Exercise session #11

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Problem 1

Introduce the concept of the lottery. Find $E(x)$ and $E(U(x))$ for the cases below.

a. The lottery outcome is a discrete variable that takes on non-negative values $x = \{x_1, x_2, ..., x_N\}$ with respective probabilities $P = \{p_1, p_2, ..., p_N\}$.

b. The lottery outcomes are $x = \{1, 4, 6\}$ and $P = \{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$. Draw the density and distribution functions.

c. The lottery outcomes are $x = \{4, 9\}$ and $P = \{\frac{1}{2}, \frac{1}{2}\}$. Compare $E(x)$ and $E(U(x))$ when $U(x) = x$, $U(x) = \sqrt{x}$, $U(x) = x^2$. Define conditions for when the agent is risk-averse, risk-neutral and risk-loving.

d. Define the coefficient of absolute risk aversion (ARA), r_A. Find such a functional form of the utility function that gives constant ARA.

e. The lottery outcome is a continuous variable x on the support $[0, +\infty)$ with continuous and differentiable distribution function $F(x)$.

Problem 2

An object is to be sold on an auction. There are two bidders with unknown private values of that object, v_i, $i = 1, 2$. However it is known that their values are distributed according to continuous distribution function $f(x)$. Suppose that bidders are risk-neutral.

a. Find agent's expected utility from bidding r when the agent's private value is v_1.

b. Find the bidding function $b^*(v)$.

c. Show that the bidding function is strictly increasing in v.

d. Obtain the biding functions when 1. $v \sim U[0, 1]$ and 2. $v \sim Exp(\lambda)$. Plot on the graph in $(v, b(v))$ locus.

Problem 3

Suppose that all bidders simultaneously submit their sealed bids. The highest bid wins the object and every bidder pays the seller the amount of his bid. Find the bidding function and compare the bidding behavior to Problem 2.

Problem 4

Consider the following variant of a first-price auction. Sealed bids are collected. The highest bidder pays his bid but receives the object only if the outcome of the toss of a fair coin is heads. If the outcome is tails, the seller keeps the object and the high bidder's bid. Bidders are symmetric and risk-neutral.

a. Find the bidding function.

b. Do bidders bid higher or lower compared to the standard first-price auction?

Problem 5

Consider agent's attitude to risk. Plot utility functions of risk-neutral, risk-averse and risk-loving agents. Are risk-averse agents likely to bid less that risk-neutral? Are risk-loving agents likely to bid more that risk-neutral agents?

Problem 6

Suppose that the agent is risk-averse with $U(v, r) = (v - b(r))^{\frac{1}{\alpha}}$, $\alpha > 1$, and v are independent values drawn from the continuous distribution $F(v)$.

a. Show that the maximization problem of this risk-averse agent is equivalent to the maximization problem of the risk-neutral agent, whose values are distributed according to $F^{\alpha}(v)$.

b. Find the optimal bidding function.

c. Show that $b^*(v)$ is increasing in α. Interpret your results.

Problem 7

Given the bidding function $b^*(v)$ compute the expected revenue of the seller in Problems $1 - 3$ and 5.

Problem 8

Suppose that there are N risk-neutral bidders. Re-derive $b^*(v)$ from Problem 2.