

HOMEWORK 1

Example 1. Show that $SS^k = S^{k+1}$. For that consider the map $f : S^k \times I \rightarrow S^{k+1}$:

$$f(x, t) = (\sqrt{1 - (2t - 1)^2}x, 2t - 1)$$

Example 2. Let the pair (X, A) satisfy HEP and let $g : A \rightarrow Y$. Show that $(X \cup_g Y, Y)$ satisfies HEP.

Example 3. Let $f : X \rightarrow Y$.

- (1) Prove that Y is a deformation retract of M_f . In particular describe the retraction $r : M_f \rightarrow Y$
- (2) Given the standard inclusion $i_Y : Y \rightarrow M_f$, prove that $i_Y \circ f \sim i_X$.

Example 4. Show that the cone CX of a space X is homotopy equivalent to a point.