# HOMEWORK 6

(1) Compute the homology and cohomology groups with coefficiets in $\mathbb{Z}, \mathbb{Z}/2\mathbb{Z}, \mathbb{Z}/5\mathbb{Z}$ for the Klein bottle and space with genus $n$ (n–holes).

(2) What space do we get when we make a hole in torus and glue Möbius band in it? Compute its cohomology with coefficients in $\mathbb{Z}/2\mathbb{Z}$.

(3) Give an example of two spaces CW–complexes $X, Y$ that have nontrivial homology groups in dimesion 2 (with coeffs in $\mathbb{Z}$) such that:
  (a) $H_*(X, \mathbb{Z}) \cong H_*(Y, \mathbb{Z})$ and $H_*(X, \mathbb{Z}_2) \not\cong H_*(Y, \mathbb{Z}_2)$.
  (b) $H_*(X, \mathbb{Z}_2) \cong H_*(Y, \mathbb{Z}_2)$ and $H_*(X, \mathbb{Z}) \not\cong H_*(Y, \mathbb{Z})$.

(4) Let us take the following exact sequence $C_*$ where all maps are 0:

$$\to \mathbb{Z}_n \to \cdots \to \mathbb{Z}_3 \to \mathbb{Z}_2 \to 0 \to 0$$

Compute its homology and cohomology groups with coefficients in $\mathbb{Z}, \mathbb{Z}_p$ for $p \in \mathbb{N}$.

---