**Homework assignment #2**

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Due date: May 15, 2012 (before first lecture)

**Problem 1**

You are given the following simultaneous move game:

| | $L$ | $C$ | $R$ |
|---|---|---|---|
| $U$ | 2, 0 | 1, 3 | 3, 2 |
| $M$ | 4, 1 | 2, 2 | 1, 4 |
| $D$ | 1, 2 | 4, 4 | 2, 3 |

1. Draw the extensive form of this game.

2. Find the Nash equilibrium by eliminating strictly dominated strategies.

**Problem 2**

There are two producers, each having marginal costs $c$. The demand function is $p(Q) = (a + Q)^{-3}$, $a > 1$.

1. Both producers are playing the Cournot quantity competition game. Find $q_1$, $q_2$, $p_1$, $p_2$.

2. Now suppose that both producers are playing the Stackelberg game, producer 1 being the leader and producer 2 being the follower. Find $q_1$, $q_2$, $p_1$, $p_2$.

**Problem 3**

Consumers are located on interval $[0, 1]$, and each wants to buy one unit of the homogenous product produced by firms. Each firm is situated on the edges of the interval. When a consumer buys a unit from the firm located at distance $d$ from him, his utility is $U = d - p$ (i.e. the consumer prefers goods located further away from him).

1. The two firms produce at 0 costs and compete in prices. Find an equilibrium in prices and the market shares of each firms.

2. Now a third firm enters in the middle of the interval, i.e. its location is $x = 0.5$. Given that the two firms at the edges charge the same price $p$, find the best response of the middle firm.

3. Given that one of the edge firms charges $p$ and the middle one charges $q > p + 0.5$, what is the best response of the other edge firm?

**Problem 4**.

Problems 4.1.*a* and 4.4. on pages 245 and 248 respectively from Gibbons (1992), *A primer in game theory*.

**Problem 5**.

The worker's talent is measured by his type $\eta$ which is known to the worker. However, firms cannot observe it; from their perspective, it is uniformly distributed from 0 to 1, with higher types values being associated with better talents. Hiring a worker of type $\eta$ would produce output $10\eta$ to either firm. Because of competition between firms, the worker will be offered a wage equal to his conditionally expected productive value, given what is publicly observable about him. The workers education does not affect his productivity, but his educational achievement is publicly observable. Suppose that the effective cost of getting $e$ years of education is $\frac{e}{\eta}$ for a worker of type $\eta$.

a. Find a separating Perfect Bayesian equilibrium in which the worker gets 0 years of education if $\eta \in [0, 0.5)$ and is offered a low wage $w_L$, but some positive number $E$ years of education if $\eta \in [0.5, 1]$ and is offered a high wage $w_H$. Find $E$, $w_L$, $w_H$ and describe firms' beliefs.

b. Find a separating Perfect Bayesian equilibrium in which the worker gets a different level of education $e(\eta)$ for every $\eta$, and is paid $10\eta$. Be sure to specify the equilibrium value of $e(\eta)$ and firms' beliefs.