Exercise sessions 12

Course: Mathematical methods in Economics

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Problem 1

An object is to be sold on an auction. There are two risk-neutral bidders with independent private values of the object, v_1 and v_2, distributed according to a continuous distribution function $F(\cdot)$. A bidder with the highest bid gets the object.

1. Find agent's expected utility from bidding r when the agent's private value is v_1.
2. Find the bidding function $b^*(v)$.
3. Show that the bidding function is strictly increasing in v.
4. Obtain the biding functions when 1. $v \sim U[0,1]$ and 2. $v \sim exp(\lambda)$. Plot them on the graph in $(v, b(v))$ locus.

Problem 2

Consider a modification to the setup of Problem 1. The highest bid wins the object and every bidder pays the seller the amount of his bid. Find the bidding function and compare the bidding behavior to Problem 1.

Problem 3

Consider the following modification of a first-price auction. The highest bidder pays his bid but receives the object only if the outcome of the toss of a fair coin is heads. If the outcome is tails, the seller keeps the object and the high bidder's bid. Find the bidding function and compare it to $b^*(v)$ from Problem 1.

Problem 4

In the first-price sealed bid auction with two bidders find the seller's revenue.

Problem 5

Suppose that there are N risk-neutral bidders with independent private valuations. Rederive $b^*(v)$ and seller's revenue. Do the comparative statics exercise.

Problem 6

Introduce the second-price sealed bid auction. Show that bidders bid their true value, $b^{sp}(v) = v$. Compare $b^{sp}(v)$ to $b^*(v)$.

Problem 7

Show the equivalence of expected revenues in the first and second-price sealed-bid auctions.