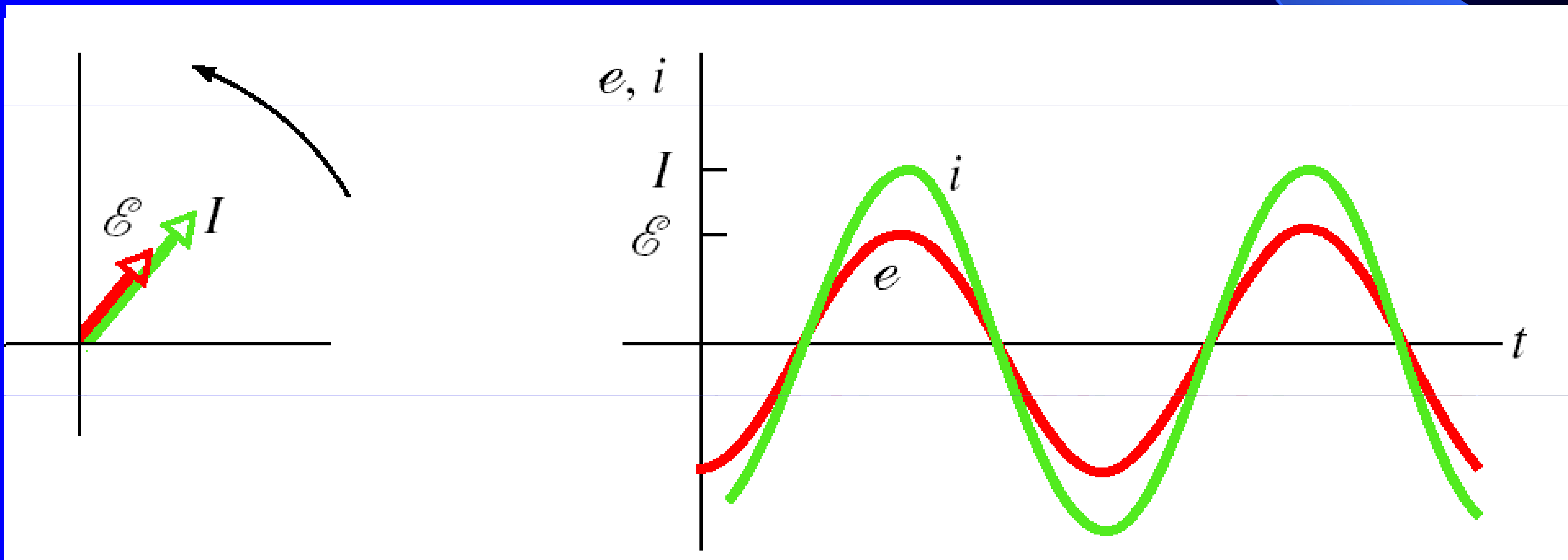
amplituda proudu:

$$I = \frac{\mathscr{E}}{\sqrt{R^2 + (X_L - X_C)^2}} .$$

impedance



$$\operatorname{tg}\varphi = \frac{X_L - X_C}{R} \qquad \text{(fázový posun).}$$



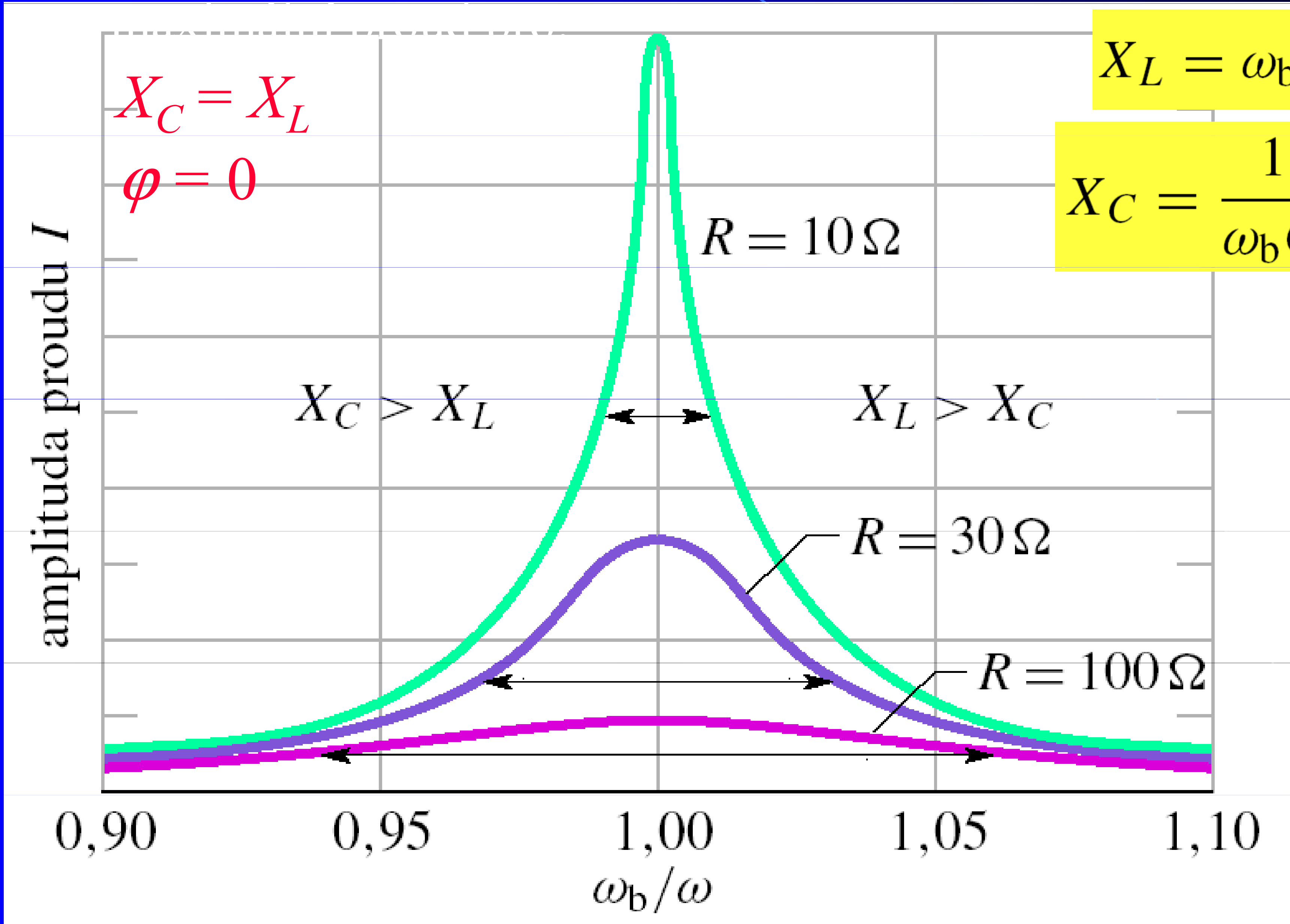
nulová φ

Rezonance

$$\omega_b = \omega = \frac{1}{\sqrt{LC}} \qquad \text{(rezonanční frekvence).}$$

$$X_L = \omega_b L$$

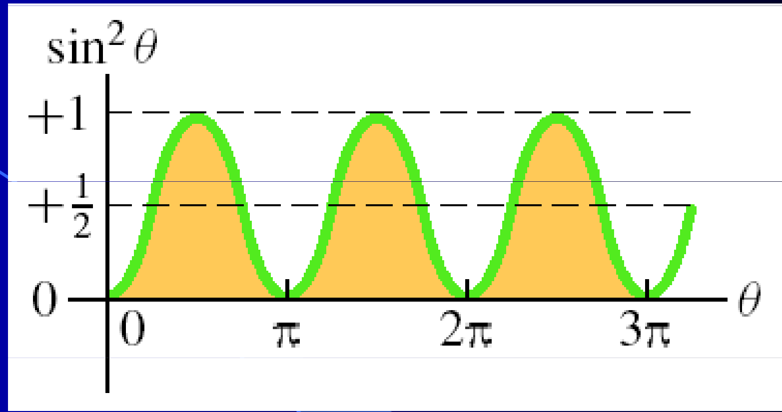
$$X_C = \frac{1}{\omega_b C}$$



Výkon v obvodech se střídavým proudem

okamžitý výkon v rezistoru:

$$P = i^2 R = I^2 R \sin^2(\omega_b t - \varphi).$$



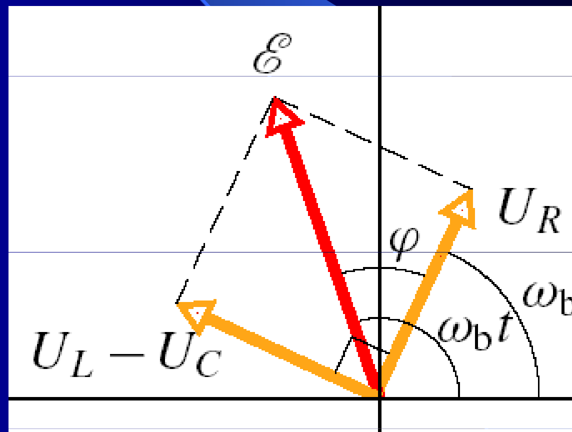
střední výkon:

$$\overline{P} = \frac{I^2 R}{2} = \left(\frac{I}{\sqrt{2}}\right)^2 R.$$

I_{ef}

$$U_{ef} = \frac{U}{\sqrt{2}} \quad \text{a} \quad \mathscr{E}_{ef} = \frac{\mathscr{E}}{\sqrt{2}}$$

(efektivní napětí a emn)



$$\cos\varphi = \frac{U_R}{\mathscr{E}} = \frac{IR}{IZ} = \frac{R}{Z}.$$

$$\overline{P} = \frac{\mathscr{E}_{ef}}{Z} I_{ef} R = \mathscr{E}_{ef} I_{ef} \frac{R}{Z} = \mathscr{E}_{ef} I_{ef} \cos\varphi$$