Exercise sessions 8 - 9

Course: Mathematical methods in Economics

Lecturer: Dmytro Vikhrov

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Problem 1

Suppose there are two countries A and B with respective endowments of capital and labor: K^A, L^A and K^B, L^B. A single good is produced competitively with a constant returns to scale technology $F(K, L)$ that satisfies the neoclassical assumptions. Derive the rental rates r^A, r^B and the wage rates w^A, w^B. Under what conditions do owners of capital wish to reallocate their capital? Under what conditions do individual workers wish to move?

Problem 2

Assume that country A is capital abundant and factor mobility is allowed. Define country A's welfare function as the income that accrues to its production factors. Consider now two policy experiments.

1. Country A liberalizes the export of capital but bans the import of labor. Write down the welfare of country A.

2. Country A liberalizes the import of labor but bans the export of capital. Write down the welfare of country A.

3. Show that the labor importing policy of country A is welfare superior to the capital exporting experiment. Which policy is preferred by Country B?

Problem 3

For $F(K, L) = K^{\alpha}L^{1-\alpha}$ address the questions below.

1. How much capital or labor will move in the competitive outcome (result from Problem 1)? First assume zero movement costs, then suggest a distribution to proxy the costs.

2. Using the welfare function defined in Problem 2, derive the welfare maximizing level of factor imports for country A.

3. In analogous manner write down the welfare of country B. What volumes of production factors does it wish to export to maximize its welfare?

4. Assume the social planner cares about the world welfare. Derive the volume of production factors that change location.

5. Compare results in 1 – 4.

Problem 4 (gravity model)

Suppose there are N countries in the world. Each country produces competitively only one good which it uses to trade with the remaining $(N-1)$ countries. Consumers in country j have CES preferences defined over all goods:

$$U(m_{1j},...,m_{Nj}) = \left[\sum_{i}^{N} \beta_i^{(1-\sigma)/\sigma} m_{ij}^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)},$$

where i is the country of origin of the good.

1. Setup the consumer maximization problem and solve for optimal m_{ij}^*.
2. Derive country j's price index. Introduce bilateral trade barriers and costs. Interpret your findings.
3. Log-linearize the demand and suggest a regression specification for the cross-section data.
4. Extend the regression specification to account for the panel structure of data. Suggest an estimator.
5. Discuss possible issues (e.g. selection, measurement error in variables, omitted variables) that might arise while estimating the log-linearized equation.