HOMEWORK 5

Exercises $2, 3$ can be foud in Hatcher's book p.133 ex.28. Those, who are interested in homotopy equivalence of pairs might be also interested in ex.27.

Exercise 1. Let $f : S^n \to S^n$ be a map of degree 2. Prove that f has a fixed point (there exists $x \in S^n$ such that f(x) = x).

Hint: Use contradiction nad antipodal map.

Exercise 2. Compute the local homology groups of the following space: take the edges of the tetrahedron (see them for example as $[v_i, v_j], 0 \leq i < j \leq 3$ and we have vertices v_0, v_1, v_2, v_3). Now we add a vertex p into the barycentre, we connect the point with all the other vertices and get edges $[v_i, p]$. To finish, we add 2-simplices $[v_i, v_j, p]\, 0 \leq i < j \leq 3$.

Compute the local homotopy groups of this space.

Exercise 3. We denote the space from the previous example X. We define $\partial X =$ set of points x of X such that $H_n(X, X - \{x_0\}) = 0$. Compute the local homology groups of ∂X.

Exercise 4. Let space X have the following (reduced) homology groups:

$$\begin{aligned} \bar{H}_1(X) &= \mathbb{Z} \oplus \mathbb{Z}_2 \\ \bar{H}_{10}(X) &= \mathbb{Z}^{2013} \oplus \mathbb{Z}_{42} \\ \bar{H}_i(X) &= 0, i \notin \{1, 10\} \end{aligned}$$

Create two spaces Y, Z such that they have the same homology groups as X and such that Y is not homeomorphic to Z and prove that. Hint: Ignore $\bar{H}_{10}$, and think of ways how to glue D^2 to $S^1 \vee S^1$.