Exercise sessions 10

Course: Mathematical methods in Economics

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Problem 1 (Prisoner's dilemma)

You are given the following one-shot game: $(L, L) = (2, 2)$, $(L, R) = (0, 5)$, $(R, L) = (5, 0)$, $(R, R) = (1, 1)$. Find the Nash equilibrium. Is it the maximum payoffs the players can get? Draw the extensive from of the game.

Problem 2 (Cournot quantity competition)

Given the demand function $p(q)$ and marginal costs equal to c, firms simultaneously decide on quantities, q_1 and q_2.

1. Define the Nash equilibrium.
2. Show that the resulting price $p^* > p^{pc}$, where p^{pc} is the perfect competition price.
3. Show that $q_1^* + q_2^* > q^m$, where q^m is the monopoly quantity.
4. For $p(q) = a - q_1 - q_2$ find q_1^*, q_2^*, p^*, π_1^* and π_2^*.

Problem 3 (*entry game*).

A potential producer (entrant) decides whether to enter the market, where the existing producer (incumbent) is already selling. If he does not enter the payoffs of both producers are $(0, 2)$. The incumbent decides whether to oppose the entrance of the new producer ($Fight$) or to agree with it ($Accommodate$). If he fights, the payoffs are $(-3, -1)$, whereas if he accommodates, the payoffs are $(2, 1)$. Draw the normal form of the game and find all Nash equilibria. Which equilibrium does not give a reasonable prediction? Draw the extensive form of the game.

Problem 4 (Stackelberg quantity competition in duopoly)

For the demand function $p(q) = a - q_1 - q_2$ and marginal costs c, let producer 1 be the Stackelberg leader and producer 2 - the Stackelberg follower. Find quantities produced of both producers, the market price and compare them to the Cournot game.

Problem 5 (Bertrand price competition)

Assume competition in prices and differentiated goods. Given that $MC_1 = MC_2 = c$, and the demand function from the previous problem, find the Nash equilibrium.

Problem 6 (linear city model, location choice)

Consumers are uniformly distributed on the unit interval (a beach, for example). Each consumer wants to buy only one unit of a homogeneous good (ice-cream). Ice-cream sellers are located at various points on the beach. All sell at the same price and choose location. A consumer pays linear transportation costs to get to a seller.

1. What are the objectives of consumers and producers?
2. Assume two sellers on the beach. What is their equilibrium location?
3. How does the equilibrium location change for three sellers?

Problem 7 (linear city model, choice of location and then price)

Description of the game is as follows: two firms first choose location simultaneously. Given their location observed, both firms choose prices simultaneously (q and p). After the prices are known, payoffs are revealed. For simplicity assume zero production costs. On top of the price consumers pay the transportation costs tx for x distance travelled.

1. Draw the extensive from of the game.
2. Setup the profit function of each firm, and find the equilibrium prices and market shares.
3. Show that the profit function is decreasing w.r.t. the distance between the firms. Interpret the principle of minimal differentiation.
4. Can the firms undercut each other? Find the undercutting price for each firm.
5. How does the outcome change if the transportation costs are quadratic, tx^2 for x distance traveled? Interpret the principle of maximum differentiation.

Problem 8 (vertical differentiation)

Let $[0, \bar{q}]$ be a range of possible qualities. Suppose that the range of consumers' preference for quality is $[a, b]$. Each consumer buys one single good and has the following utility: $U(\theta) = \theta q - p$, $\theta \in [a, b]$ and $q \in [0, \bar{q}]$.

1. If $p_1 < p_2$, will q_2 be sold?
2. For the remainder of the problem assume that better quality is more expensive. Setup the maximization problem of producer q_1 and find his market share. Under what restrictions on a will q_1 and q_2 to be sold?
3. Setup the maximization problem of producer i. What is the range of qualities sold?

Problem 9 (informative advertising)

Two firms are located at two ends of a linear city of length 1. Consumers are uniformly distributed along the city. If a consumer located at x buys from firm 1, her net utility is $5 - x - p_1$. Similarly, her utility is $5 - (1 - x) - p_1$ if she buys from firm 2. Consumers are unaware of the existence of producers, who can inform them via advertising. The cost of advertising at intensity ϕ is $\frac{a\phi}{2}$, $a > 0$.

1. Find a symmetric equilibrium in price and advertising intensity.
2. Find the socially optimal level of advertising and compare it to the market level.