Homework assignment #1

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Due date: April 3, 2012 (before midterm)

Problem 1

If we graph Jana's indifference curve with avocados on the horizontal and grapefruits on the vertical axis, then whenever she has more grapefruits than avocados, the slope of the indifference curve is -2. Whenever she has more avocados than fruits, the slope is $-\frac{1}{2}$. Jana is indifferent between the bundle with 24 avocados and 36 grapefruits and another bundle with 32 avocados and x grapefruits. Find x and plot Jana's indifference curve.

Problem 2

Clara's utility function is $U(x_1, x_2) = (x_1 + 2)(x_2 + 1)$. If her marginal rate of substitution is -4 and she is consuming 14 units of x_1, how many x_2 is she consuming?

Problem 3

You know that $U(x_1, x_2) = x_1 - x_2^{-1}$, prices of goods x_1 and x_2 are p_1 and p_2, and w is the available budget.

a. Find the Marshallian demand functions for both goods and check whether they are normal or inferior, Giffen or non-Giffen. Are both goods complements or substitutes?

b. Compute elasticities of each good with respect to p_1 and p_2. Is any of the goods luxury?

c. Obtain the indirect utility function, derive the cost function and the Hicksian demand.

d. Setup the cost minimization problem and verify that the Hicksian demand in the point above is correct. Compute the income and substitution effects for both goods separately and show that the Slutsky equation holds. How is x_1 different from x_2. Draw IE and SE for both goods in separate graphs.

Problem 4

Suppose that the economy is populated by two consumers: A and B with respective utility functions: $U^A(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$ and $U^B(x_1, x_2) = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$. Endowments are $(e_1^A, e_2^A) = (1, 1)$ and $(e_1^B, e_2^B) = (3, 2)$.

a. Setup the maximization problem of each consumer and solve it.

b. Setup the market clearing conditions (2 equations, 2 unknowns).

c. Define the competitive equilibrium. Is the price vector unique in this equilibrium? Given the price vector, is the allocation for both consumers unique?

d. Draw your findings in the Edgeworth box and provide respective analysis.

Problem 5

Consumer's utility function is $U(C, L) = \frac{C^\gamma}{\gamma} - L$, where C is consumption and L is labor.

a. Setup the budget constraint when the time endowment is 1.

b. Derive the optimal labor supply L^* and show it in a graph.

c. Re-derive L^*_τ when τ is a per-unit tax imposed on the wage. How does L^*_τ compare to L^*?

d. Now let us model the government - its objective is to maximize the tax revenue. Setup the government's maximization problem.

e. Find the revenue maximizing tax rate. Define the competitive equilibrium here.

Problem 6

You are given the total cost function: $TC(q) = Aq^3 - Bq^2 + Cq + D$.

a. Find $MC(q)$ and $AC(q)$. What restrictions can you impose on the parameters A, B, C, D? Depict all three cost functions in a graph.

b. Find q^*_{MC} that minimizes the marginal costs and q^*_{AC} that minimizes the average costs. Is q^*_{MC} equal to q^*_{AC}? Why or why not?

Problem 7

Assume a company producing two goods under perfect competition. Its total cost function is $TC(q_1, q_2) = 2q_1^2 + q_1q_2 + 2q_2^2$.

a. For this structure of the market, what are the choice variables for the company? Setup the profit function and take the first-order conditions.

b. Derive the profit-maximizing quantities q_1^* and q_2^*. How can you make sure that these quantities really maximize the firm's profit? Show that the profit function is concave in q_1^* and q_2^*.

Now suppose that the same company is a monopolist on both markets. The demand functions for goods are: $q_1 = 40 - 2p_1 + p_2$, $q_2 = 15 + 2p_1 - p_2$.

c. Are the goods substitutes or complements?

c. Setup the monopolist's profit function and find the equilibrium quantities and prices.

c. Compute the deadweight loss of the monopoly in both markets.

Problem 8

A monopolist faces a linear marginal revenue and a fixed marginal cost, $MR(q) = a - bq$, $MC(q) = c$. There is also a fixed cost of production f.

a. Find the total revenue, demand function, total costs and average cost.

b. Find the profit maximizing output and the maximized profit.

Suppose that the government imposes a per-unit tax on the sale price, τ.

a. Setup the new profit function. Find the optimal quantity sold. What is the effect of the tax on the sales volume?

b. Can you find the government's Laffer curve here? Obtain the tax rate that maximizes the tax revenue.