Exercise session 5

Course: Mathematical methods in Economics

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Problem 1 *(pure exchange economy)*

Suppose that the economy is populated by two consumers: A and B with respective utility functions: $U^A(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$ and $U^B(x_1, x_2) = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$. Endowments are $(e_1^A, e_2^A) = (1, 0)$ and $(e_1^B, e_2^B) = (0, 1)$.

1. Setup the maximization problem of each consumer and solve it.
2. Setup the market clearing conditions (2 equations, 2 unknowns).
3. Define a competitive equilibrium $(x^*,\ p)$. Is the price vector unique in this equilibrium? Given the price vector, is the allocation for both consumers unique?
4. Draw your findings in the Edgeworth box and provide respective analysis.

Problem 2 *(application of the Second Welfare Theorem)*

Suppose the government wants to achieve a more egalitarian distribution of allocations $(y,\ p)$. Find such transfers that can sustain $(y,\ p)$ as a competitive equilibrium.

Problem 3 *(exchange with uncertainty)*

Farmer's Bernoulli utility function in each state of the is $u(x) = log(x)$, where x if farmer's output. There are two equally likely states of the world: rainy (R) and dry weather (D). In state 1, it rains and those near rivers do poorly and those in the plains do well. In state 2, it is dry and those near rivers do well, while those in the plains do poorly. There are two classes of farmers: river dwellers A are endowed with output 1 in the rainy state and 2 in the dry state; plain dweller B are endowed with output 2 in the rainy state and 1 in the dry state.

1. Express each individual's utility function over consumption in the two states.
2. Now let there be a market for weather insurance. What will be the price of consumption in rainy weather if the price of consumption in the dry weather is normalized to 1? What are the equilibrium consumption levels? Is this outcome efficient? Illustrate all findings in the Edgeworth box.
3. Repeat your analysis for the case when the plain dwellers B are endowed with 3 units of output in the rainy state instead. How do your findings differ from point 2 above? Give intuitive interpretation.

Problem 4 (*externalities*)

The lake is freely accessed by fishermen. The cost of sending a boat out on the lake is $r > 0$. When b boats are sent out onto the lake $f(b)$ fish are caught in total, where $f'(b) > 0$ and $f''(b) < 0$ at all $b \geq 0$. The price of fish is $p > 0$, which is unaffected by the level of the catch from lake.

1. If each individual decides on his / her own whether to send a boat, many boats will be sent in total?

2. If the lake is owned by a company, how many boats will be allowed to fish?

3. Define a tax that would equalize the amounts in 1. and 2.

Problem 5 (*public good*)

Consider a two-consumer economy in which there are two types of goods: a private good and a public good. The two consumers have identical preferences given by

$$u(x_i, g) = x_i + 4log(g)$$

where x_i is the consumer i's consumption of the private good, i = 1, 2, and g is the quantity of the public good. The total amount of the public good g is determined by the sum of the individual contributions, i.e., $g = g_1 + g_2$. Each consumer is endowed with 10 units of the private good that can be converted into the public good in a 1-for-1 fashion.

1. What is the Pareto efficient amount of the public good?

2. Suppose that the public good is provided by private contributions. What is the equilibrium amount of the public good and how much does each consumer contribute?

3. Compare the equilibrium amount to the Pareto efficient amount. Do they differ and why?