Exercise session #1

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Problem 1

You are given a one-period Cobb-Douglas utility function in the form $U(x_1, x_2) = Ax_1^{\alpha}x_2^{1-\alpha}$, where x_1 and x_2 are the consumption goods, α is the intensity of x_1 in the utility, $0 < \alpha < 1$, and A is a parameter, $A > 0$.

1. Setup the consumer's maximization problem. Suggest a monotonic transformation of the objective function to simplify the derivations.
2. Setup the Lagrangian function and take the first order necessary conditions.
3. Derive the optimal demand functions for each good and depict the solution graphically in (x_1, x_2) locus. Show graphically how the solution changes if (i) the budget increases; (ii) prices increase in turn.
4. Verify if one of the goods is inferior or Giffen, check whether the goods are the substitutes or complements and analyze the degree of homogeneity in p and w.
5. Define the equilibrium in this particular problem.
6. Derive the indirect utility function and check its properties. By how much will the utility change if A changes by a small amount.

Problem 2

The studied utility maximization framework is also used to analyze how economic agents (*i.e.* individuals) supply labor. Agent's utility consists of how much labor, L, to supply and how many other goods, x, to buy: $U(x, L) = \frac{x^{\gamma}}{\gamma} - L$, $\gamma \neq 0$. The price of labor is the wage,w, and the price of the consumption good is p. The total endowment of time is 1.

1. Analyze the objective function - which goods are "good" and which are "bad"?
2. Define the agent's constraint and setup the maximization problem. Graphically represent the budget constraint and infer the location of the indifference curve for the following cases:

    i. The minimum number of hours worked is 8 hours per day.

ii. The maximum number of hours worked is 40 hours per week.

iii. The agent is offered to work overtime at a higher wage, $w' > w$.

3. Derive the optimal allocation of the labor supply and consumption.

4. Define the equilibrium in this model.

5. Suppose that the labor income is taxed at the rate τ. Re-derive the optimal labor supply and analyze the effect of the taxation.

Problem 3

Suppose now a dynamic setup. The agent lives for only two periods and cares about his consumption, C and leisure, $1 - L$, so the utility function is implicitly given by $U(C, 1 - L)$. At the beginning of each period the agent gets an endowment, e_1 in period 1, and e_2 in period 2. Each period the agent can choose to invest in bonds, b, which are then paid off the next period with an interest R. The discount factor between periods is β.

1. Setup the agents maximization problem and interpret the parameter β. Define possible boundaries for all parameters in the model.

2. Derive the Euler equation for consumption and labor. Give economic intuition behind it.

3. How will the intertemporal choice of consumption and labor be influenced by a small change in p_1, p_2, w_1, w_2, R and β?

4. Assume that $U(C, 1 - L) = ln(C) + ln(1 - L)$. Derive the allocations of consumption and labor in both periods.

5. Define the competitive equilibrium in the economy.

Problem 4

Suppose that in Problem 3 the two period model is replaced by the infinite time horizon, *i.e.* the utility function is given by $\sum_{t=0}^{\infty} U(C_t, 1 - L_t)$. Show that the solution remains the same.