Exercise session #2

Lecturer: Dmytro Vikhrov

Date: February 28, 2012

Problem 1

Suppose Karel's utility function is $U(x_1, x_2) = x_1 x_2$.

1. Fix the utility at $\overline{U} = 12$ and draw his indifference curve in (x_1, x_2) locus. On the graph explain the notion of the **marginal utility** and **total utility**. Draw both graphs.

2. Suppose that we are at the point $(2, 6)$, and x_2 suddenly drops to $x_2 = 5$. By how much must the consumption of x_1 increase to keep Karel's utility constant at the level $\overline{U}$?

3. Take the total derivative of the utility function and formalize the concept of the **marginal rate of substitution**.

Problem 2

In the above Problem 1 prices of goods were absent. Let us extend the setup to include prices, p_1 for good x_1 and p_2 for good x_2.

1. Reproduce the first-order conditions of the utility maximization problem from Lecture 1. Compute the optimal consumption bundle for $(p_1, p_2) = (\frac{4}{3}, 1)$ and $w = 8$. Compute the elasticity of demand with respect to prices.

2. Suppose that p_1 increases to $p_1^{'} = 2$. Compute the utility loss.

3. By how much should the initial budget increase for Karel to get back on his initial indifference curve, $\overline{U} = 12$? Setup a system of three equations with three unknowns. Interpret your findings.

4. Generalize the notion of **compensating variation** (CV).

5. Now suppose that the budget decrease is measured in terms of old prices. Generalize the notion of **equivalent variation** (EV).

6. How can the CV/EV framework be applied to the case of the recent tax increase in the Czech Republic?

Problem 3

Veronika's utility function is $U(x, y) = min\{x, y\}$. Her monthly salary is 150 units of currency. The price of both x and y is 1. Her boss is thinking of sending her to work to another town, where everything is the same, but $p_y = 2$. The boss offers no rise in pay and the transportation costs are zero. Veronika says that for her the move is as bad as a cut in pay of A units. She also does not mind moving if she gets a rise in pay equivalent to B units. What are the values of A and B?

Problem 4 (left-over from the previous exercise session)

The studied utility maximization framework is also used to analyze how economic agents (*i.e.* individuals) supply labor. Agent's utility consists of how much labor to supply and how many other goods, x, to buy: $U(x, L) = \frac{x^\gamma}{\gamma} + L$, where L is free time, i.e. leisure, and $\gamma \neq 0$. The price of labor is the wage,w, and the price of the consumption good is p. The total endowment of time is 1.

1. Define the agent's constraint and setup the utility maximization problem. Graphically represent the budget constraint and discuss the location of the indifference curve for the following cases:

   i. The minimum number of hours worked is 8 hours per day.

   ii. The maximum number of hours worked is 40 hours per week.

   iii. The agent is offered to work overtime at a higher wage, $w' > w$.

2. Derive the optimal allocation of the labor supply and consumption. Are leisure (or labor) and consumption substitutes or complements? Are they normal or inferior goods?

3. Define the equilibrium in this model.

4. Suppose that the labor income is taxed at a per-unit rate τ. Re-derive the optimal labor supply and analyze the effect of the taxation.

5. Now think that, unlike to the previous point, there is a lump-sum tax on labor T. Re-derive the optimal labor supply and compare it with the labor supply under per-unit tax.