



ZPRACOVÁNÍ A ANALÝZA BIOSIGNÁLŮ



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VLNKOVÁ TRANSFORMACE MOTIVACE ANEB O CO JDE?



LITERATURA

- ☑ Polikar R.: The Wavelet Tutorial, Part I, 2, III, IV

<http://users.rowan.edu/~polikar/WAVELETS/WTpart1.html>

<http://users.rowan.edu/~polikar/WAVELETS/WTpart2.html>

<http://users.rowan.edu/~polikar/WAVELETS/WTpart3.html>

<http://users.rowan.edu/~polikar/WAVELETS/WTpart4.html>

- ☑ Selesnick. I.W.: Wavelet Transforms – A Quick Study

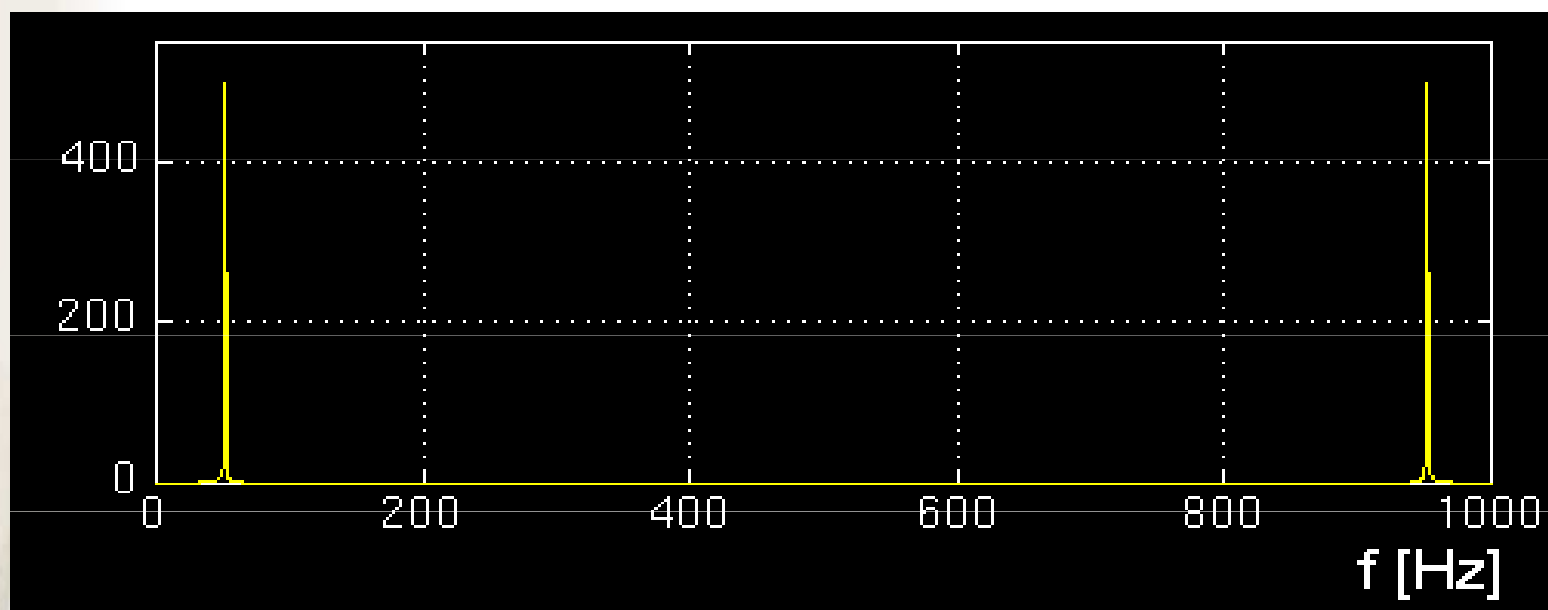
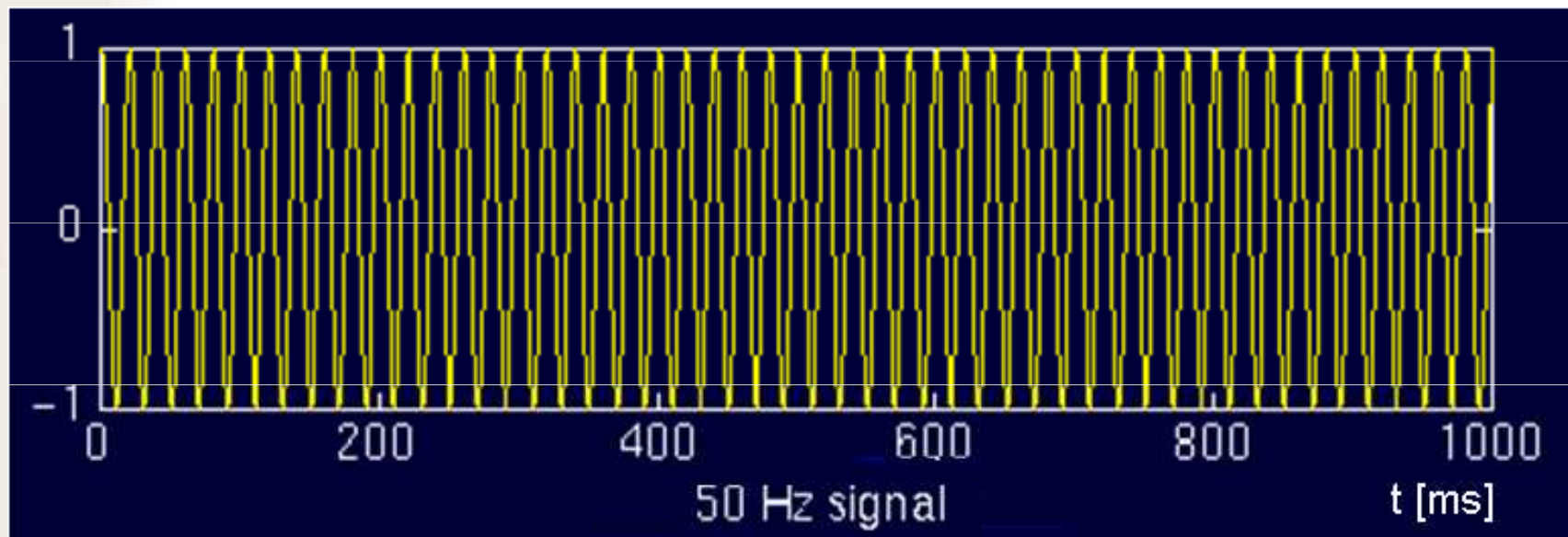
http://eeweb.poly.edu/iselesni/lecture_notes/WaveletQuickStudy_expanded.pdf

- ☑ wavelet.org <http://www.wavelet.org/phpBB2/gallery.php?c=Tutorial>

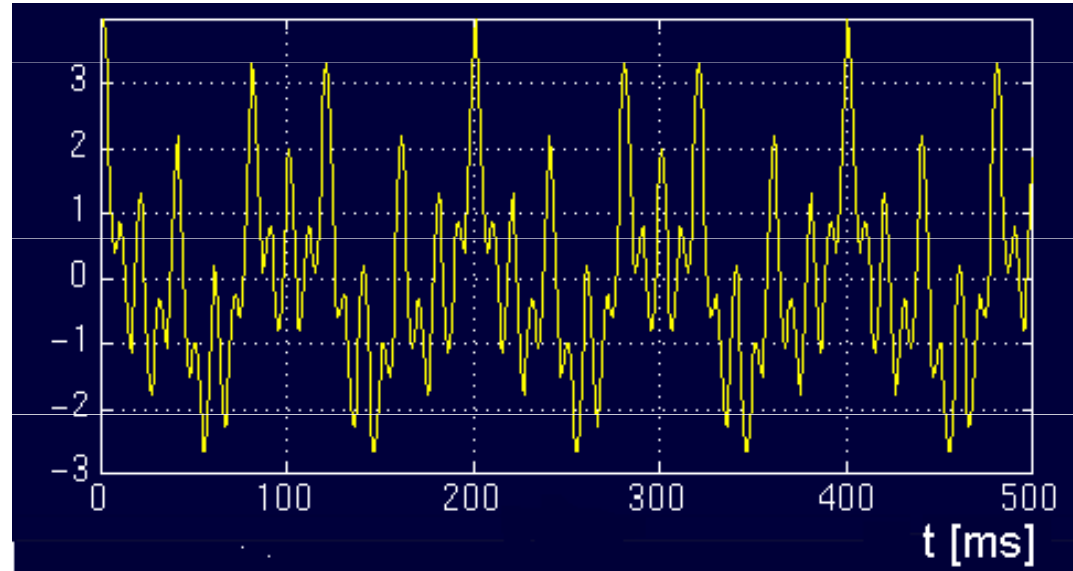
- ☑ Valens, C.: A Really Friendly Guide to Wavelets.

<http://math.ecnu.edu.cn/~qgu/friendintro.pdf>

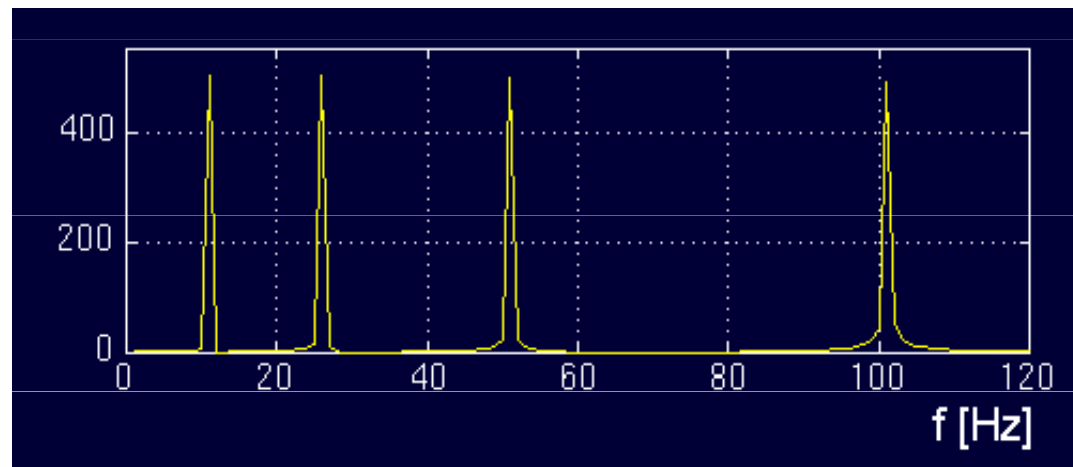
FOURIEROVA TRANSFORMACE X X VLNKOVÁ TRANSFORMACE



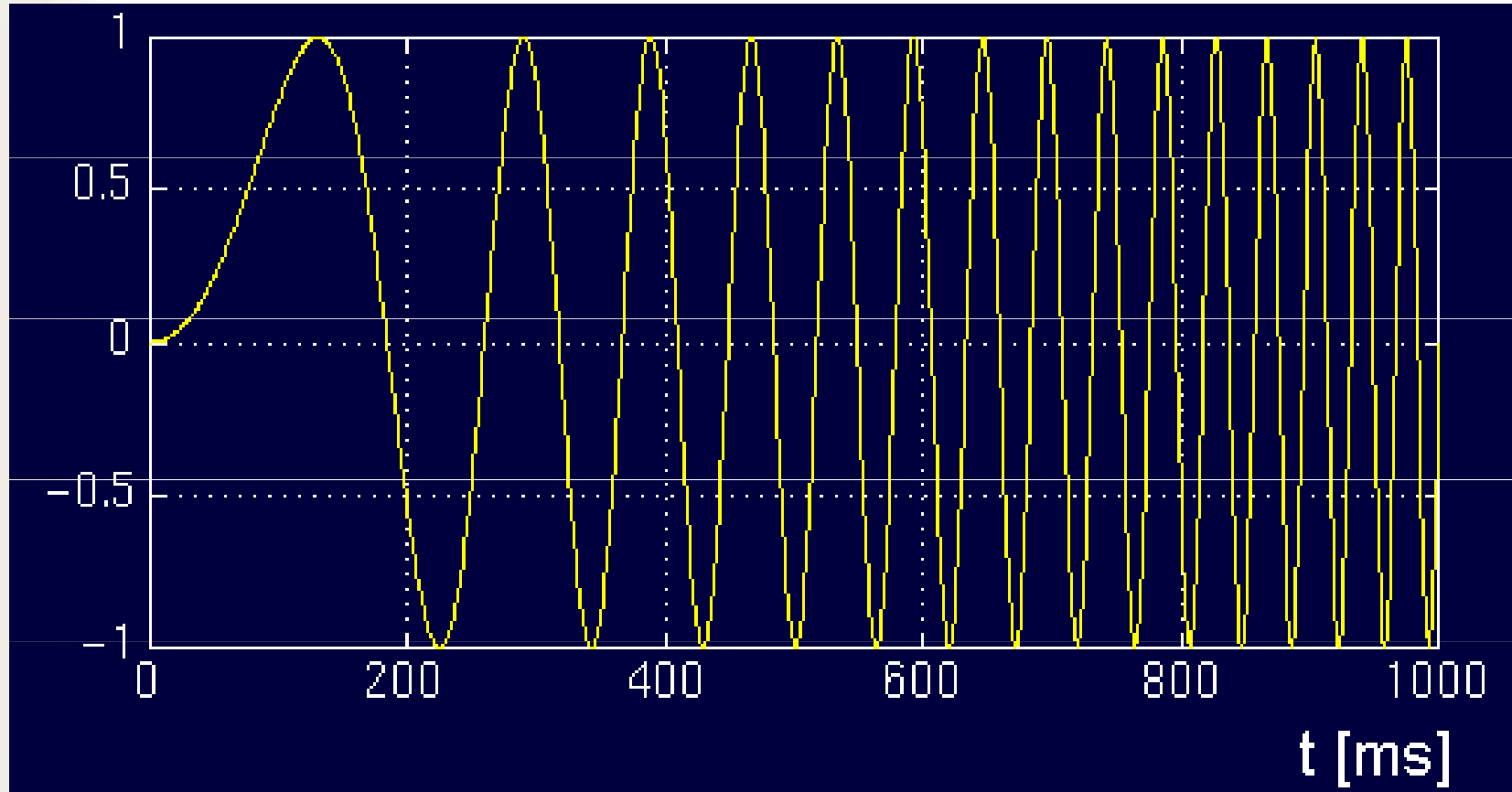
FOURIEROVA TRANSFORMACE X X VLNKOVÁ TRANSFORMACE



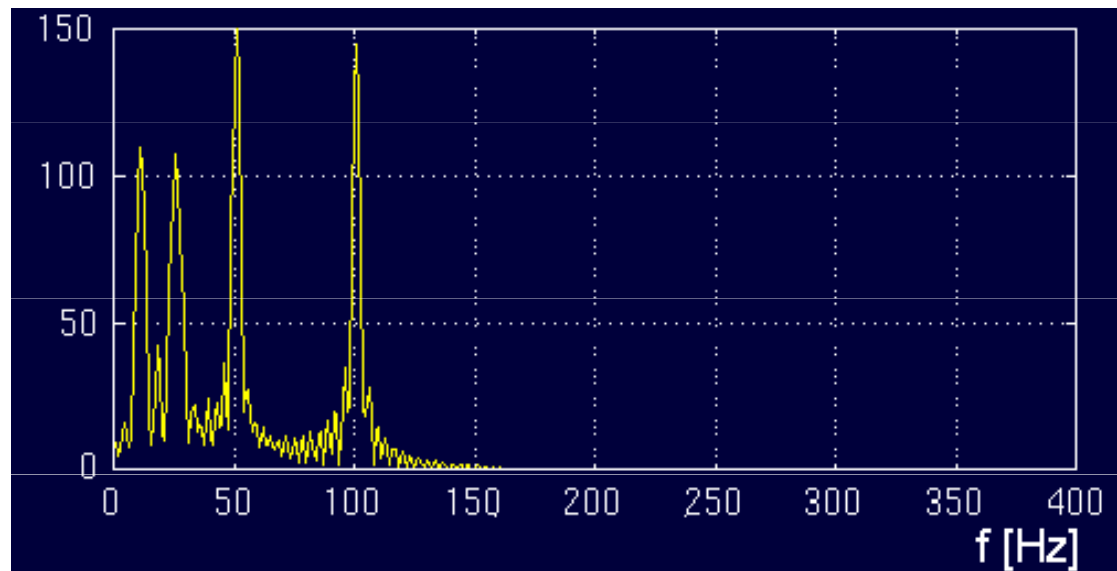
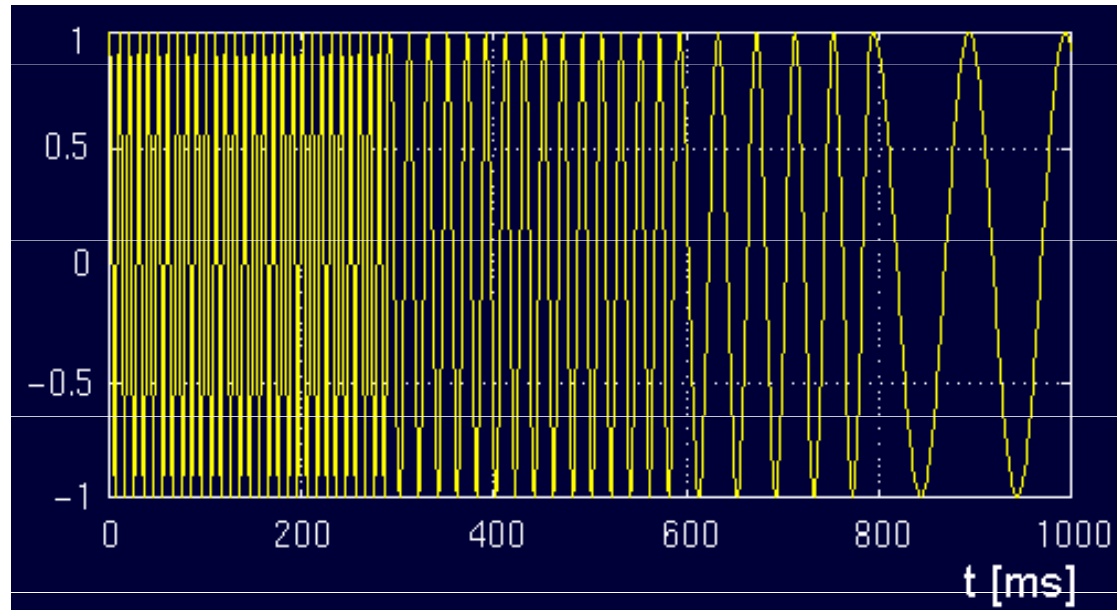
$$x(t) = \cos(2\pi 10t) + \cos(2\pi 25t) + \cos(2\pi 50t) + \cos(2\pi 100t)$$



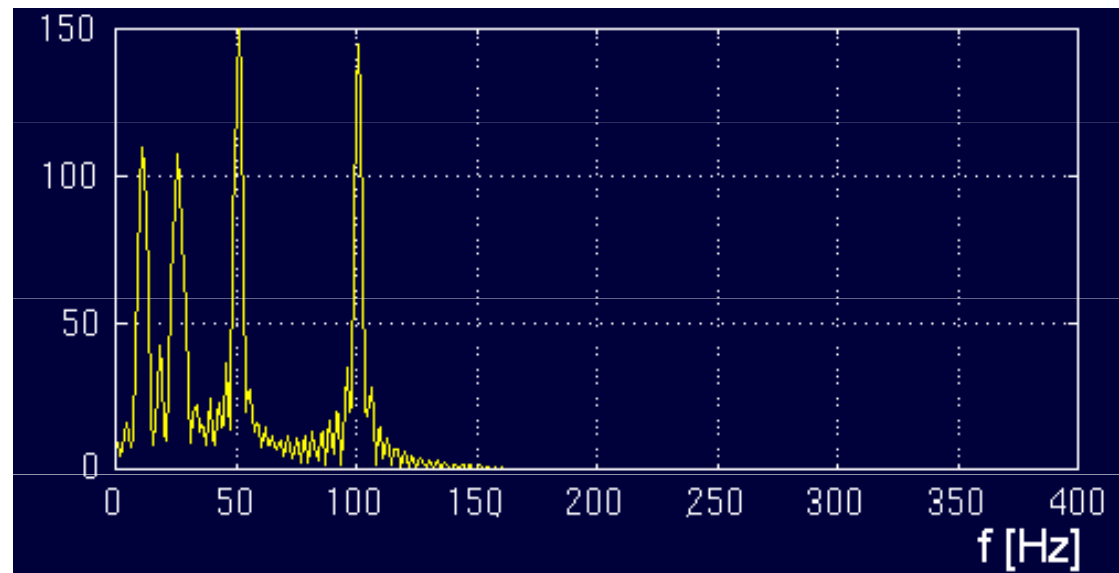
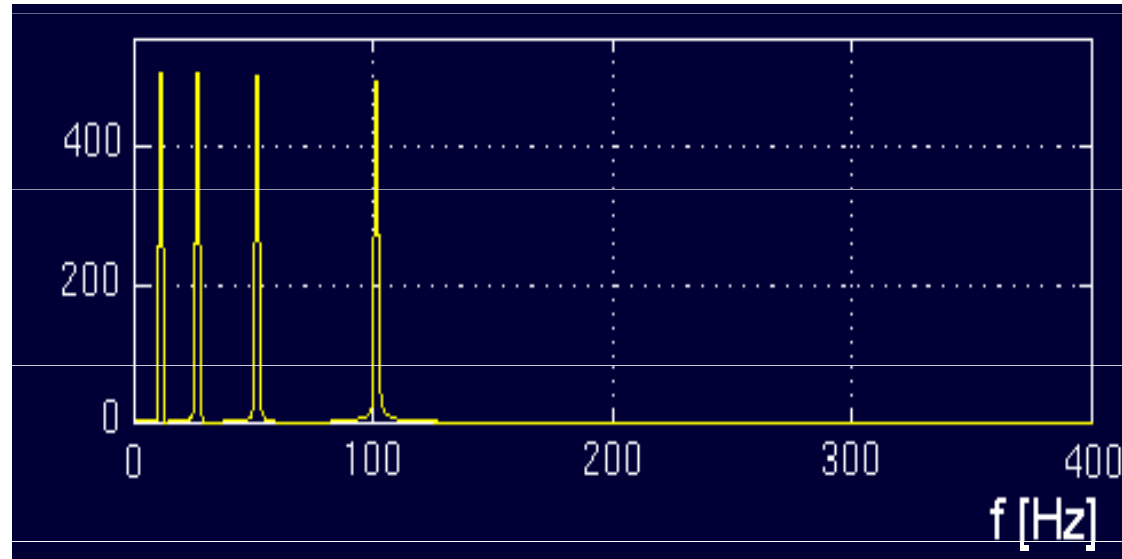
FOURIEROVA TRANSFORMACE X X VLNKOVÁ TRANSFORMACE



FOURIEROVA TRANSFORMACE X X VLNKOVÁ TRANSFORMACE



FOURIEROVA TRANSFORMACE X X VLNKOVÁ TRANSFORMACE



KRÁTKODOBÁ FOURIEROVA TRANSFORMACE (SHORT TIME FOURIER TRANSFORM – STFT)

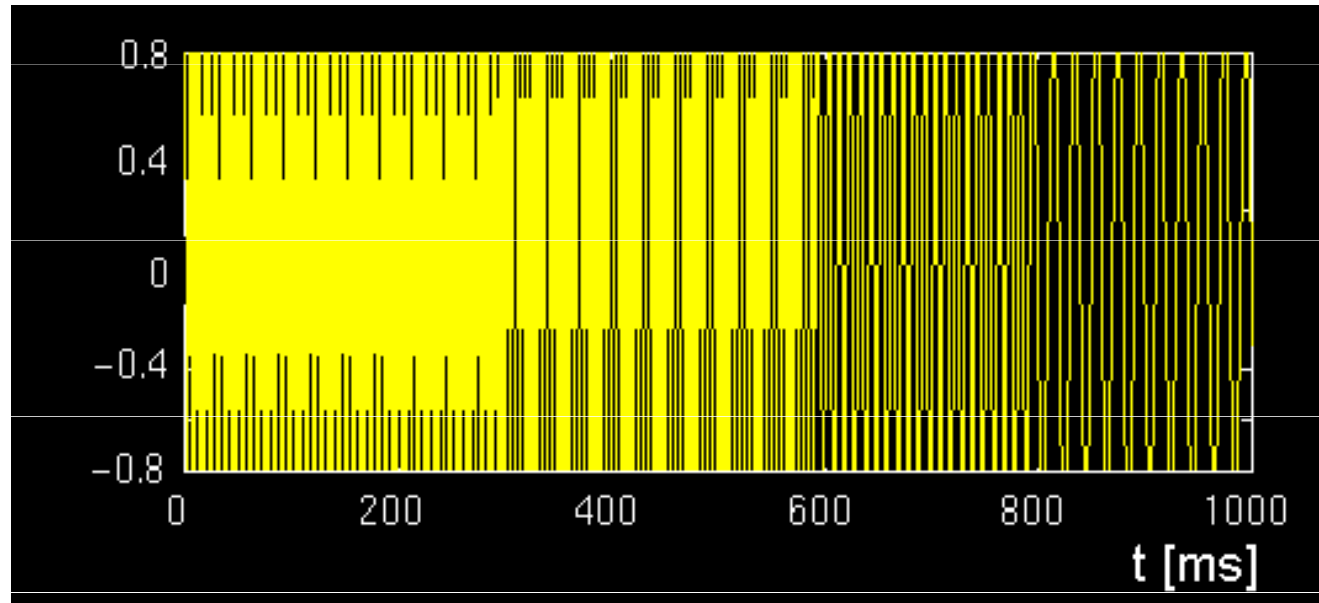
- ✓ Fourierova transformace

$$X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft} dt$$

- ✓ krátkodobá Fourierova transformace

$$X_{\text{STFT}}(t', f) = \int_t x(t) \cdot w(t - t') \cdot e^{-j2\pi ft} dt$$

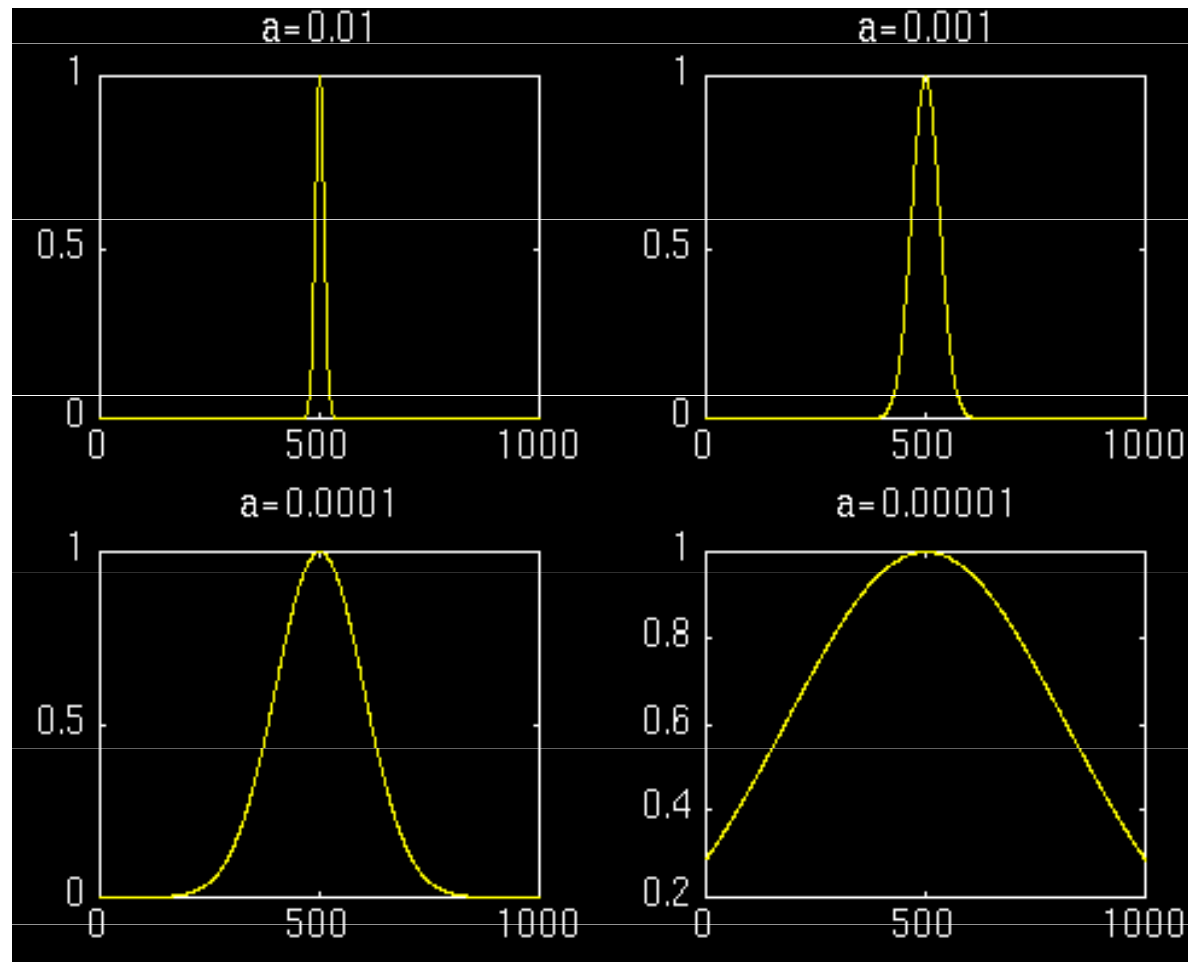
KRÁTKODOBÁ FOURIEROVA TRANSFORMACE (SHORT TIME FOURIER TRANSFORM – STFT)



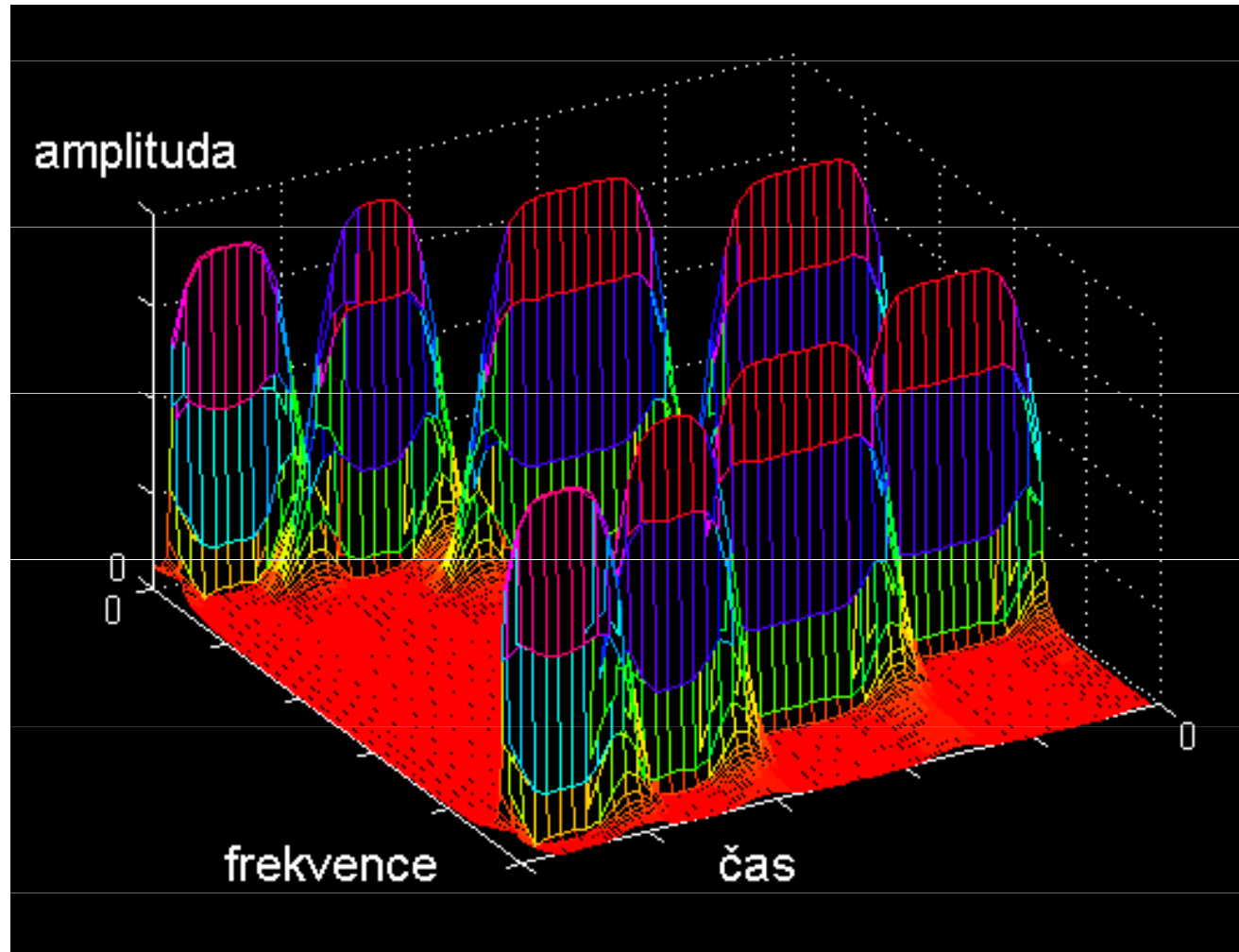
0 – 300 ms: $f = 300$ Hz
300 – 600 ms: $f = 200$ Hz
600 – 800 ms: $f = 100$ Hz
800 – 1000 ms: $f = 50$ Hz

KRÁTKODOBÁ FOURIEROVA TRANSFORMACE (SHORT TIME FOURIER TRANSFORM – STFT)

Gaussovo okno: $w(t) = \exp(-a.t^2/2)$

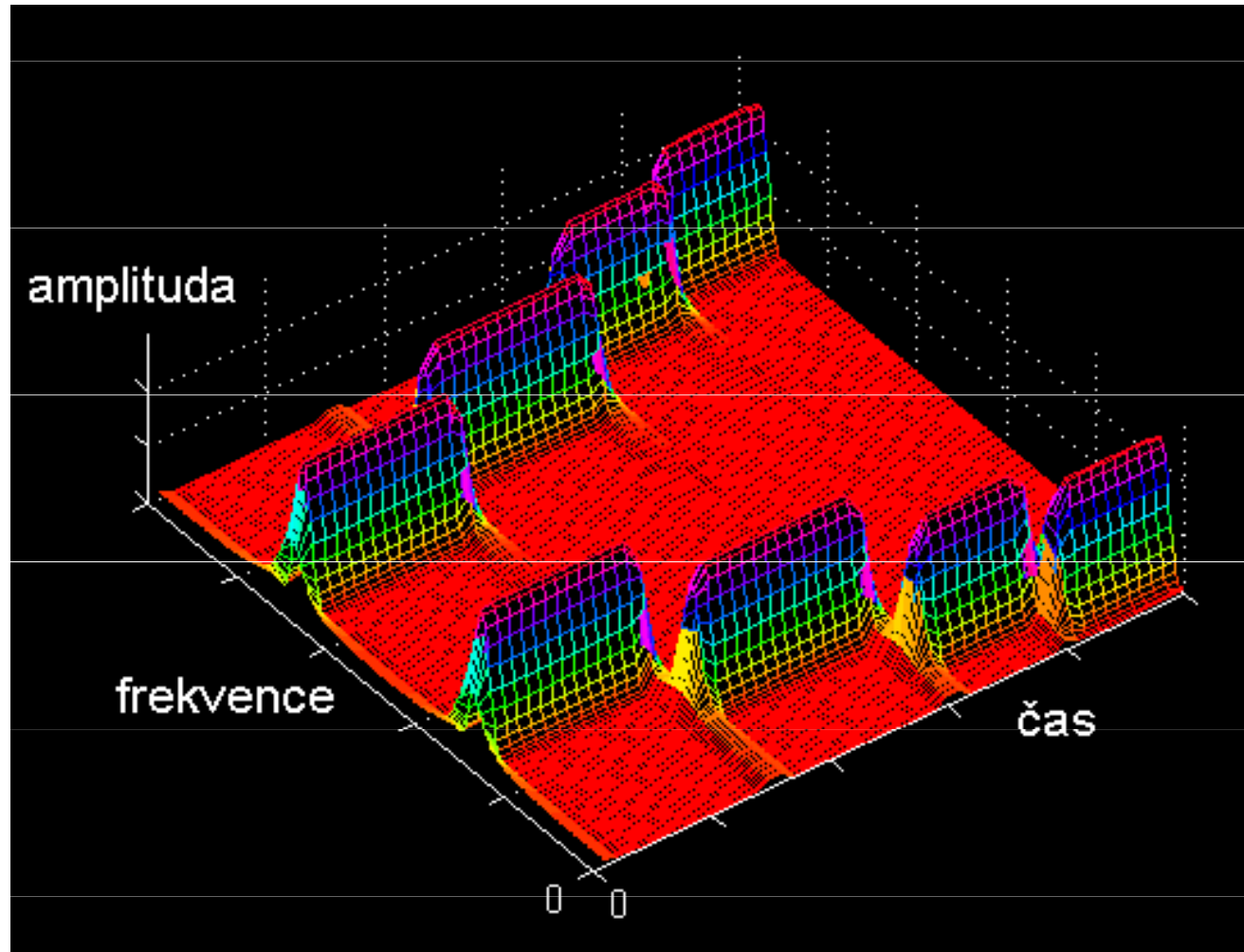


KRÁTKODOBÁ FOURIEROVA TRANSFORMACE (SHORT TIME FOURIER TRANSFORM – STFT)



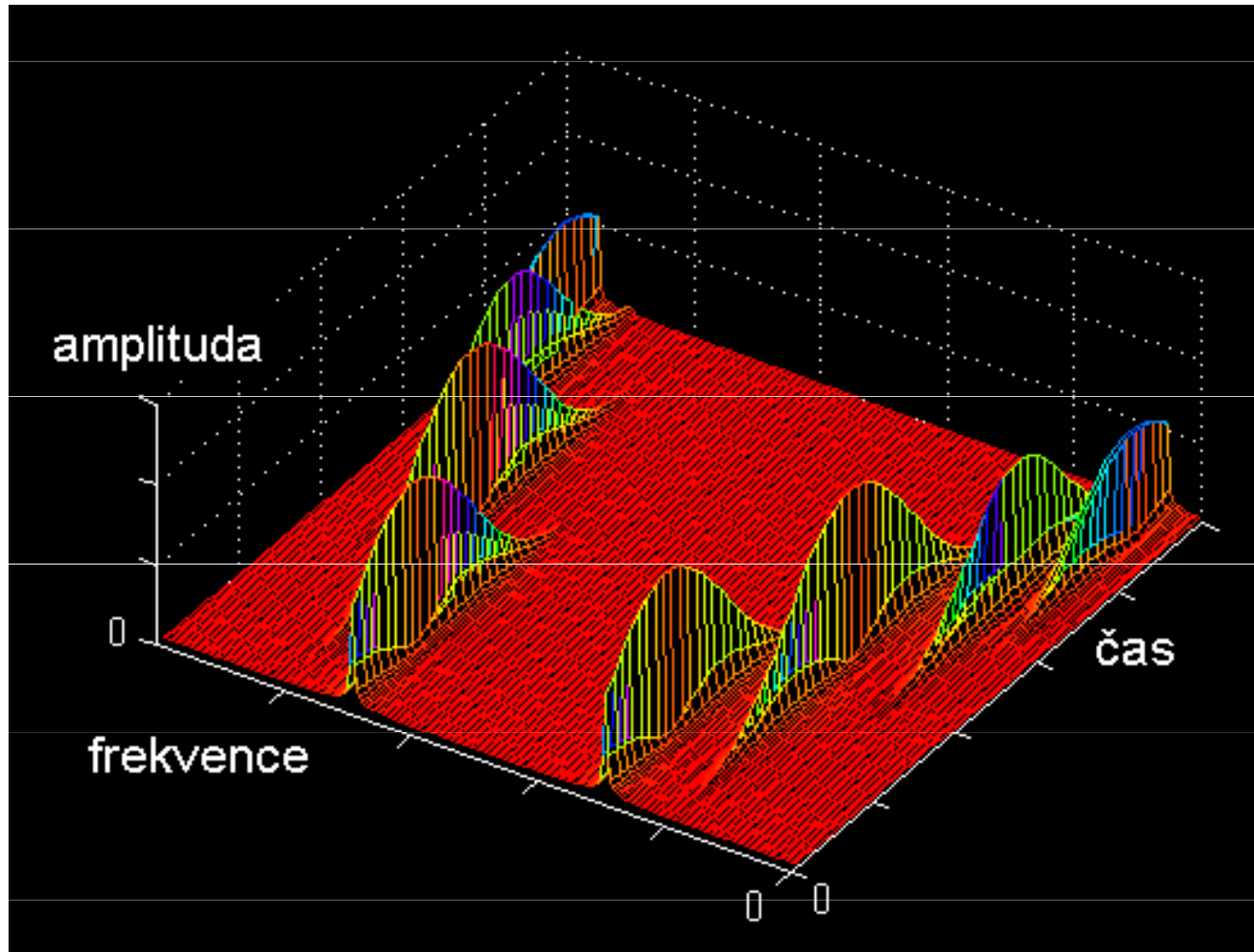
$$a = 0,001$$

KRÁTKODOBÁ FOURIEROVA TRANSFORMACE (SHORT TIME FOURIER TRANSFORM – STFT)



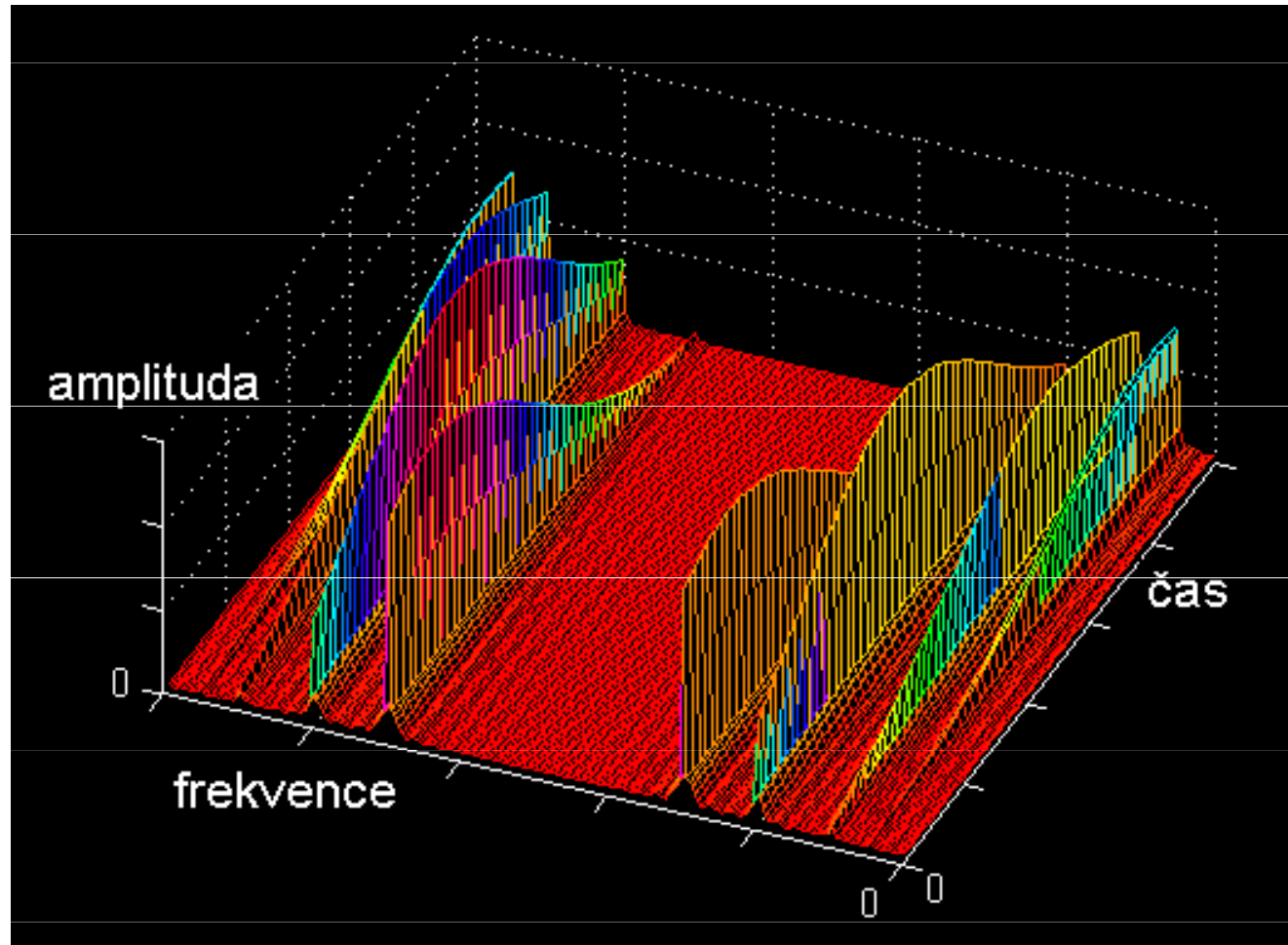
$$a = 0,01$$

KRÁTKODOBÁ FOURIEROVA TRANSFORMACE (SHORT TIME FOURIER TRANSFORM – STFT)



$$a = 0,0001$$

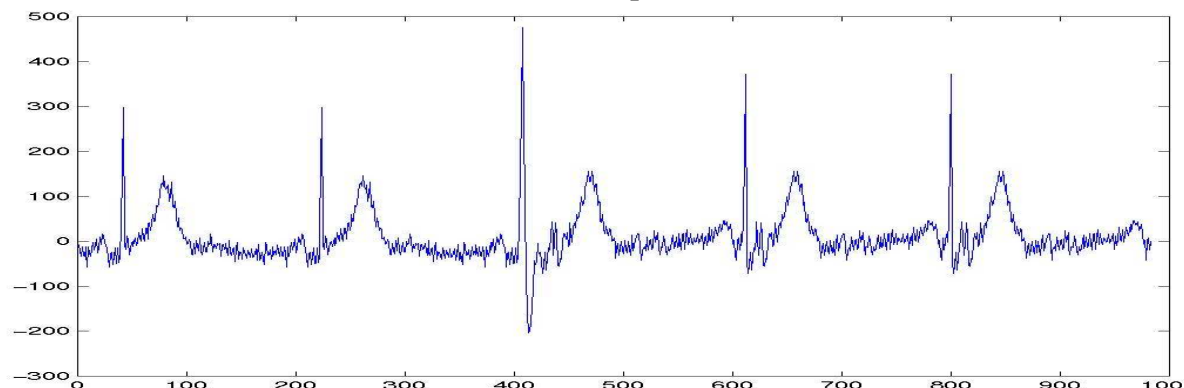
KRÁTKODOBÁ FOURIEROVA TRANSFORMACE (SHORT TIME FOURIER TRANSFORM – STFT)



$$a = 0,00001$$

MULTIREZOLUČNÍ ANALÝZA

- ☑ signál je analyzován s různým rozlišením (přesností vyjádření) pro různé frekvence
- ☑ je to tak, že je dobré rozlišení v čase a horší frekvenční rozlišení na vysokých frekvencích – to je šikovní především tehdy, pokud zpracováváný signál obsahuje vysoké frekvence po krátkou dobu trvání a nízkofrekvenční složky delší dobu



VLNKOVÁ TRANSFORMACE

$$X_{\text{CWT}}(\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t) \psi\left(\frac{t - \tau}{s}\right) dt$$

☑ parametry

→ τ - časový posun

→ s - měřítko (jako na mapě, čím menší číslo, tím větší detaily), inverzní vazba na frekvence (nízká frekvence - velké měřítko a vice versa, ale u vlnek je to naopak, protože s je ve jmenovateli)

☑ $\psi(\bullet)$ - mateřská vlnka (jsou používány různé typy vlnek)

ZÁKLADNÍ OPERACE SE SIGNÁLY

OPERACE S JEDNOU FUNKCÍ

☑ **změna časového měřítka**

$$x(t) \sim x(mt),$$

kde m je kladné reálné číslo

$m > 1$ – časová komprese;

$m < 1$ – časová expanze

$m = 1$ – nic se neděje

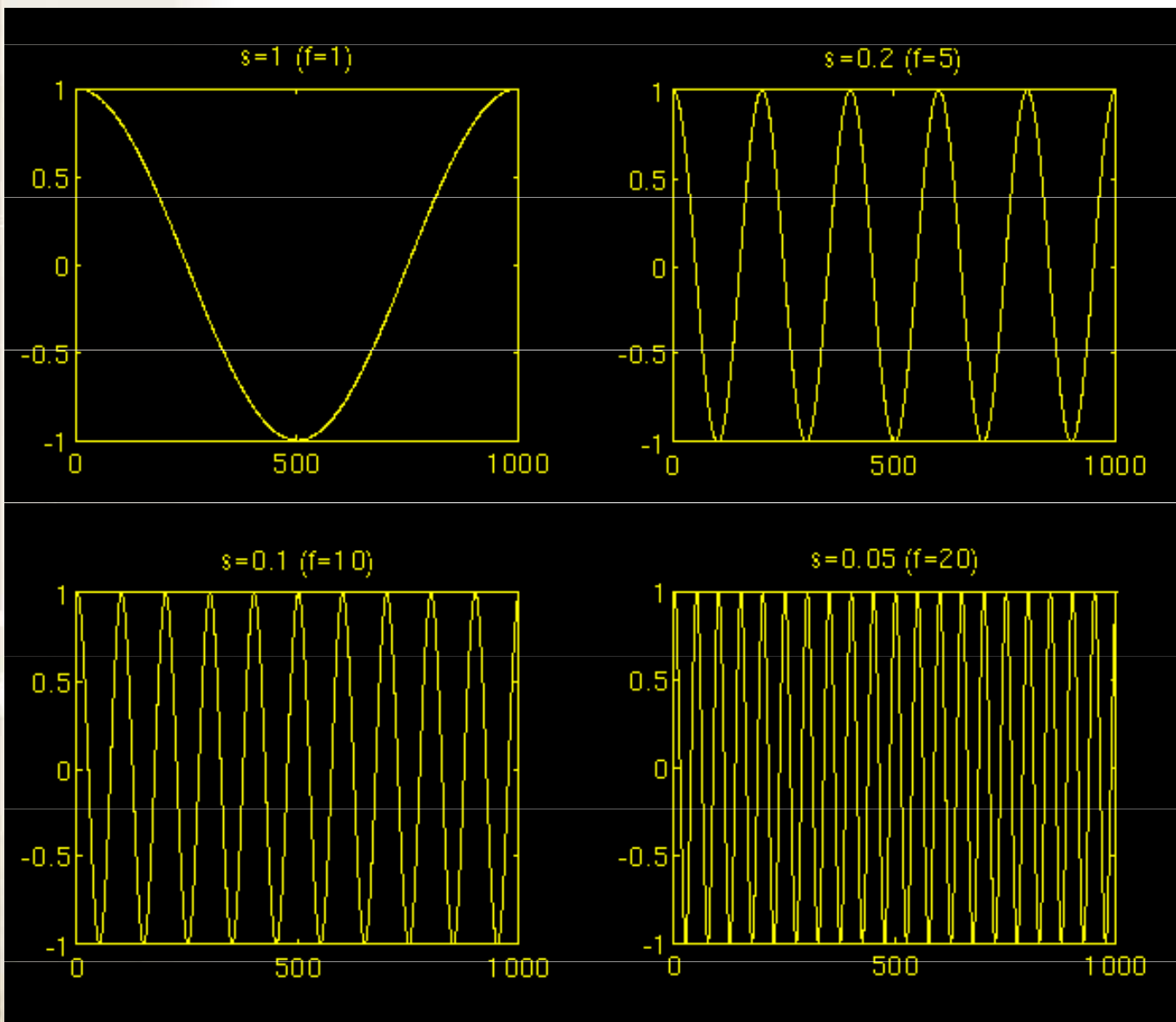
u vlnek

$\sim x(t/m)$, takže

$m < 1$ – časová komprese;

$m > 1$ – časová expanze, dilatace časové osy

MĚŘÍTKO



VÝPOČET

$$X_{\text{CWT}}(\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t) \psi\left(\frac{t - \tau}{s}\right) dt$$

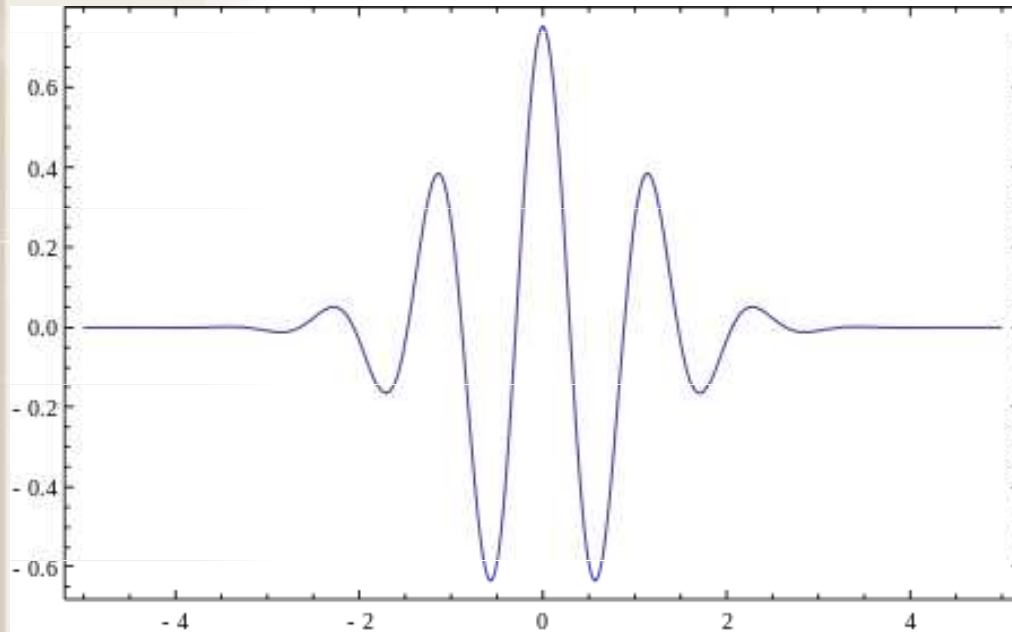
korelační funkce:

$$R_{xy}(\tau) = \frac{1}{T} \int_0^T x(t) y(t + \tau) dt,$$

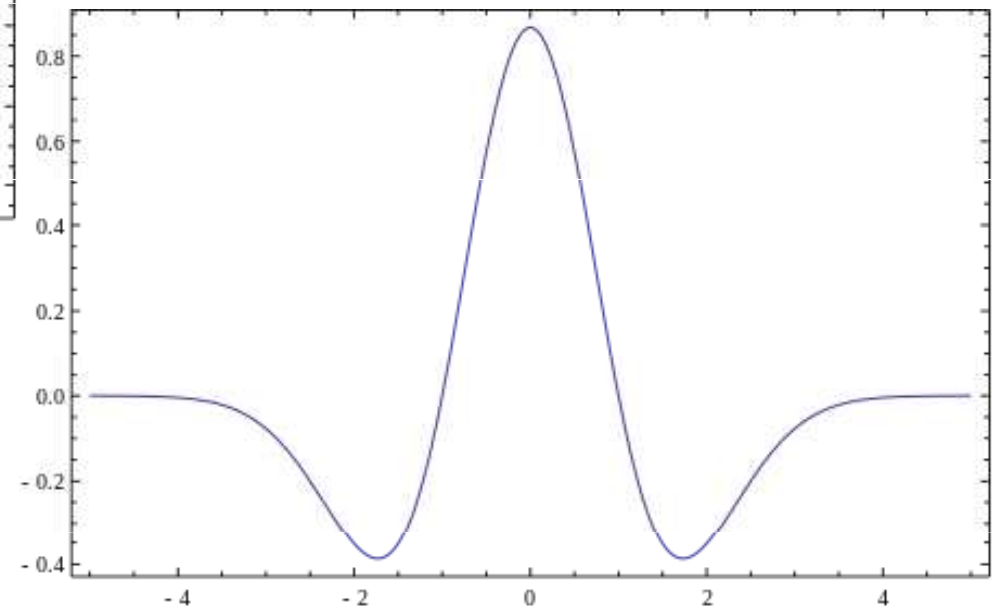
$$\text{resp. } R_{xy}(\tau) = \frac{1}{T} \int_0^T x(t) y(t - \tau) dt$$

EJHLE !

RŮZNÉ TYPY MATEŘSKÝCH VLNEK

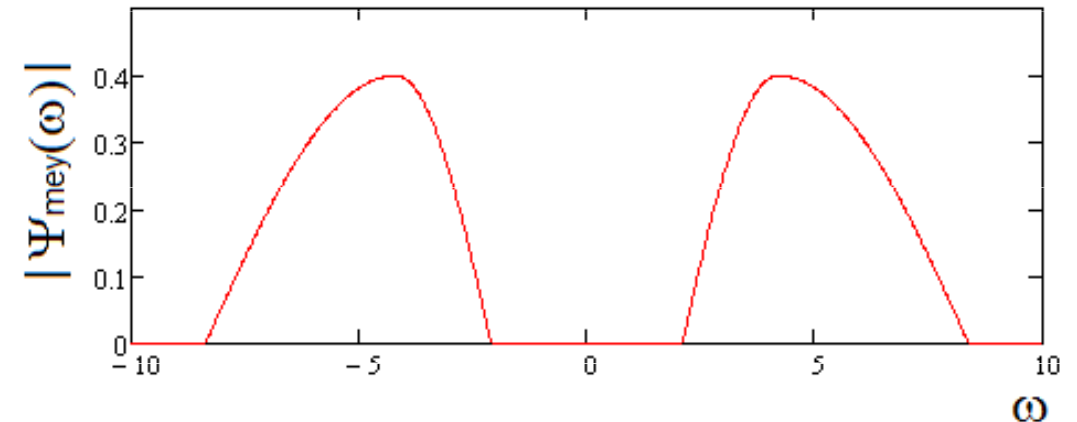
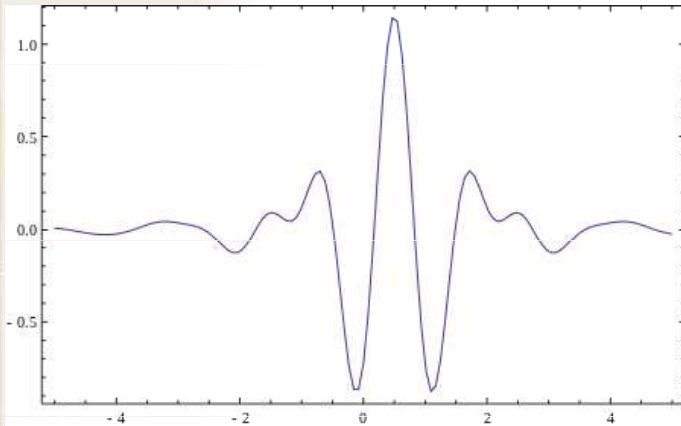


Morletova vlnka



vlnka tvaru mexický klobouk

RŮZNÉ TYPY MATEŘSKÝCH VLNEK



Meyerova vlnka (reálná část)

$$\Psi(\omega) := \begin{cases} \frac{1}{\sqrt{2\pi}} \sin\left(\frac{\pi}{2}\nu\left(\frac{3|\omega|}{2\pi} - 1\right)\right) e^{j\omega/2} & \text{if } 2\pi/3 < |\omega| < 4\pi/3, \\ \frac{1}{\sqrt{2\pi}} \cos\left(\frac{\pi}{2}\nu\left(\frac{3|\omega|}{4\pi} - 1\right)\right) e^{j\omega/2} & \text{if } 4\pi/3 < |\omega| < 8\pi/3, \\ 0 & \text{otherwise,} \end{cases}$$

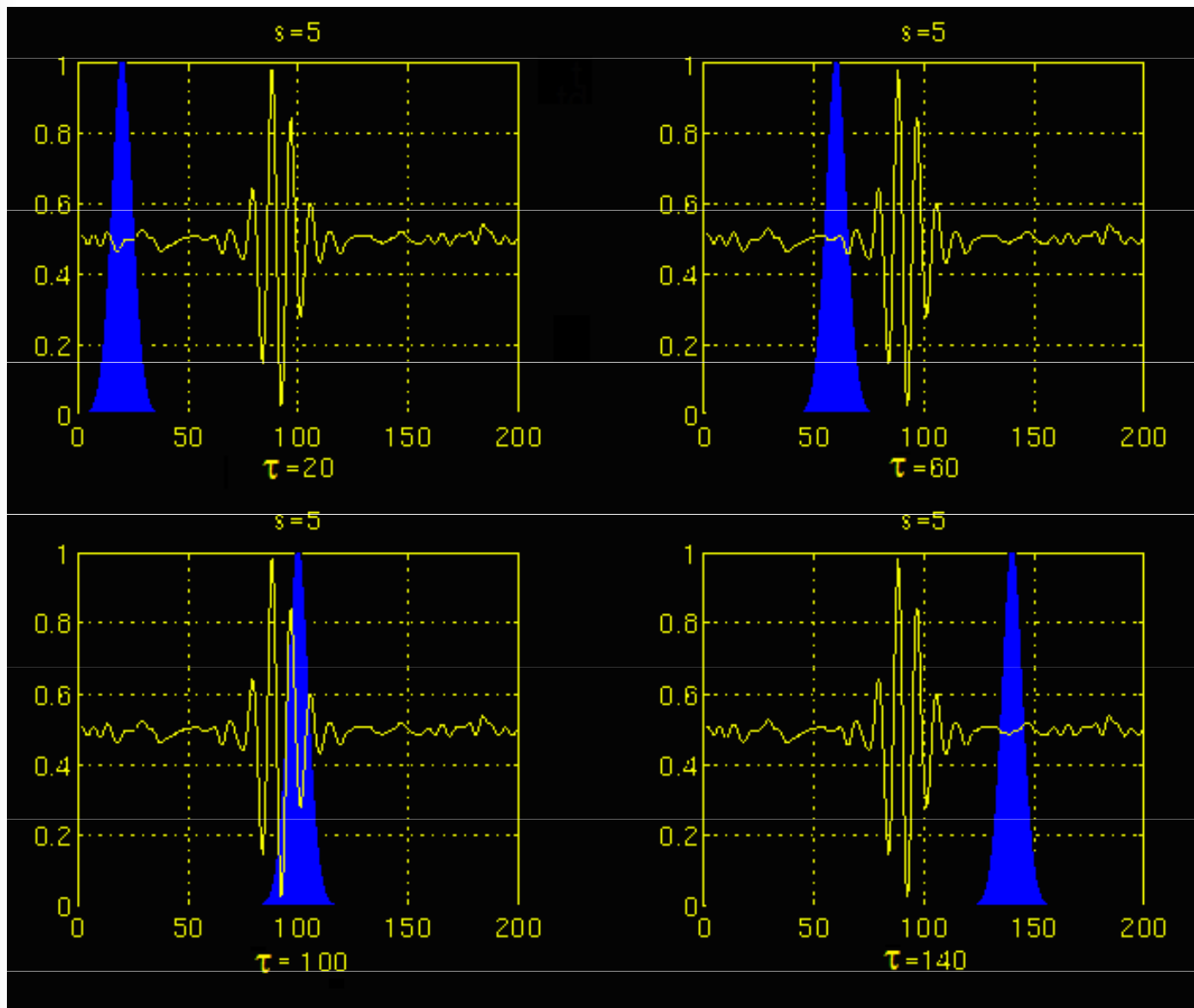
kde

$$\nu(x) := \begin{cases} 0 & \text{if } x < 0, \\ x & \text{if } 0 < x < 1, \\ 1 & \text{if } x > 1. \end{cases} \quad \text{nebo třeba} \quad \nu(x) := \begin{cases} x^4(35 - 84x + 70x^2 - 20x^3) & \text{if } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

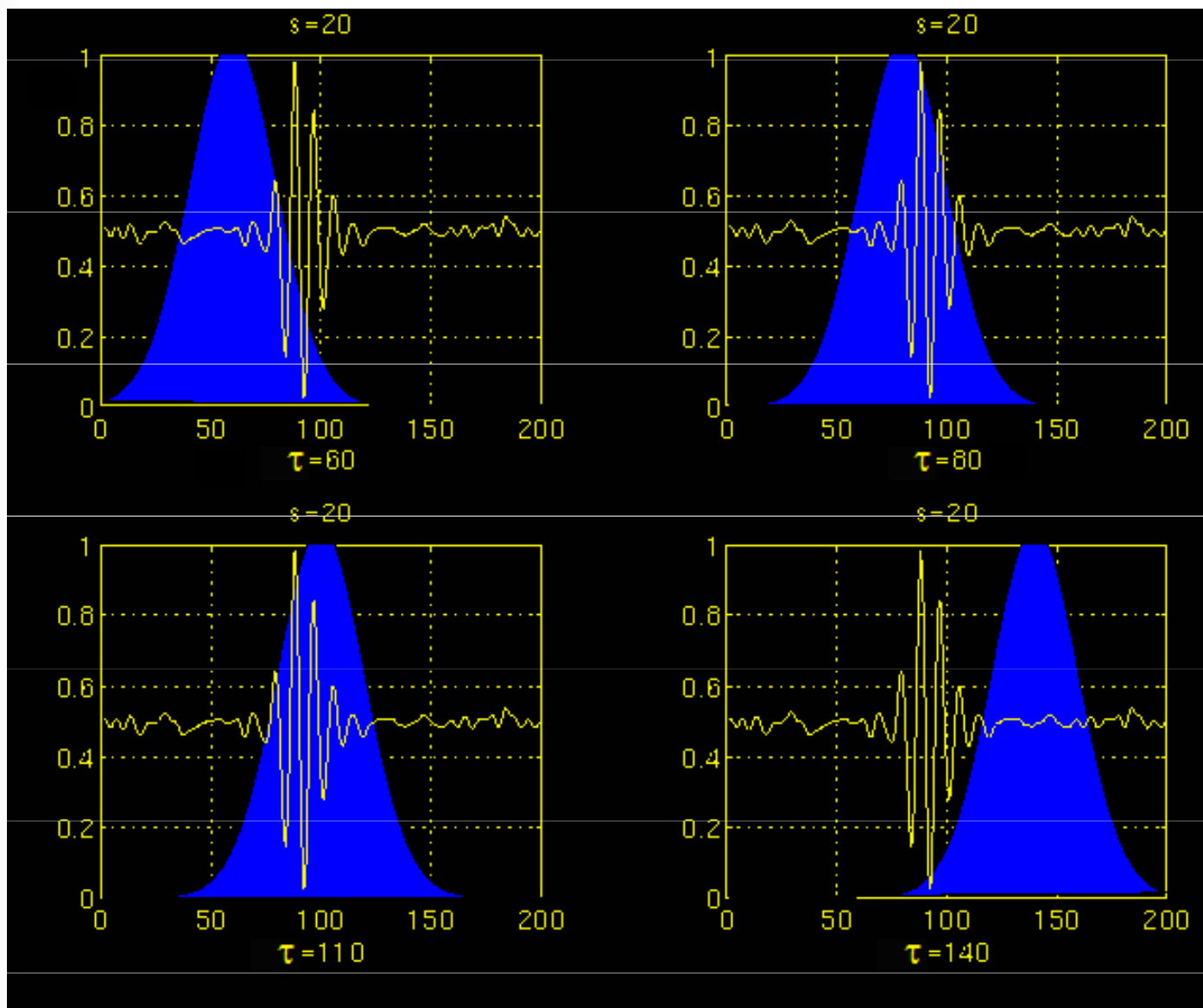
měřítková funkce:

$$\Phi(\omega) := \begin{cases} \frac{1}{\sqrt{2\pi}} & \text{if } |\omega| < 2\pi/3, \\ \frac{1}{\sqrt{2\pi}} \cos\left(\frac{\pi}{2}\nu\left(\frac{3|\omega|}{2\pi} - 1\right)\right) e^{j\omega/2} & \text{if } 2\pi/3 < |\omega| < 4\pi/3, \\ 0 & \text{otherwise.} \end{cases}$$

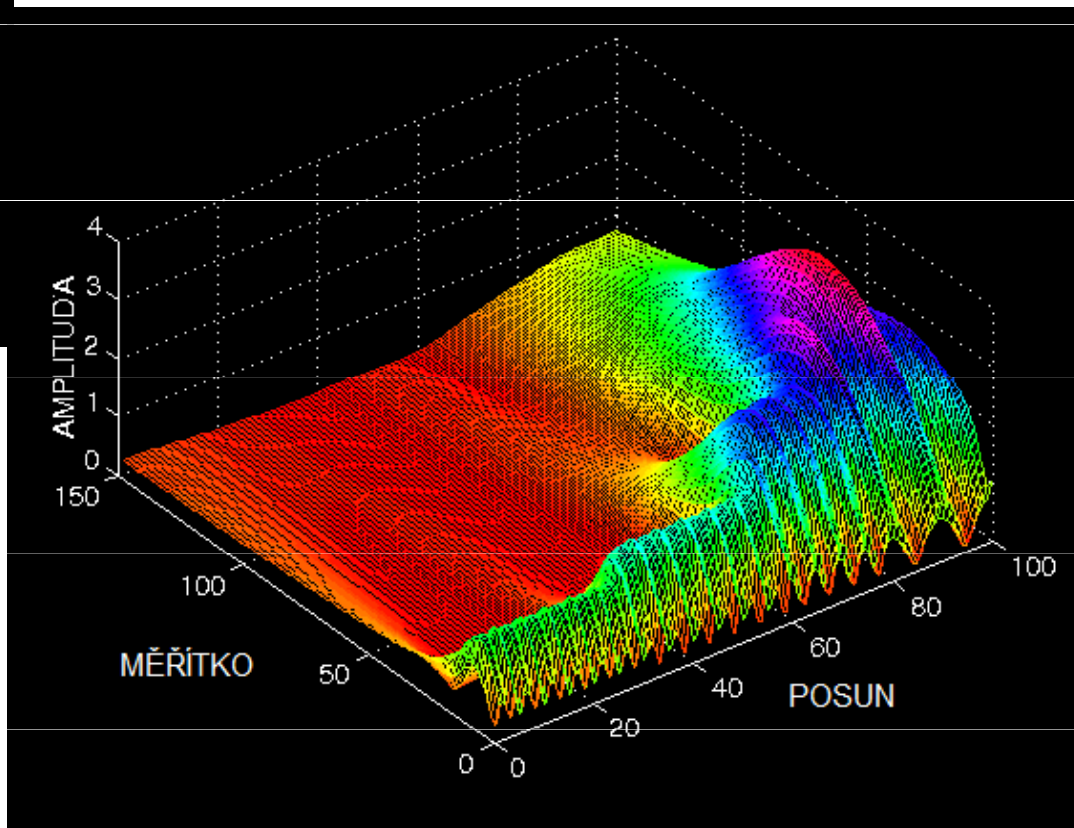
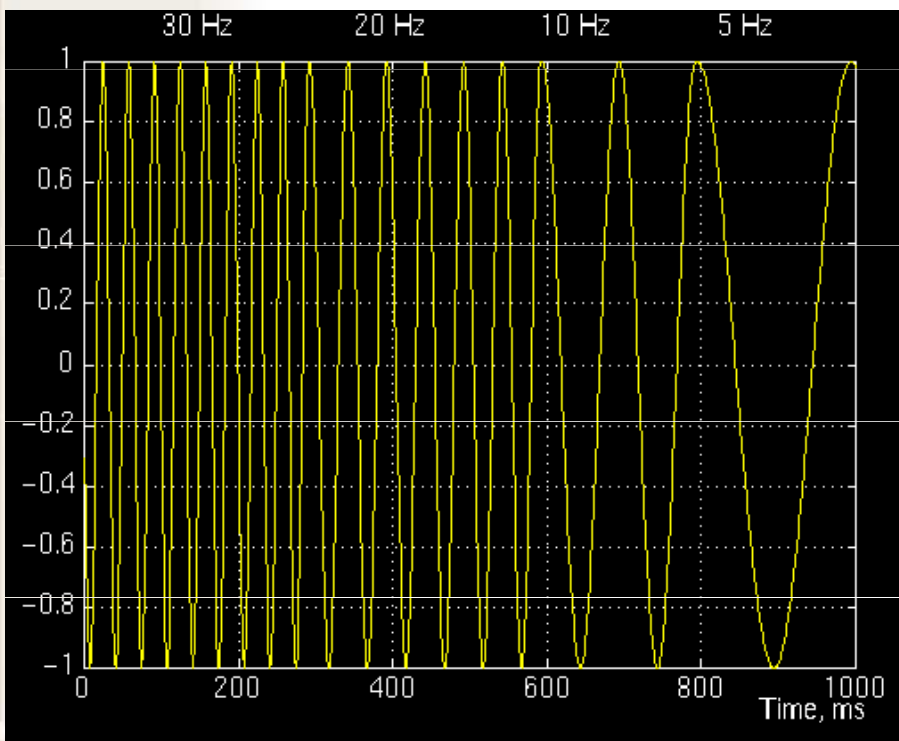
VÝPOČET



VÝPOČET



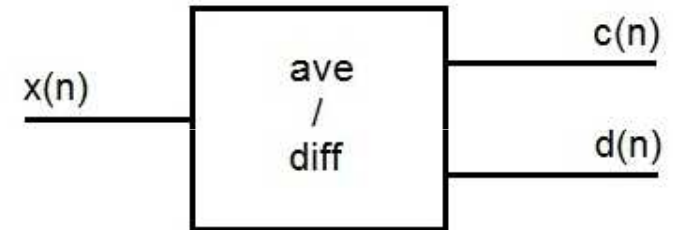
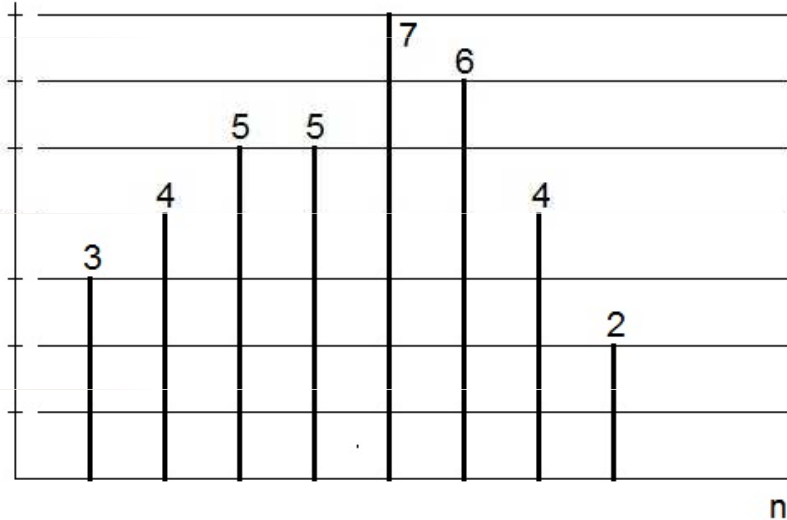
VÝPOČET



VÝPOČET

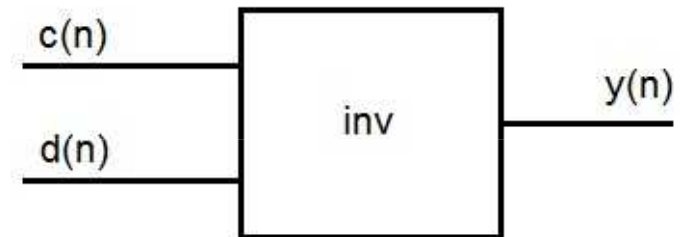
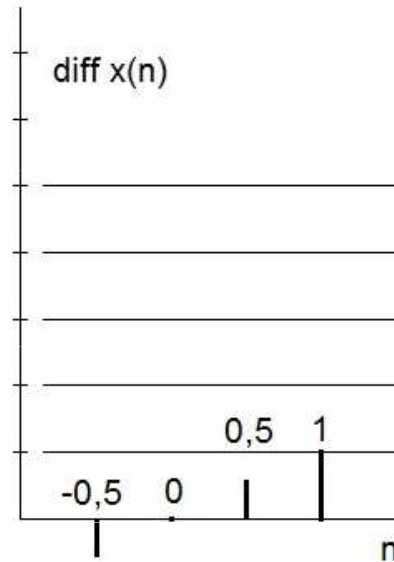
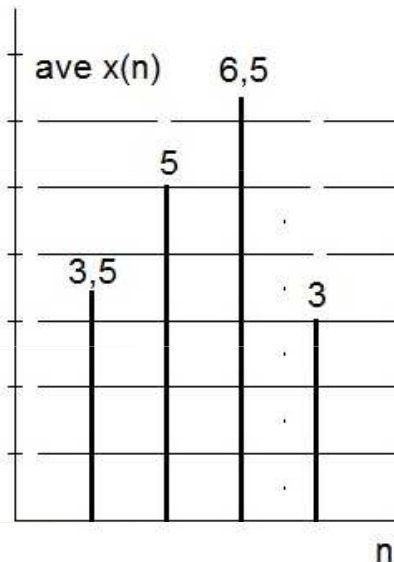
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DISKRÉTNÍ WT



$$c(n) = 0,5 \cdot x(2n) + 0,5 \cdot x(2n+1)$$

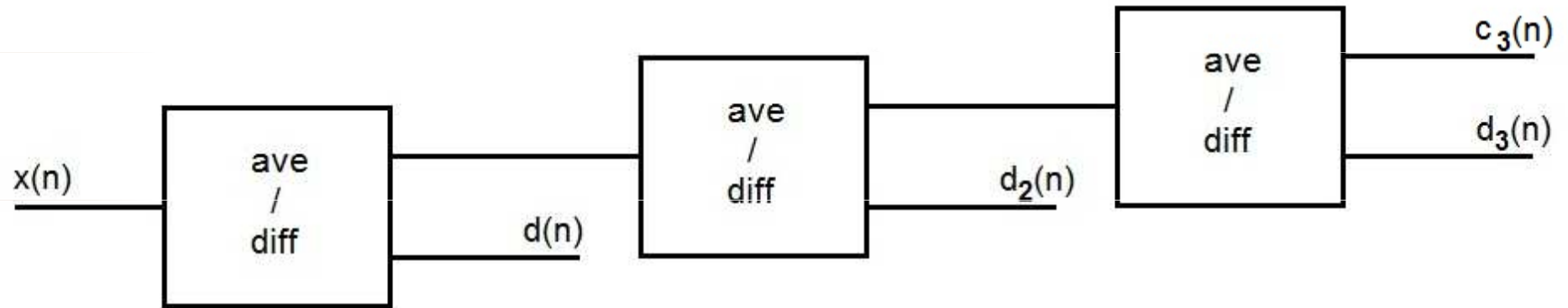
$$d(n) = 0,5 \cdot x(2n) - 0,5 \cdot x(2n+1)$$



$$y(2n) = c(n) + d(n)$$

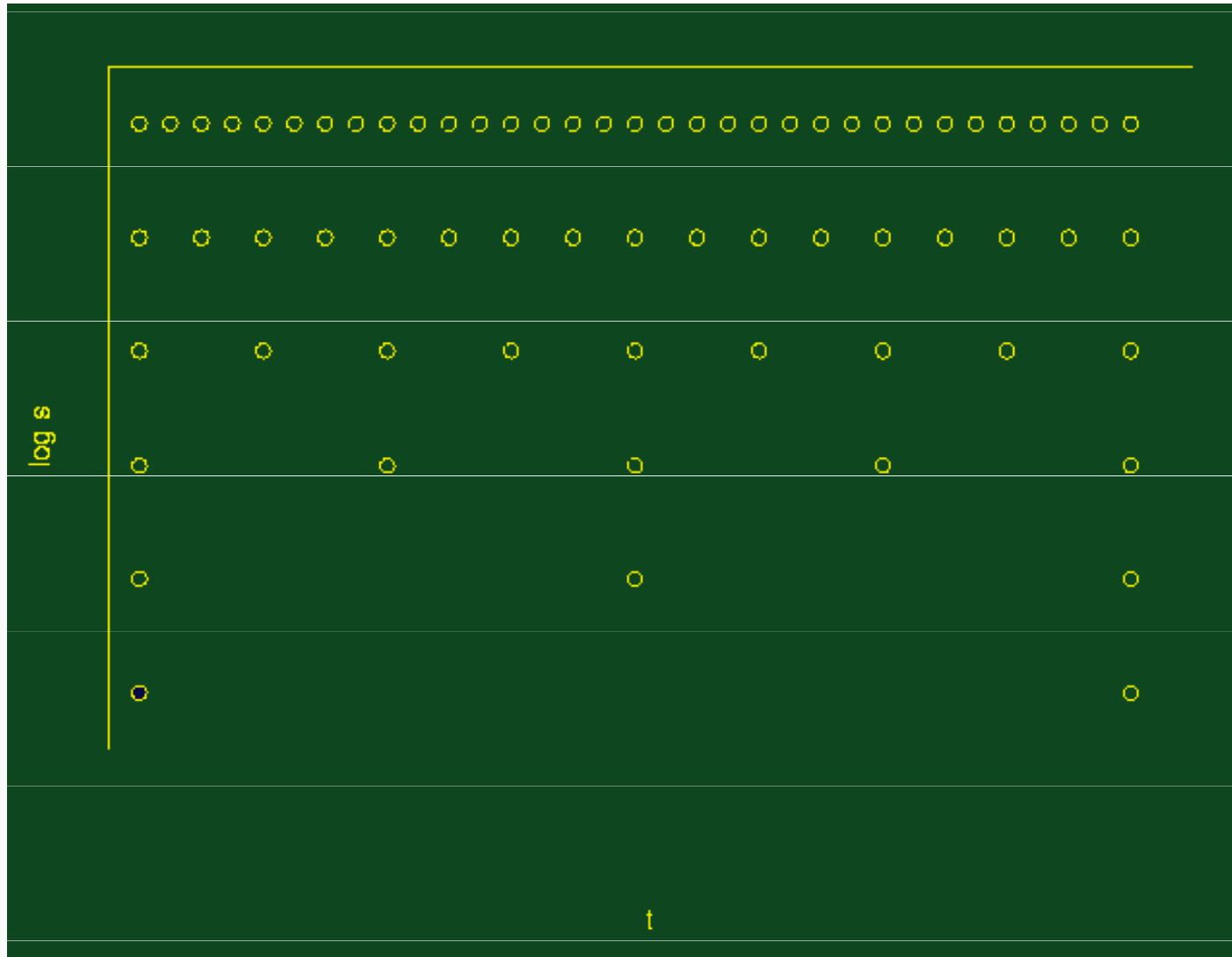
$$y(2n+1) = c(n) - d(n)$$

DISKRÉTNÍ WT

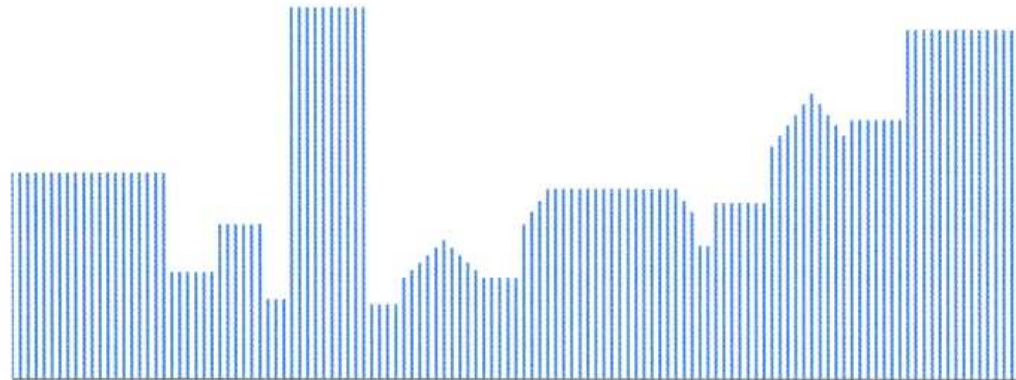


$$\begin{aligned}c_3 &= [4, 5] \\d_3 &= [-0,25] \\d_2 &= [-0,75 \ 1,75] \\d &= [-0,5 \ 0 \ 0,5 \ 1]\end{aligned}$$

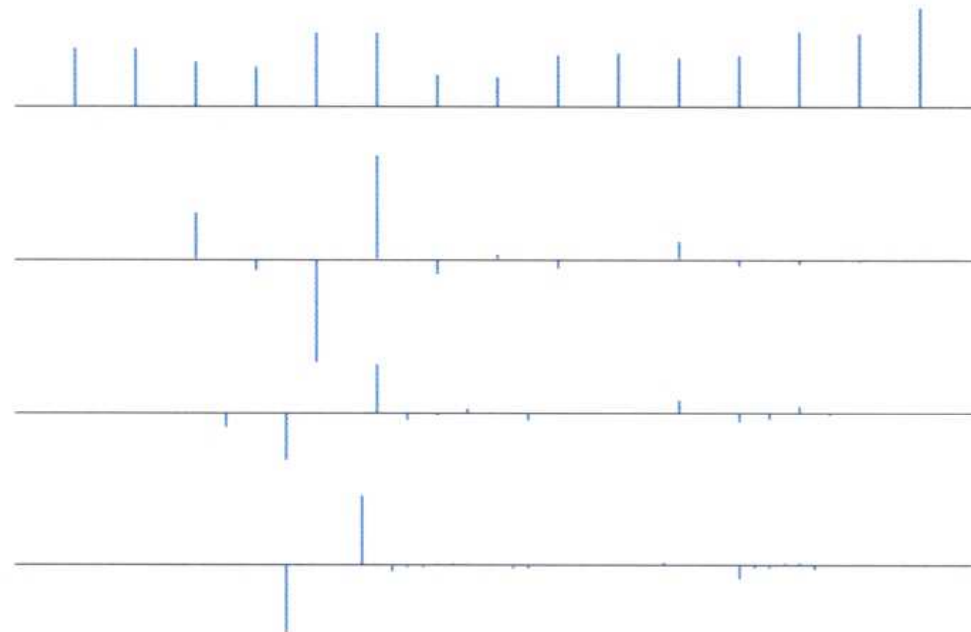
DISKRETIZACE



DISKRÉTNÍ WT



3 úrovnňová
Haarova
transformace



DISKRÉTNÍ WT

$$c(n) = h_0x(2n) + h_1x(2n+1) + h_2x(2n+2) + h_3x(2n+3)$$

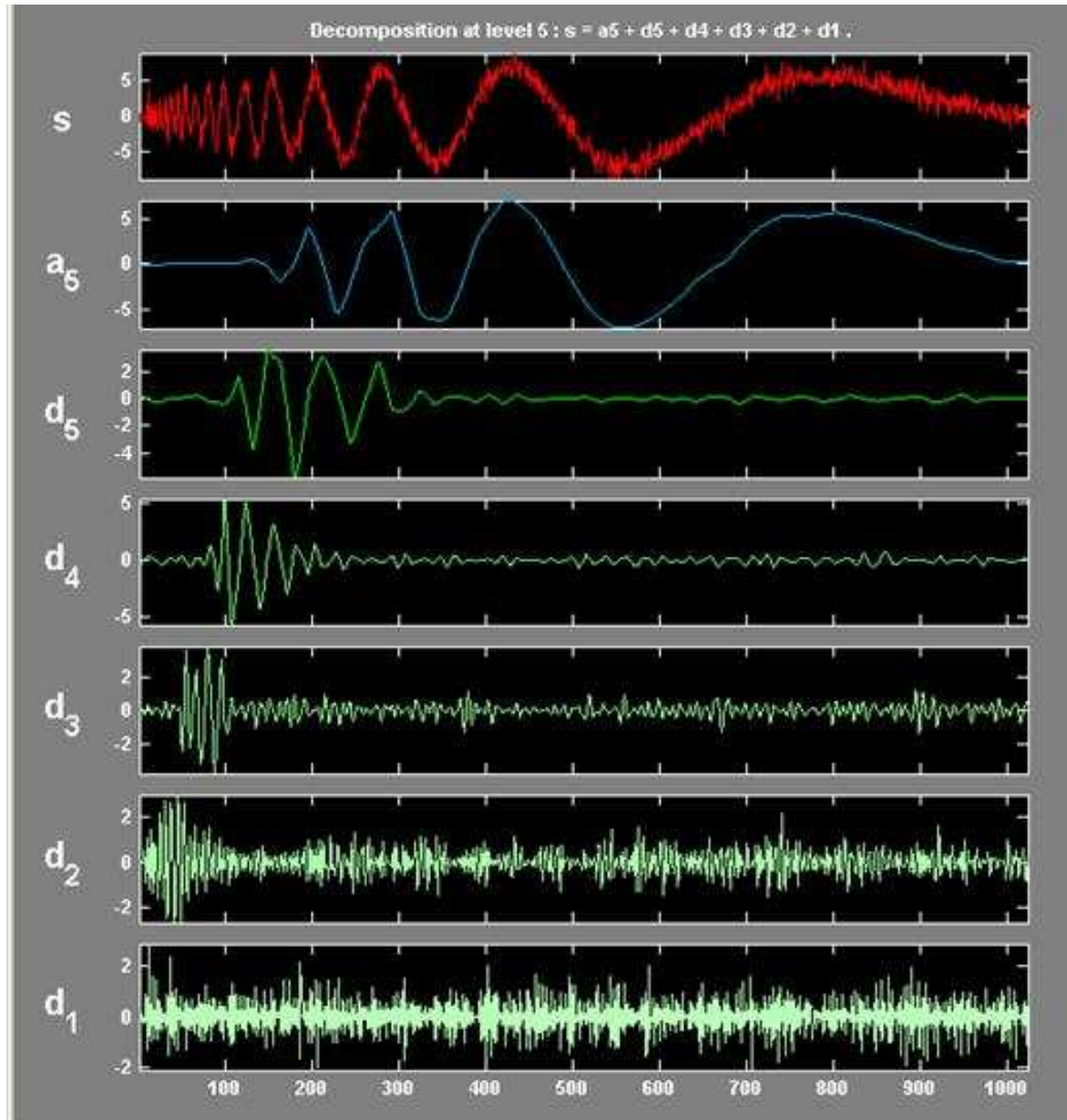
$$d(n) = h_3x(2n) - h_2x(2n+1) + h_1x(2n+2) - h_0x(2n+3)$$

$$h_0 = \frac{1 + \sqrt{3}}{4\sqrt{2}}; \quad h_1 = \frac{3 + \sqrt{3}}{4\sqrt{2}}; \quad h_2 = \frac{3 - \sqrt{3}}{4\sqrt{2}}; \quad h_3 = \frac{1 - \sqrt{3}}{4\sqrt{2}};$$

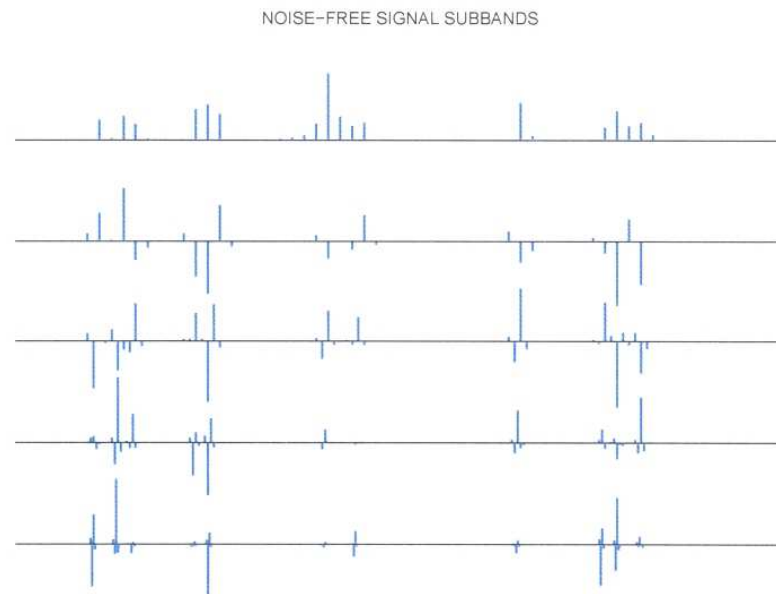
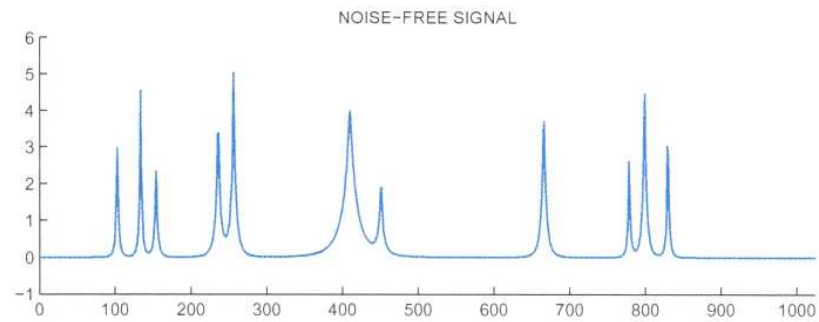
$$y(2n) = h_0c(n) + h_2c(n-1) + h_3d(n) + h_1d(n-1)$$

$$y(2n+1) = h_1c(n) + h_3c(n-1) - h_2d(n) - h_0d(n-1)$$

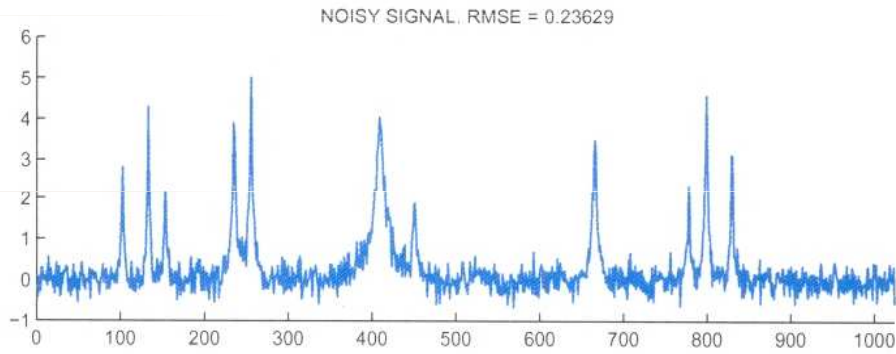
DISKRÉTNÍ WT



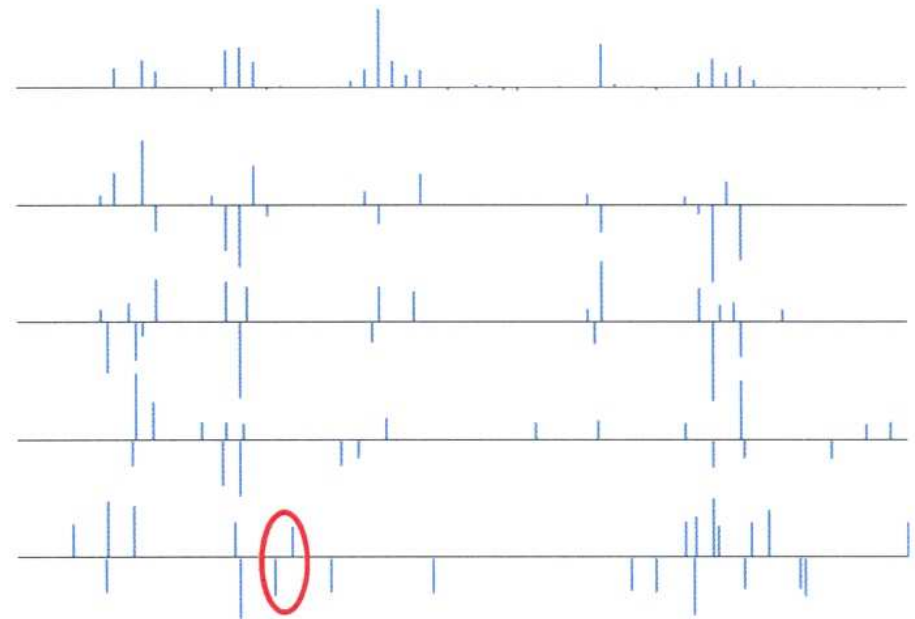
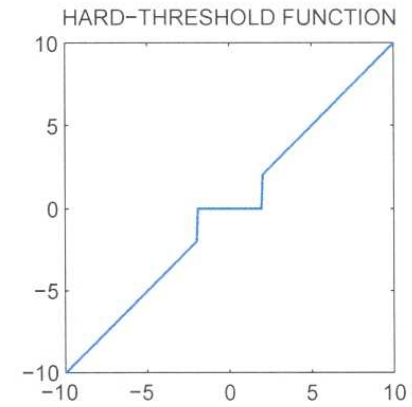
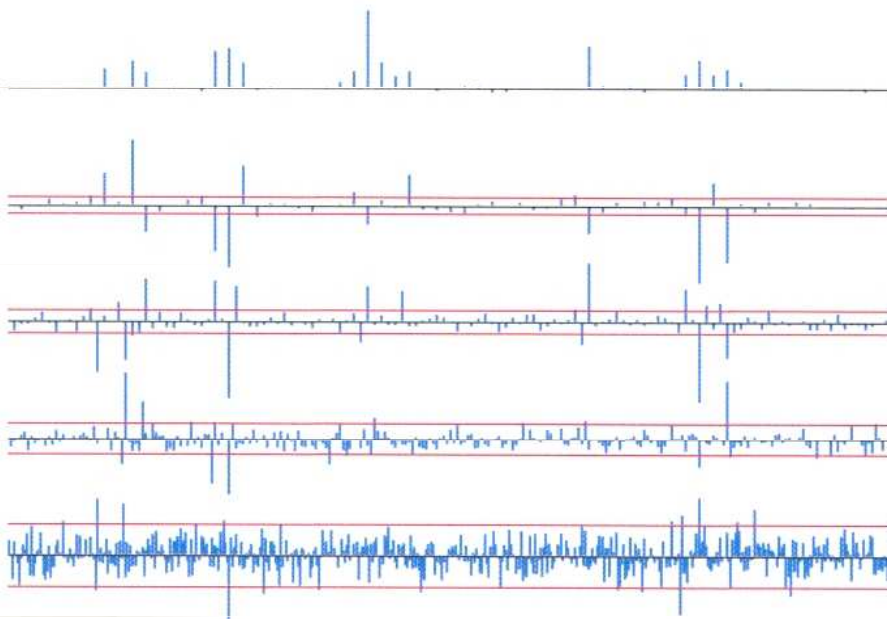
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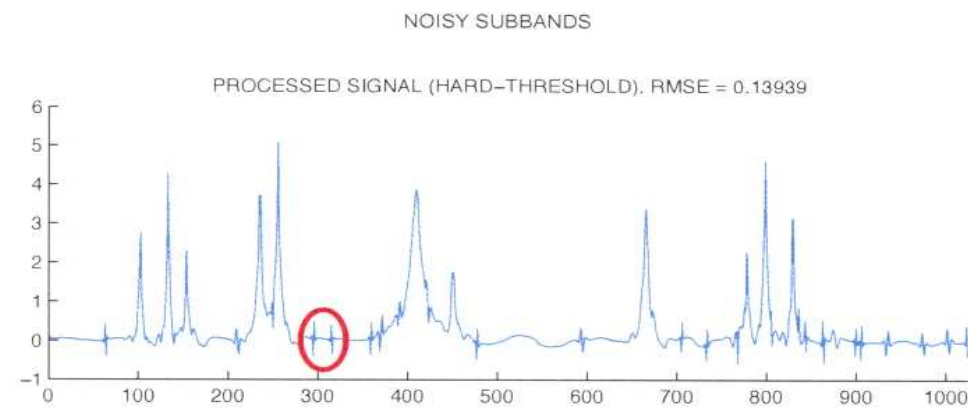
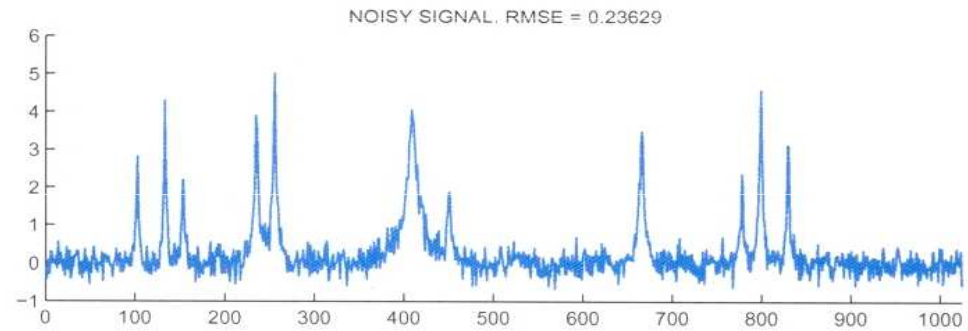
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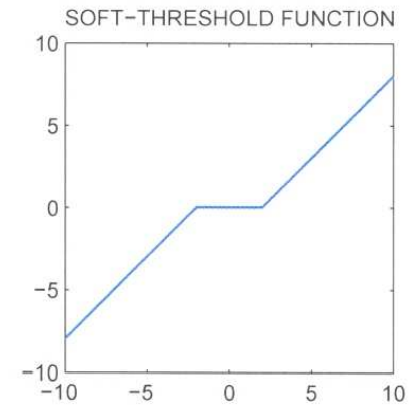
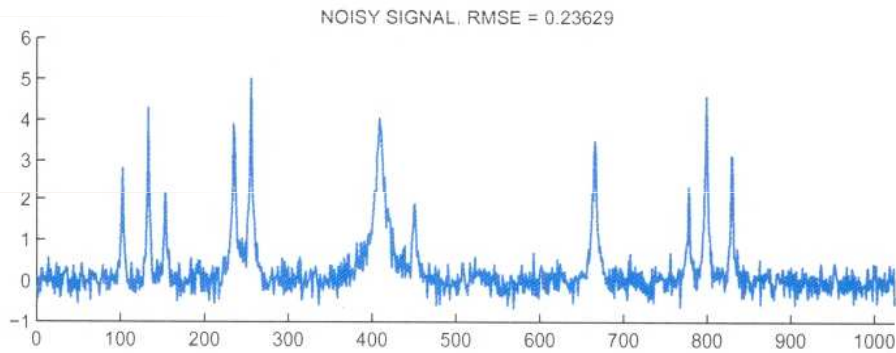
NOISY SUBBANDS



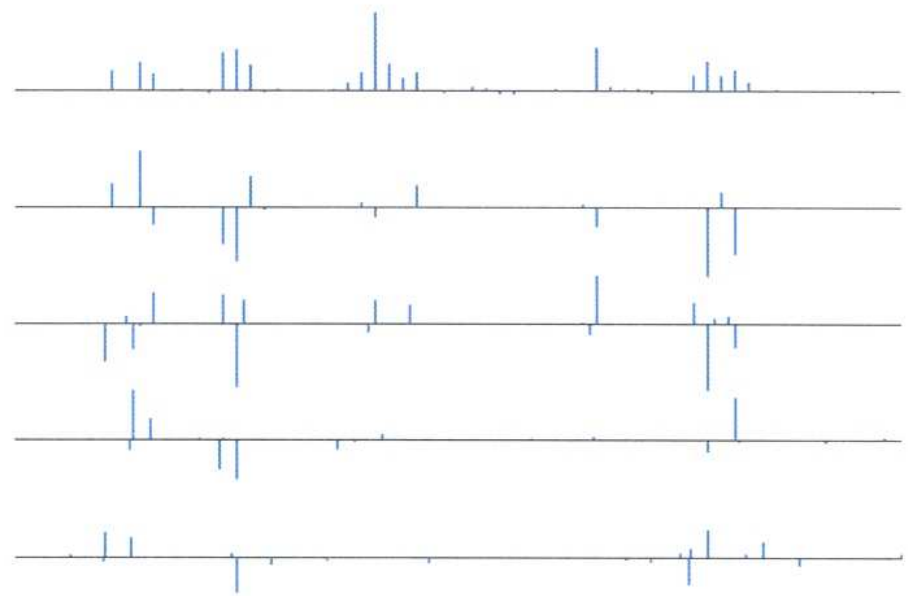
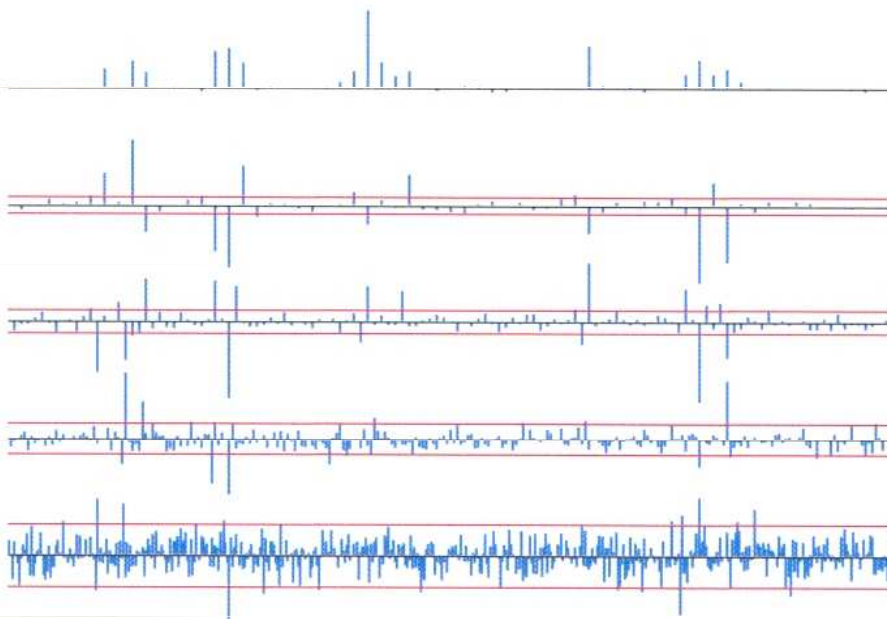
DISKRÉTNÍ WT



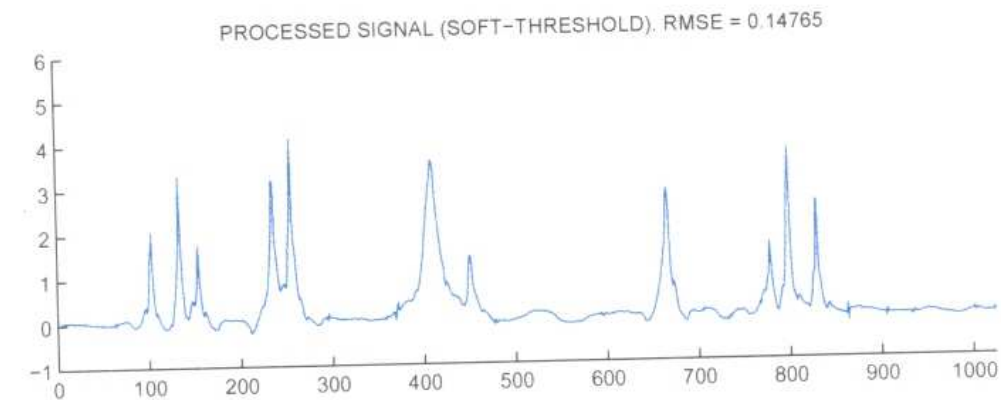
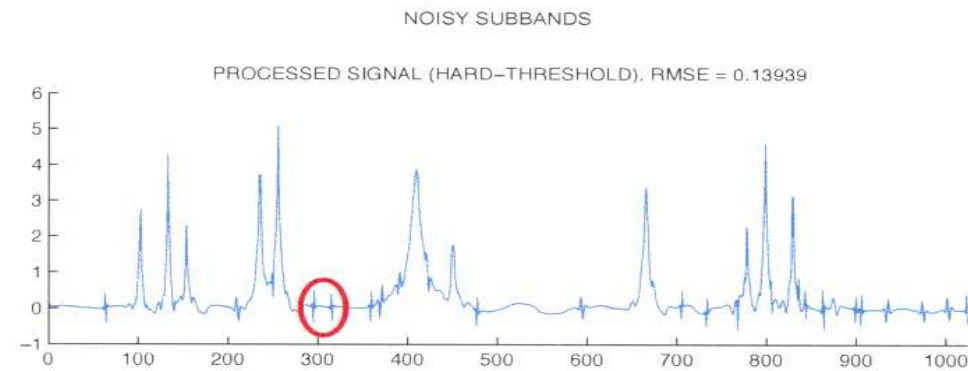
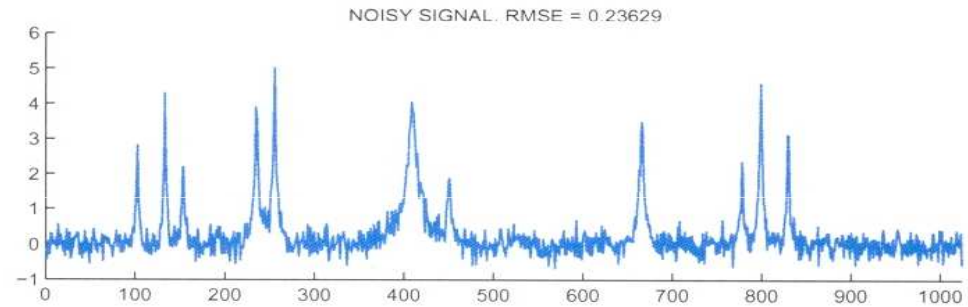
DISKRÉTNÍ WT



NOISY SUBBANDS



DISKRÉTNÍ WT



DISKRÉTNÍ WT - EKG

