Exercise session #8

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Problem 1. In the following one-shot game find the Nash equilibrium by elimination of strictly dominated strategies.

	b_1	b_2	b_3
a_1	0 2	1 1	2 4
a_2	4 3	2 1	3 2
a_3	3 1	2 0	0 3

Problem 2 (*battle of sexes*). Draw the normal form of the battle of sexes game and find its equilibria.

Problem 3 (*problem of the commons*). The lake is freely accessed by fishermen. The cost of sending a boat out on the lake is $r > 0$. When b boats are sent out onto the lake $f(b)$ fish are caught in total (so each boat catches $f(b)/b$ fish), where $f'(b) > 0$ and $f''(b) < 0$ at all $b \geq 0$. The price of fish is $p > 0$, which is unaffected by the level of the catch from lake.

a. If each individual decides on his / her own whether to send a boat, many boats will be sent in total?

b. If the lake is owned by a company, how many boats will be allowed to fish?

c. Define a tax that would equalize the amounts in a and b.

Problem 4 (*linear city model*). Suppose consumers are distributed uniformly on interval [0, 1]. Two producers ($N = 2$) are deciding where to locate. Each consumer buys from the closest producer. Find the equilibrium location of the producers. How does it change when $N = 3$? Suppose the transportation costs for each consumer are t per unit of distance. Find quantities and prices of both producers.

Problem 5 (*matching pennies game, mixed strategies*). Two players are tossing a coin. The payoffs are (player 1, player 2): $(H, H) = (-1, 1)$, $(H, T) = (1, -1)$, $(T, H) = (1, -1)$, $(T, T) = (-1, 1)$. Find equilibria in pure and mixed strategies.

Problem 6. Consider the following three-stage sequential game: stage one: player 1 chooses L or R. If L is chosen the game ends with payoffs $(2, 0)$. If R is chosen, Player 2 gets to choose between L' and R': if L' is chosen, the game ends with payoffs $(1, 1)$. If R' is chosen, Players 1 gets to choose between L'' and R''. If he chooses L'' the game ends with payoffs $(3, 0)$ and if he chooses R'', the game ends with payoffs $(0, 2)$.

a. Draw the extensive form of the game and use the backward induction to find the equilibrium.

b. Assume that it is common knowledge that player 2 is irrational. What could be outcomes of the game? Discuss outcomes when player 1 is irrational.

Problem 7 (*entry game*). A potential producer (entrant) decides whether to enter the market, where the existing producer (incumbent) is already selling. If he does not enter the payoffs of both producers are $(0, 2)$. The incumbent decides whether to oppose the entrance of the new producer ($Fight$) or to agree with it ($Accommodate$). If he fights, the payoffs are $(-3, -1)$, whereas if he accommodates, the payoffs are $(2, 1)$.

a. Draw the normal form of the game and find all Nash equilibria. Which equilibrium does not give a reasonable prediction? Draw the extensive form of the game.

b. Extend the game to account for imperfect information. Find the perfect Bayesian equilibrium.

Problem 8. You need to hire one employee for your firm. You advertise the job opening and 3 prospective employees apply for the position. Your policy is never to hire an employee before interviewing her. For any of the three applicants, you believe that the value to you of hiring this particular applicant is continuously uniformly distributed in the interval from 100 to 200. You will hire at most one of the prospective employees, and your objective is to maximize your value.

a. What should your hiring strategy be?

b. How will your hiring strategy change if the constant cost of interviewing is 20 per interview.

Problem 9 (*Stackelberg duopoly*). For the demand function $p(q) = A - Bq$ and marginal costs c, let producer 1 be the Stackelberg leader and producer 2 - the Stackelberg follower. Find quantities produced of both producers, the market price and compare them to the cases of monopoly and perfect competition.