Exercise session #4

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Problem 1
Agent's A optimal consumption is $x^* = \frac{w}{2p}$ (derived last class). Suppose that Agent B supplies this good according to $x = x_s(p)$, $x_s'(p) > 0$.

1. Depict this situation in the supply-demand graph and show graphically the excess demand and supply for a given price $\overline{p}$.
2. In your opinion, what should be the value of excess demand / supply in equilibrium? Suggest ways of achieving it.
3. Generalize the intuition in terms of (i) agent's optimal allocations and (ii) market clearing conditions.

Problem 2 (pure exchange economy)
Suppose that the economy is populated by two consumers: A and B with respective utility functions: $U^A(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$ and $U^B(x_1, x_2) = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$. Endowments are $(e_1^A, e_2^A) = (1, 0)$ and $(e_1^B, e_2^B) = (0, 1)$.

1. Setup the maximization problem of each consumer and solve it.
2. Setup the market clearing conditions (2 equations, 2 unknowns).
3. Define the competitive equilibrium. Is the price vector unique in this equilibrium? Given the price vector, is the allocation for both consumers unique?
4. Draw your findings in the Edgeworth box and provide respective analysis.

Problem 3
Now we introduce risk/uncertainty to the setup. Suppose there is one good x and two states of the world: high (H) and low (L) with respective probabilities of occurrence p_H and p_L.

1. For three time periods $(t_0 - t_2)$ draw a probability tree. What is $Prob(H, H, H)$ and Prob(L, H, L)? How many state contingent goods are there? What are spot and forward markets here?

2. Generalize the intuition into Arrow-Debreu equilibrium with zero time trading?

Problem 4 (exchange economy with uncertainty)
Farmer's Bernoulli utility function in each state of the is $u(x) = lnx$, where x if farmer's output. There are two equally likely states of the world: rainy (R) and dry weather (D). In state 1, it rains and those near rivers do poorly and those in the plains do well. In state 2, it is dry and those near rivers do well, while those in the plains do poorly. There are two classes of farmers: river dwellers A are endowed with output 1 in the rainy state and 2 in the dry state; plain dweller B are endowed with output 2 in the rainy state and 1 in the dry state.

1. Express each individual's utility function over consumption in the two states. Is it Cobb-Douglas?

2. Now let there be a market for weather insurance. What will be the price of consumption in rainy weather if the price of consumption in the dry weather is normalized to 1? What are the equilibrium consumption levels? Is this outcome efficient? Illustrate all findings in the Edgeworth box.

3. Repeat your analysis for the case when the plain dwellers B are endowed with 3 units of output in the rainy state instead. How do your findings differ from point 2 above? Give intuitive interpretation.

Problem 5 (application of GE - the Laffer curve)
Consumer's utility function is $U(C, L) = 2\sqrt{C} - L$, where C is consumption and L is labor.

1. Derive the optimal labor supply when τ is a per-unit tax rate imposed on the wage.

2. Now let us model the government - its objective is to maximize the tax revenue. Setup the government's maximization problem.

3. Draw the reaction curves.

4. Find the revenue maximizing tax rate. Define the competitive equilibrium here.

5. Provide intuition on what the Laffer curve is when there is only a lump-sum tax.