

### Exercise session 3

Course: Mathematical methods in Economics

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#### Problem 1

For the clarity of exposition, assume the following utility function  $U(x_1, x_2) = x_1x_2$ . Construct the indirect utility function,  $V(p, w)$  and check that the Roy's identity holds.

#### Problem 2

For the purpose of consistency assume the utility function as in Problem 1. Carefully setup the consumer's minimization problem and find the Hicksian demand. Check its properties and the Shephard's lemma.

#### Problem 3

1. Show the relationship between the indirect utility and expenditure functions.
2. From the relationship between the Hicksian and Marshallian demands derive the Slutsky equation. Define and interpret the income and substitution effects. Draw them for all types of goods.

#### Problem 4 (*application - taxation of labor*)

On a simple supply - demand graph show how a tax on worker,  $\tau_w$ , and a tax on firm,  $\tau_f$  introduces a distortion. How is the division of the tax wedge between firm and worker changes as the labor supply becomes more / less elastic?

**Problem 5** Assume the utility function from the last class, i.e.  $U(c, l) = c - \frac{l^{1+\epsilon}}{1+\epsilon}$ , where  $c$  is consumption and  $l$  is labor.

1. Suppose a lump-sum tax  $T$  is imposed on labor. Show how the optimal labor supply changes.
2. Suppose a per-unit tax  $\tau$  is imposed on worker's wage. Re-derive the optimal allocation. Show how the tax distorts the labor supply.
3. Derive the income and substitution effect of a marginal change in the tax rate  $\tau$ . Provide your interpretation.

#### Problem 6

The tax revenues are collected by the government. Construct the government revenue function. Which tax rate  $\tau^*$  maximizes the government revenue?