Exercise session #10

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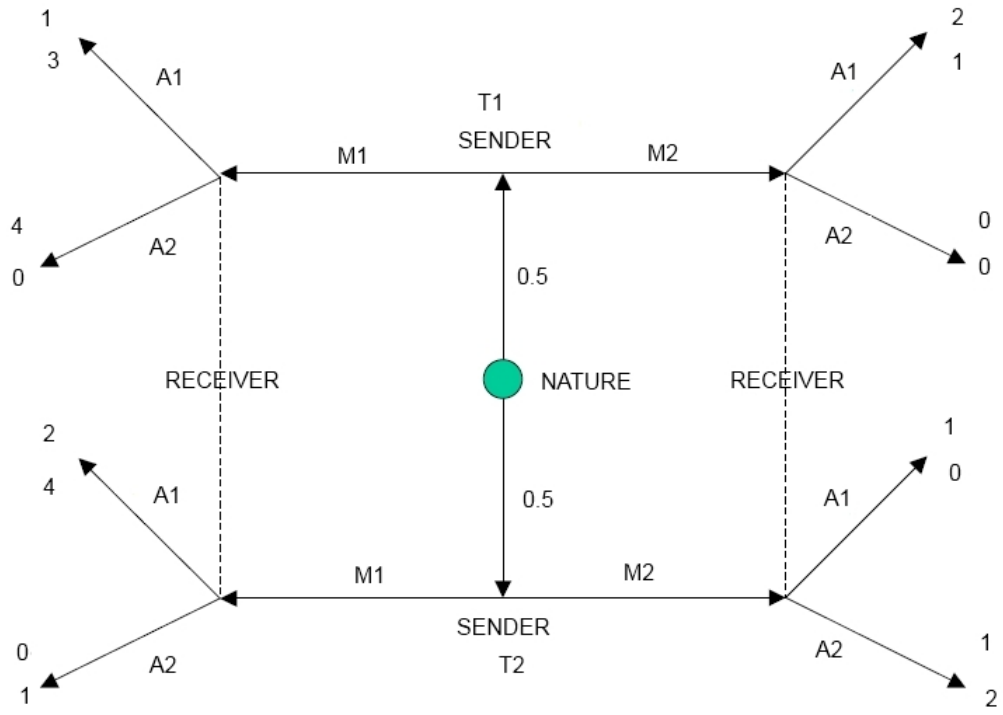
Date: April 24, 2012

Problem 1 *(signaling game 1)*

Find all separating and pooling equilibria in the following signaling game.



Problem 2 *(signaling game 2)*

Mr. Brown is involved in an accident with Mrs. Smith and knows whether he has been negligent or not (for instance, he knows if he has used his cell phone or not while driving). Mrs. Smith does not know for sure, but has a prior belief that the probability that he has been negligent is $p \in (0, 1)$. She has the opportunity to go to a court where it can be determined whether Mr. Brown was negligent or not. Mr. Brown can try to settle the case without going to a court by making a take-it-or-leave-it pre-trial compensation offer. Suppose the offer can be either $5,000$ (high offer) or $3,000$ (low offer). Mrs. Smith can accept or reject this offer. If she accepts, then she takes the money and does not take the case to a court. If she rejects, then she takes the case to a court. If Mr. Brown is found negligent in the trial, he has to compensate Mrs. Smith with an amount of $5,000$. If he is found innocent (i.e. not negligent), then Mrs. Smith receives nothing. In either case, Mr. Brown ends up paying $6,000$ for the trial expenses (so, in case of trial, he pays $11,000$ in total if he is found negligent, $6,000$ in total if he is not found negligent).

a. Draw the extensive form of the game.

b. Show that no separating equilibrium exists.

c. Show that there are two pooling equilibria. Fully characterize them.

Problem 3 *(education signaling 1)*

An agent is born with a skill type η, $\eta = \{L, H\}$ and she decides whether to obtain education and, if yes, how many units. The cost function is given by $c(\eta, e)$.

a. Impose reasonable restrictions on the cost function. Setup the agent's utility function.

b. Setup the firm's maximization problem.

c. Find the separating equilibrium, in which the employer can perfectly distinguish types. Make sure to describe the out-of-equilibrium beliefs and draw a graph where necessary.

d. Find the pooling equilibrium, in which the employer cannot distinguish types. Make sure to describe the out-of-equilibrium beliefs and and draw a graph where necessary.

Problem 4 *(education signaling 2)*

There are two identical firms who would like to hire a worker. There is a single worker in the market whose productivity may be high (H) or low (L). Suppose that the worker's output is 2 if $\eta = H$, and is 1 if $\eta = L$. The worker may signal his actual productivity by investing in education e. The cost of education level e depends on the type η of the worker and is given by $c(\eta, e) = \frac{e}{\eta}$. The utility of the worker is linear in wage net of the education costs.

a. What would be the optimal education level chosen by the worker if the firms knew the actual value of η?

Now suppose that the worker's productivity is unobservable by firms, but his educational level is. The prior belief of the firms that the worker is H-type is $\lambda \in (0, 1)$. Furthermore, suppose that firms believe that $e \geq \overline{e}$ is a signal of high productivity, while $e < \overline{e}$ is a signal of low productivity.

a. What are the optimal wages under these beliefs?

b. Given your answer above, calculate the educational level that each type of the worker will choose (this will depend on the value of $\overline{e}$).

c. Find the necessary condition on $\overline{e}$ so that education is an effective signal of productivity (i.e. for what values of e does a separating Perfect Bayesian equilibrium exist?).

Problem 5 *(education signaling 3)*

Find a separating Perfect Bayesian equilibrium in which the worker gets 0 years of education if $\eta \in [0, \eta^*)$ for some $\eta \in (0, \eta^*)$ and is offered a low wage w_L, but 4 years of education if $\eta \in [\eta^*, 1]$ and is offered a high wage w_H. Find η^*, w_L, w_H and describe firms' beliefs. If hired, worker's output is 10η and costs of obtaining education are $c(\eta, e) = \frac{e}{\eta}$.

Problem 6 *(market for lemons)*

There is a seller trying to sell his car, and there is only one potential buyer. The buyer is uncertain about the quality of the car, which the seller knows. The buyer's belief about the seller's value of keeping the car is described by a uniform distribution on $[0, 1000]$. The actual value of the car to the buyer is 50% more than its value to the seller.

a. Suppose the buyer offers to buy the car for $p_B = 600$. What is the probability that the seller will accept this offer? What is the buyers expected profit from bidding 600?

b. Find the bid for the buyer that maximizes her expected profit.

c. Now consider the game in which the seller can offer to sell his car at any price and then the buyer decides to accept or decline this offer. Find such a price R the buyer will surely agree to buy the car, but not for any higher price.

Now suppose the buyer's belief about the seller's value of the car is described by a uniform distribution on $[250, 1000]$. Still consider the game in which the seller can offer to sell his car at any price and then the buyer decides to accept or decline this offer.

Find an equilibrium in which the buyer would surely accept a price of 500, would have some probability more than 0 but less than 1 of accepting a price of 1100, but would reject any other price over 500.