Exercise session 2

Course: Mathematical methods in Economics

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Date: February 26, 2013.

Problem 1

Consider the constant elasticity of substitution utility function $U(x_1, x_2) = (\beta x_1^\rho + (1-\beta)x_2^\rho)^{\frac{1}{\rho}}$. The elasticity of substitution, defined as $\sigma = \frac{1}{1-\rho}$, is constant. Show that:

1. when $\sigma \to \infty$, $U(x_1, x_2)$ is linear;

2. when $0 < \sigma < \infty$, $U(x_1, x_2)$ is of the Cobb-Douglas form;

3. when $\sigma = 0$, $U(x_1, x_2)$ is of the Leontief form.

Problem 2

For two cases, when the utility is linear $U(x_1, x_2) = ax_1 + bx_2$ and of the Leontief form, $U(x_1, x_2) = min\{ax_1, bx_2\}$, derive the optimal consumption bundles. Consider all cases.

Problem 3

Using the usual budget constraint, derive the Cournot and Engel aggregations.

Problem 4

For the clarity of exposition, assume the following utility function $U(x_1, x_2) = x_1 x_2$. Construct the indirect utility function, $V(p, w)$ and check that the Roy's identity holds.

Problem 5

For the purpose of consistency assume the utility function as in Problem 4. Carefully setup the consumer's minimization problem and find the Hicksian demand. Check its properties and the Shephard's lemma.

Problem 6

1. Discuss the relationship between the indirect utility and the expenditure functions. Check on the example above that it holds.

2. From the relationship between the Hicksian and Marshallian demands derive the Slutsky equation. Define and interpret the income and substitution effects. Draw them for all types of goods.

Application - individual labor supply.

Problem 7

Consider an individual, whose utility is defined over consumption, c and labor l, $U(c,l) = c - \frac{l^{1+\epsilon}}{1+\epsilon}$. Individual's time endowment is l_0 and her non-labor income is m.

1. Explain the elasticity parameter ϵ.
2. Write down individual's budget constraint and draw it in (l, c) locus. Show the location of the indifference curve.
3. Show how the graph changes when:
   - mim / max hours worked is H.
   - wage rate is $\bar{w} > w$ for hours worked $l > H$.
   - per-unit tax τ is imposed on worker's wage.
   - minimum wage rate is $\underline{w}$.

Problem 8

Consider the setup of the above Problem 7. In what follows assume the demand for labor is competitive.

1. Setup the individual's maximization problem. Derive the optimal allocation of consumption and labor, (c^*, l^*), and the highest utility level attained.
2. You are given that the labor demand function is $\left(\frac{A}{w}\right)^{\frac{1}{\mu}}$, where A is a technology parameter and μ is elasticity of demand. Obtain the equilibrium wage rate w^*.
3. Suppose the government introduces the minimum wage rate $\underline{w}$. Consider its effect on the equilibrium employment when $\underline{w} < w^*$ and $\underline{w} > w^*$. Can $\underline{w}$ be a source of inefficiency?

Problem 9

Suppose there is only one monopsonist, who demands labor. His offer is $w^M = \left(\frac{A}{1+\epsilon}\right)^{\frac{\epsilon}{\mu+\epsilon}}$. Consider the effect of introducing a minimum wage $\underline{w} > w^M$.

Problem 10

Consider Card and Krueger (1994) paper, who use a natural experiment to infer the effects of introduction of the minimum wage on the labor market.

1. Describe the underlying assumptions behind the division into treatment and control groups. Define the population of interest in the survey?
2. Discuss the presented estimation results.