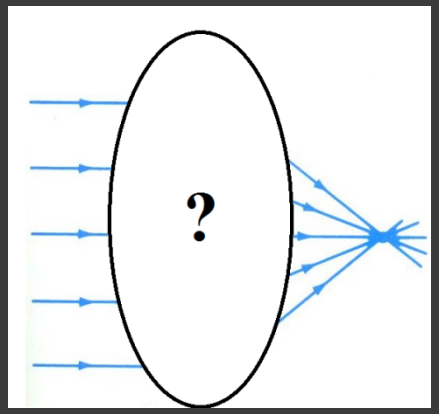
Paraxiálna aproximácia pre elektrostatickú šošovku

ELEKTRÓNOVÁ OPTIKA

Miroslav Jagelka

Elektrostatická šošovka



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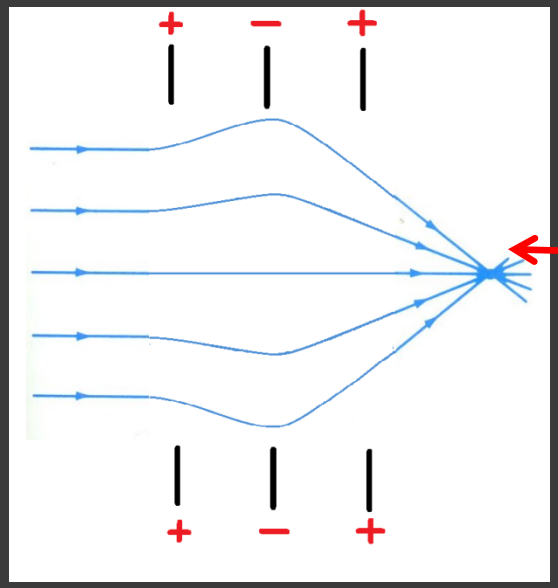

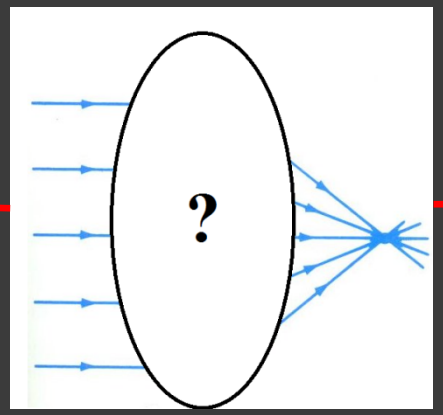

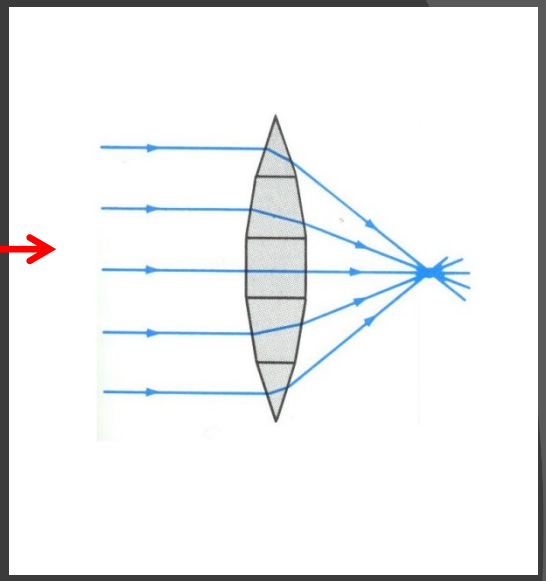

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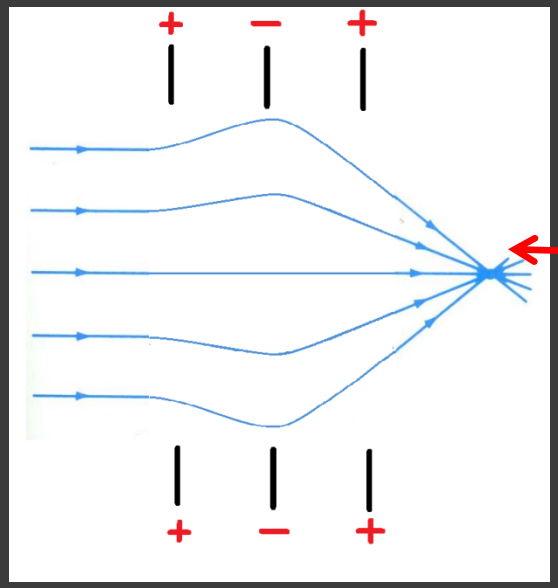
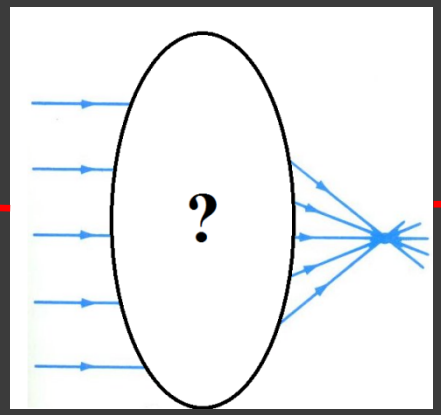
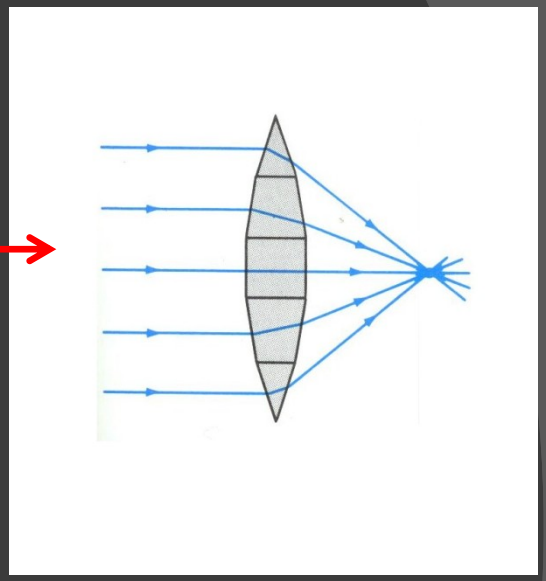






- spojitý index lomu
- Laplaceova rovnica/Poissonova rovnica

$$\Delta\varphi = \frac{\rho}{\varepsilon_0}$$

- sústava katód a anód, prípadne zložitejší mechanizmus
- elektróny

$$\lambda = \frac{hc}{E_k}\frac{1}{\sqrt{1+\frac{2m_e}{E_k}}}$$

- diskrétny index lomu - mení sa na rozhraní
- Snellov zákon/zákon mimimálneho času

$$\sin\theta_1 n_1 = \sin\theta_2 n_2$$

- spojky alebo rozptylky z rôznych homogénnych materiálov
- fotóny

$$\lambda = \frac{hc}{E}$$

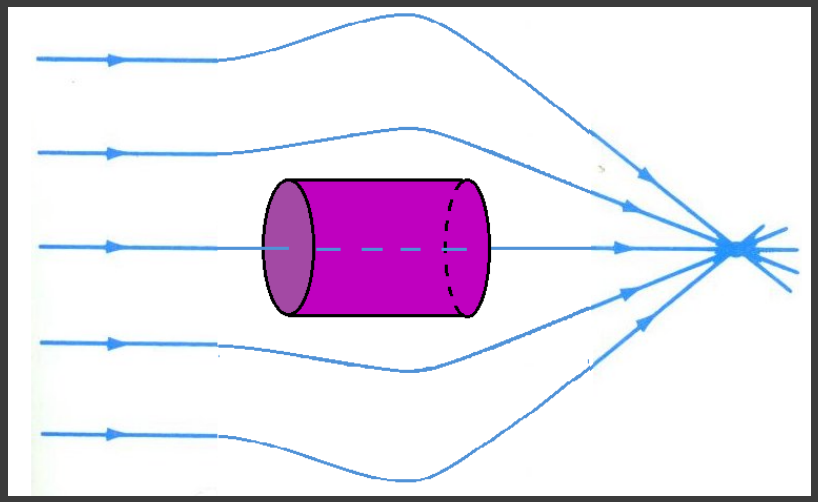
Paraxiálna oblasť

$$\rho = 0$$

$$\Delta\varphi = \frac{\rho}{\varepsilon_0}$$

$$\Delta\varphi = \nabla^2\varphi$$

$$\nabla^2\varphi = div(grad\varphi)$$



$$\oint_S \vec{E}.d\vec{S} = \int_V div\vec{E}.d\vec{V}$$

$$\vec{E} = -grad\varphi$$

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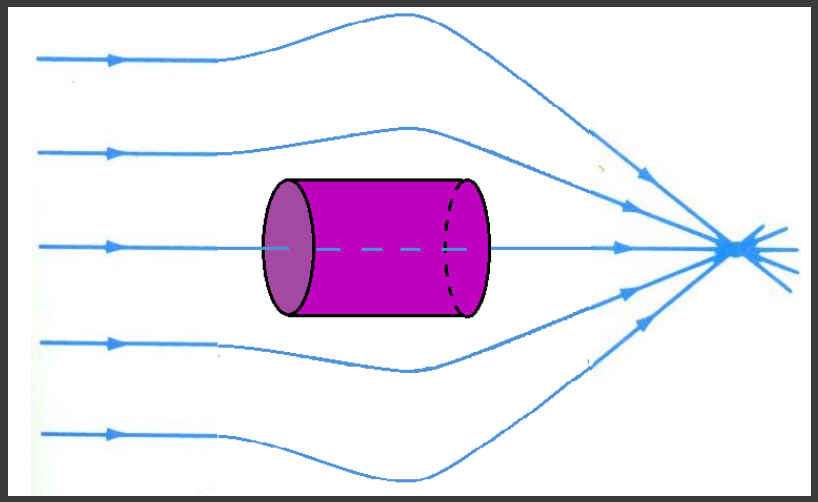

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Gaussova rovnica

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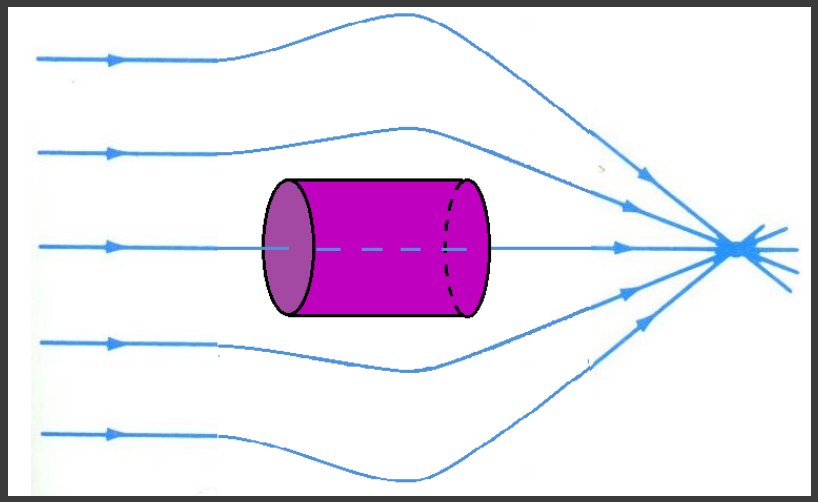

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Gaussova rovnica

$$\oint_S \vec{E}.d\vec{S} = E_r.2\pi r dl - E_z.\pi r^2 + \left(E_z + \frac{dE_z}{dz}dl\right).\pi r^2 = 0$$

$$\boxed{E_r = -\frac{r}{2}\frac{dE_z}{dz}}$$

Paraxiálna paprsková rovnica

$$\ddot{r} = \frac{d}{dt}\left(\frac{dr}{dt}\right) = \frac{dz}{dt}\frac{d}{dz}\left(\frac{dz}{dt}\frac{dr}{dz}\right) = v_z\frac{d}{dz}\left(v_z\frac{dr}{dz}\right)$$

$$ma(r,z) = eE(r,z)$$

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[redacted]

Redukovaná paprsková rovnica

Substitúcia parametrom $R = r\varphi^{\frac{1}{4}}$

- po aplikovaní na paraxiálnu paprskovú rovnicu dostaneme tvar:

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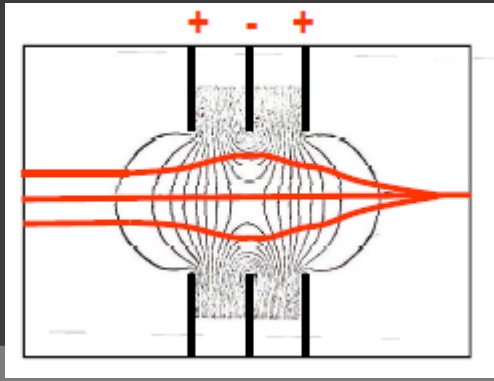
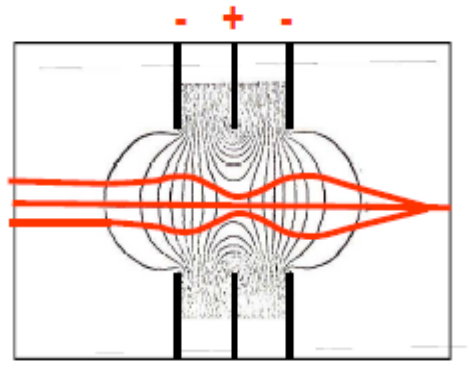
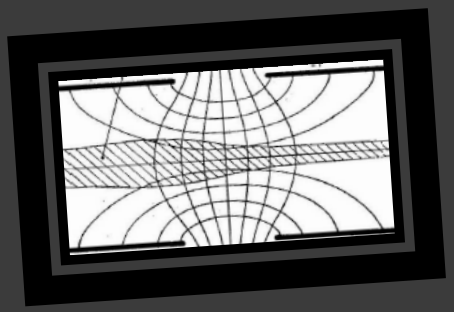
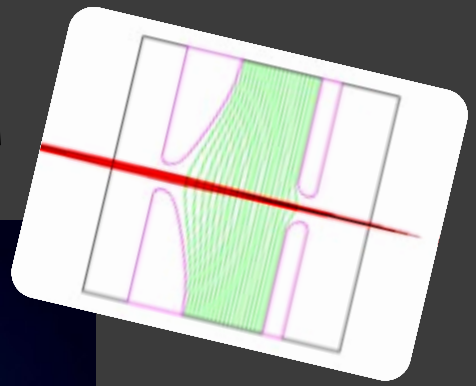
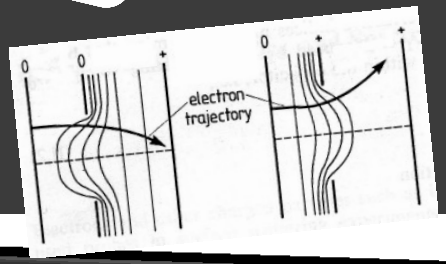
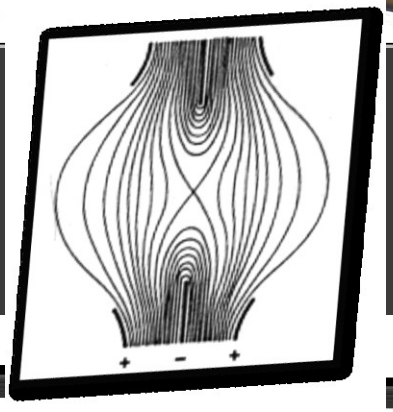
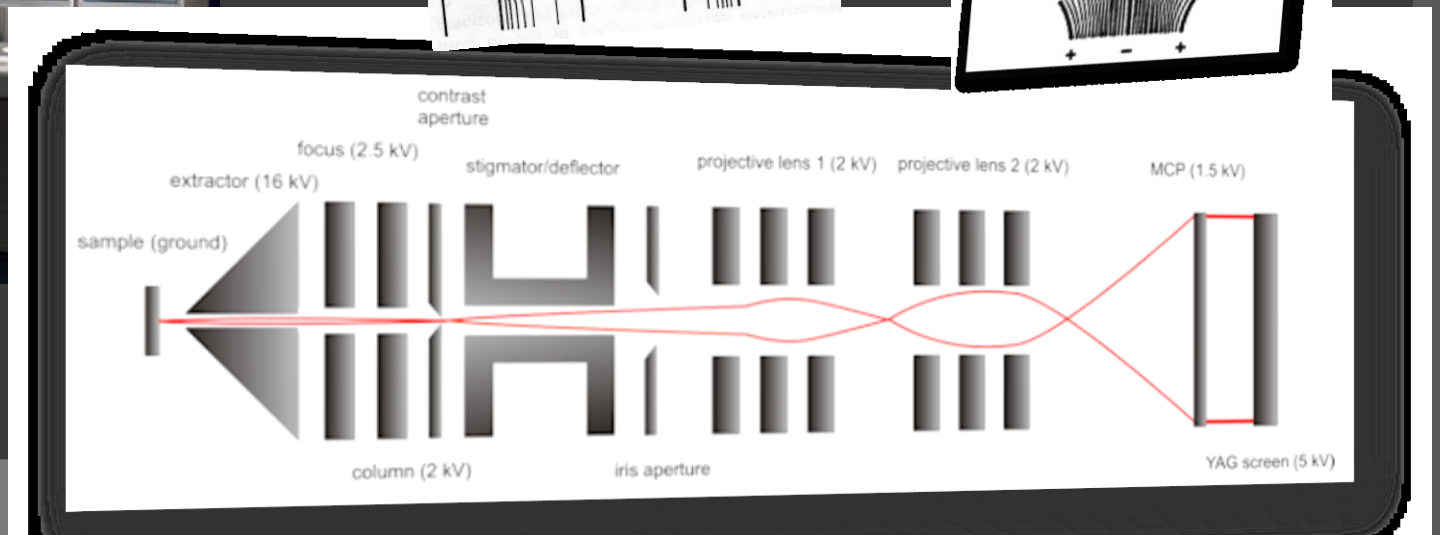
- po aplikovaní na paraxiálnu paprskovú rovnicu dostaneme tvar:

- $R \approx r$ – konkávne, klesá k ose $\rightarrow$ **spojka**
- optická mohutnosť závisí na pomere φ'/φ:

$$\frac{1}{f} = \frac{3}{16}\left(\frac{\varphi}{\varphi_0}\right)^{\frac{1}{4}} \int_{-\infty}^{\infty} \left(\frac{\varphi'}{\varphi}\right)^2 dz$$

- R'' je rovnaké, či je pomer kladný alebo záporný, ohnisko je rovnaké, rozdiel je len v trajektórii





Ukážky z praxe