

## Exercise session #6

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### Problem 1

- Define production costs: fixed costs ( $FC$ ), variable costs ( $VC$ ), total costs ( $TC$ ), average costs ( $AC$ ), marginal costs ( $MC$ ). Draw them in a (quantity, cost) graph.
- Assume a quadratic revenue function  $R(q) = Aq - Bq^2$ ,  $A, B > 0$ , and a total cost function,  $TC(q) = Cq + D$ . Setup the profit function  $\pi(q)$ , derive and interpret the first order conditions.
- Derive the second order conditions and suggest the curvature of the revenue and the cost functions. Depict your analysis in  $(q, TC)$ ,  $(q, \pi)$ ,  $(q, MC)$  graphs.

### Problem 2

- Setup the firm's profit function in the case of perfect competition. What are the choice variables?
- Derive the first order conditions and interpret your findings.

### Problem 3

- Suppose that each consumer has her own private valuations of the good. Given the distribution of valuation, setup the demand function and characterize its main properties.
- Setup the monopolist's profit function and discuss how it is different from the perfect competition case. Obtain the elasticity of demand with respect to price and show that the monopolist operate in the area  $|\varepsilon_{dp}| > 1$ .
- Compute the producer and consumer surpluses for the case of the perfect competition and the monopoly. Find the monopoly deadweight loss.
- Go over points  $a. - c.$  for the inverse demand function  $p(q) = 1 - q$ .

### Problem 4

Farmers produce corn from land and labor. The labor cost to produce  $q$  bushels of corn is  $C(q) = q^2$ . There are 100 identical farms and they behave competitively.

- a. Derive the supply curve of each farmer. What is the market supply of corn?
- b. Suppose the demand curve for corn is  $D(p) = 200 - 50p$ . Obtain the equilibrium price and quantity sold.
- c. Compute the profits of each firm. What is the equilibrium rent on the land?

### Problem 5

There are 100 boat builders on an island with an individual cost function  $C(q_i) = 11 + iq_i$ , where  $i$  is the number of a boat builder,  $i = \overline{1, 100}$ . Each of them can build up to 12 boats per year and each builder maximizes his profits at the given market price. If the price of a boat is 25, how many builders will choose to produce and what will be the total supply.

### Problem 6

There are 50 competitive firms with identical cost structure  $C(q) = \frac{q^2}{2}$ . At the same time there is one monopolist with 0 marginal costs. The demand curve for the product is  $D(p) = 1000 - 50p$ .

- a. Find the monopolist's output and price.
- b. Find the output of the competitive sector.
- c. If the monopolist and the competitive sector were to engage in a price war, how many producers would remain in the market?

### Problem 7

Consumers of Country A have the following demand function for umbrellas  $D(p) = 90 - p$ . Umbrellas are supplied by a representative producers from Country A and Country B that behaves competitively. The cost function of each producer is  $C(q) = \frac{q^2}{2}$ .

- a. Find the aggregate supply function for umbrellas.
- b. Find the equilibrium price and quantity sold.

Suppose that the domestic industry lobbies for protection, and the Parliament imposes a \$3 tariff on foreign umbrellas.

- c. What is the new price for umbrellas  $\bar{p}$ ? How many umbrellas does each producer supply?

**Problem 8**

Suppose that the monopolist can different prices to different types of consumers.

- a. Discuss a mechanism how the monopolist can fully extract the surplus from each consumer when her private valuation is known.
- b. Draw on the graph when the monopolist can set a two-part tariff - a participation fee,  $T$ , and a price per unit,  $p$ .
- c. Suppose the monopolist knows about the existence of two types of agents - wealthy (H) and poor (L), but he does not know who is who. Setup the participation and incentive-compatibility constraints.

**Problem 9**

When a person arrives at an amusement park Elvisland, her demand for rides is  $q = \max\{5 - p, 0\}$ . The owner of Elvisland is a monopolist with marginal costs \$2.

- a. Find a two-part tariff:  $T$  is the entrance fee equal to the consumer surplus,  $p$  is the price per ride.
- b. Compute the monopolist's profit and compare it to the profit under the uniform pricing.

**Problem 10**

A consumer has the utility function  $U(x_1, x_2) = 4x_1 - \frac{x_1^2}{2} + x_2$ . Assume that  $p_2 = 1$ .

- a. Suppose the monopolist produces good 1 at a constant unit cost  $c$ . Find the optimal price and quantity under no price discrimination.
- b. The monopolist offers a two-part tariff, where a consumer must pay a lump-sum amount  $T$  in order to enjoy a low price  $\bar{p}$ . Derive conditions for the consumer to switch to this tariff.