Homework number 3

Do two from the following three exercises.

Exercise 7. Consider a map $f : S^n \to X$. Prove that the cone Cf of the map f is homeomorphic to the space $D^{n+1} \cup_f X$.

Exercise 8. Prove that $H_k(D^n, S^{n-1}) \cong \bar{H}_{k-1}(S^{n-1})$.

Exercise 9. From the long exact sequence for the triple $(\Delta^n, \delta\Delta^n, \delta\Delta^n - \text{one face})$ and the excision theorem derive that

$$H_k(\Delta^n, \delta\Delta^n) \cong H_{k-1}(\Delta^{n-1}, \delta\Delta^{n-1}).$$

Show by induction that the singular simplex $\mathrm{id} : \Delta^n \to \Delta^n$ is a cycle in $C_n(\delta^n, \delta\Delta^n)$ the homology class of which determines a generator of $H_n(\Delta^n, \delta\Delta^n) \cong \mathbb{Z}$.