

Exercise sessions 11

Course: Mathematical methods in Economics

Lecturer: Dmytro Vikhrov

Date: April 30, 2013.

Problem 1 (*Mincerian wage equation*)

An individual with S years of schooling at time t has income $E(S, t)$.

1. Setup the Mincerian wage equation.
2. Linearize the equation and suggest a regression to estimate the return to schooling.

Problem 2 (*education signaling 1*)

An agent is born with a skill type η , $\eta = \{L, H\}$ and she decides whether to obtain education and, if yes, how many units. The cost function is given by $c(\eta, e)$.

1. Impose reasonable restrictions on the cost function. Setup the agent's utility function.
2. Setup the firm's maximization problem.
3. Find the separating equilibrium, in which the employer can perfectly distinguish types. Make sure to describe the out-of-equilibrium beliefs and draw a graph where necessary.
4. Find the pooling equilibrium, in which the employer cannot distinguish types. Make sure to describe the out-of-equilibrium beliefs and draw a graph where necessary.

Problem 3 (*education signaling 2*)

There are two identical firms who would like to hire a worker. There is a single worker in the market whose productivity may be high (H) or low (L). Suppose that the worker's output is 2 if $\eta = H$, and is 1 if $\eta = L$. The worker may signal his actual productivity by investing in education e . The cost of education level e depends on the type η of the worker and is given by $c(\eta, e) = \frac{e}{\eta}$. The utility of the worker is linear in wage net of the education costs.

1. What would be the optimal education level chosen by the worker if the firms knew the actual value of η ?

Now suppose that the worker's productivity is unobservable by firms, but his educational level is. The prior belief of the firms that the worker is H-type is $\lambda \in (0, 1)$. Furthermore, suppose that firms believe that $e \geq \bar{e}$ is a signal of high productivity, while $e < \bar{e}$ is a signal of low productivity.

1. What are the optimal wages under these beliefs?

2. Given your answer above, calculate the educational level that each type of the worker will choose (this will depend on the value of \bar{e}).
3. Find the necessary condition on \bar{e} so that education is an effective signal of productivity (i.e. for what values of e does a separating Perfect Bayesian equilibrium exist?).

Problem 4 (*education signaling 3*)

Find a separating equilibrium in which the worker gets 0 years of education if $\eta \in [0, \eta^*)$ for some $\eta^* \in (0, 1)$ and is offered a low wage w_L , but 4 years of education if $\eta \in [\eta^*, 1]$ and is offered a high wage w_H . Find η^* , w_L , w_H and describe firms' beliefs. If hired, worker's output is 10η and costs of obtaining education are $c(\eta, e) = \frac{e}{\eta}$.

Problem 5

Consider a monopolist that produces a single good whose quality is either high or low, $q = \{H, L\}$. The marginal cost of production is \$4 if $q = H$ and \$3 if $q = L$. Each out of N consumers is willing to pay \$10 for high and \$5 for low quality. Consumers buy a product in the first period, learn its true quality and possess complete information in the second period. Suppose, the firm can advertise for its good on TV. The cost of an ad is \$ A if $q = L$ and \$ αA if $q = H$. The discount rate is β . Find the minimum number of ads that the high monopolist needs to buy to signal its quality.

Problem 6

The setup is similar to Problem 5. In the first period consumers think that $prob(q = H) = \mu$.

1. Find a separating equilibrium, in which the monopolist can signal high quality by price under no advertising.
2. What is the minimum number of ads that the monopolist needs to buy to signal high quality?
3. Compare profits in 1. and 2. Which option does the monopolist choose?