Chapter 9

Auctions and Mechanism Design

In most real-world markets, sellers do not have perfect knowledge of market demand. Instead, sellers typically have only statistical information about market demand. Only the buyers themselves know precisely how much of the good they are willing to buy at a particular price. In this chapter, we will revisit the monopoly problem under this more typical circumstance.

Perhaps the simplest situation in which the above elements are present occurs when a single object is put up for auction. There, the seller is typically unaware of the buyers' values but may nevertheless have some information about the distribution of values across buyers. In such a setting, there are a number of standard auction forms that the seller might use to sell the good – first-price, second-price, Dutch, English. Do each of these standard auctions raise the same revenue for the seller? If not, which is best? Is there a non-standard yet even better selling mechanism for the seller? To answer these and other questions, we will introduce and employ some of the tools from the theory of *mechanism design*.

Mechanism design is a general theory about how and when the design of appropriate institutions can achieve particular goals. This theory is especially germane when the designer requires information possessed only by others to achieve his goal. The subtlety in designing a successful mechanism lies in ensuring that the mechanism gives those who possess the needed information the incentive to reveal it to the designer. This chapter provides an introduction to the theory of mechanism design. We shall begin by considering the problem of designing a revenue-maximising selling mechanism. We then move on to the problem of efficient resource allocation. In both cases, the design problem will be subject to informational constraints – the agents possessing private information will have to be incentivised to report their information truthfully.

9.1 The Four Standard Auctions

Consider a seller with a single object for sale who wishes to sell the object to one of N buyers for the highest possible price. How should the seller go about achieving this goal? One possible answer is to hold an auction. Many distinct auctions have been put to use at one time or another, but we will focus on the following four standard auctions.[1]

[1] We shall assume throughout and unless otherwise noted that in all auctions ties in bids are broken at random: each tied bidder is equally likely to be deemed the winner.

- **First-Price, Sealed-Bid:** Each bidder submits a sealed bid to the seller. The highest bidder wins and pays his bid for the good.
- **Second-Price, Sealed-Bid:** Each bidder submits a sealed bid to the seller. The highest bidder wins and pays the second-highest bid for the good.
- **Dutch Auction:** The seller begins with a very high price and begins to reduce it. The first bidder to raise his hand wins the object at the current price.
- **English Auction:** The seller begins with very low price (perhaps zero) and begins to increase it. Each bidder signals when he wishes to drop out of the auction. Once a bidder has dropped out, he cannot resume bidding later. When only one bidder remains, he is the winner and pays the current price.

Can we decide even among these four which is best for the seller? To get a handle on this problem, we must begin with a model.

9.2 THE INDEPENDENT PRIVATE VALUES MODEL

A single risk-neutral seller wishes to sell an indivisible object to one of N risk-neutral buyers. The seller values the object at zero euros.[2] Buyer i's value for the object, v_i, is drawn from the interval $[0, 1]$ according to the distribution function $F_i(v_i)$ with density function $f_i(v_i)$.[3] We shall assume that the buyers' values are mutually independent. Each buyer knows his own value but not the values of the other buyers. However, the density functions, $f_1, \ldots, f_N$, are public information and so known by the seller and all buyers. In particular, while the seller is unaware of the buyers' exact values, he knows the distribution from which each value is drawn. If buyer i's value is v_i, then if he wins the object and pays p, his payoff (i.e., von Neumann-Morgenstern utility) is $v_i - p$, whereas his payoff is $-p$ if he must pay p but does not win the object.[4]

This is known as the 'independent, private values' model. **Independent** refers to the fact that each buyer's *private information* (in this case, each buyer's value) is independent of every other buyer's private information. **Private value** refers to the fact that once a buyer employs his own private information to assess the value of the object, this assessment would be unaffected were he subsequently to learn any other buyer's private information, i.e., each buyer's *private* information is sufficient for determining his *value*.[5]

Throughout this chapter, we will assume that the setting in which our monopolist finds himself is well-represented by the independent private values model. We can now

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[2]This amounts to assuming that the object has already been produced and that the seller's use value for it is zero.

[3]Recall that $F_i(v_i)$ denotes the probability that i's value is less than or equal to v_i, and that $f_i(v_i) = F_i'(v_i)$. The latter relation can be equivalently expressed as $F_i(v_i) = \int_0^{v_i} f_i(x)dx$. Consequently, we will sometimes refer to f_i and sometimes refer to F_i since each one determines the other.

[4]Although such an outcome is not possible in any one of the four auctions above, there are other auctions (i.e., all-pay auctions) in which payments must be made whether or not one wins the object.

[5]There are more general models in which buyers with private information would potentially obtain yet *additional* information about the value of the object were they to learn *another buyer's* private information, but we shall not consider such models here.

begin to think about how the seller's profits vary with different auction formats. Note that with the production decision behind him and his own value equal to zero, profit-maximisation is equivalent to revenue-maximisation.

Before we can determine the seller's revenues in each of the four standard auctions, we must understand the bidding behaviour of the buyers across the different auction formats. Let us start with the first-price auction.

9.2.1 BIDDING BEHAVIOUR IN A FIRST-PRICE, SEALED-BID AUCTION

To understand bidding behaviour in a first-price auction, we shall, for simplicity, assume that the buyers are ex ante symmetric. That is, we shall suppose that for all buyers $i = 1, \ldots, N$, $f_i(v) = f(v)$ for all $v \in [0, 1]$.

Clearly, the main difficulty in determining the seller's revenue is in determining how the buyers, let us agree to call them *bidders* now, will bid. But note that if you are one of the bidders, then because you would prefer to win the good at a lower price rather than a higher one, you will want to bid low when the others are bidding low and you will want to bid higher when the others bid higher. Of course, you do not know the bids that the others submit because of the sealed-bid rule. Yet, *your optimal bid will depend on how the others bid.* Thus, the bidders are in a strategic setting in which the optimal action (bid) of each bidder depends on the actions of others. Consequently, to determine the behaviour of the bidders, we shall employ the game theoretic tools developed in Chapter 7.

Let us consider the problem of how to bid from the point of view of bidder i. Suppose that bidder i's value is v_i. Given this value, bidder i must submit a sealed bid, b_i. Because b_i will in general depend on i's value, let us write $b_i(v_i)$ to denote bidder i's bid when his value is v_i. Now, because bidder i must be prepared to submit a bid $b_i(v_i)$ for each of his potential values $v_i \in [0, 1]$, we may view bidder i's **strategy** as a *bidding function* $b_i\colon [0, 1] \to \mathbb{R}_+$, mapping each of his values into a (possibly different) non-negative bid.

Before we discuss payoffs, it will be helpful to focus our attention on a natural class of bidding strategies. It seems very natural to expect that bidders with higher values will place higher bids. So, let us restrict attention to *strictly increasing* bidding functions. Next, because the bidders are ex ante symmetric, it is also natural to suppose that bidders with the same value will submit the same bid. With this in mind, we shall focus on finding a strictly increasing bidding function, $\hat{b}\colon [0, 1] \to \mathbb{R}_+$, that is optimal for each bidder to employ, given that all other bidders employ this bidding function as well. That is, we wish to find a symmetric Nash equilibrium in strictly increasing bidding functions.

Now, let us suppose that we find a symmetric Nash equilibrium given by the strictly increasing bidding function $\hat{b}(\cdot)$. By definition it must be payoff-maximising for a bidder, say i, with value v to bid $\hat{b}(v)$ given that the other bidders employ the same bidding function $\hat{b}(\cdot)$. Because of this, we can usefully employ what may at first appear to be a rather mysterious exercise.

The mysterious but useful exercise is this: imagine that bidder i cannot attend the auction and that he sends a friend to bid for him. The friend knows the equilibrium bidding function $\hat{b}(\cdot)$, but he does not know bidder i's value. Now, if bidder i's value is v, bidder i would like his friend to submit the bid $\hat{b}(v)$ on his behalf. His friend can do this for him once bidder i calls him and tells him his value. Clearly, bidder i has no incentive to lie to his friend about his value. That is, among all the values $r \in [0, 1]$ that bidder i with value v can report to his friend, his payoff is maximised by reporting his true value, v, to his friend. This is because reporting the value r results in his friend submitting the bid $\hat{b}(r)$ on his behalf. But if bidder i were there himself he would submit the bid $\hat{b}(v)$.

Let us calculate bidder i's expected payoff from reporting an arbitrary value, r, to his friend when his value is v, given that all other bidders employ the bidding function $\hat{b}(\cdot)$. To calculate this expected payoff, it is necessary to notice just two things. First, bidder i will win only when the bid submitted for him is highest. That is, when $\hat{b}(r) > \hat{b}(v_j)$ for all bidders $j \neq i$. Because $\hat{b}(\cdot)$ is strictly increasing this occurs precisely when r exceeds the values of all $N - 1$ other bidders. Letting F denote the distribution function associated with f, the probability that this occurs is $(F(r))^{N-1}$ which we will denote $F^{N-1}(r)$. Second, bidder i pays only when he wins and he then pays his bid, $\hat{b}(r)$. Consequently, bidder i's expected payoff from reporting the value r to his friend when his value is v, given that all other bidders employ the bidding function $\hat{b}(\cdot)$, can be written

$$u(r, v) = F^{N-1}(r)(v - \hat{b}(r)). \tag{9.1}$$

Now, as we have already remarked, because $\hat{b}(\cdot)$ is an equilibrium, bidder i's expected payoff-maximising bid when his value is v must be $\hat{b}(v)$. Consequently, (9.1) must be maximised when $r = v$, i.e., when bidder i reports his true value, v, to his friend. So, if we differentiate the right-hand side with respect to r, the resulting derivative must be zero when $r = v$. Differentiating yields

$$\frac{dF^{N-1}(r)(v - \hat{b}(r))}{dr} = (N-1)F^{N-2}(r)f(r)(v - \hat{b}(r)) - F^{N-1}(r)\hat{b}'(r). \tag{9.2}$$

Evaluating the right-hand side at $r = v$, where it is equal to zero, and rearranging yields,

$$(N-1)F^{N-2}(v)f(v)\hat{b}(v) + F^{N-1}(v)\hat{b}'(v) = (N-1)vf(v)F^{N-2}(v). \tag{9.3}$$

Looking closely at the left-hand side of (9.3), we see that it is just the derivative of the product $F^{N-1}(v)\hat{b}(v)$ with respect to v. With this observation, we can rewrite (9.3) as

$$\frac{dF^{N-1}(v)\hat{b}(v)}{dv} = (N-1)vf(v)F^{N-2}(v). \tag{9.4}$$

Now, because (9.4) must hold for every v, it must be the case that

$$F^{N-1}(v)\hat{b}(v) = (N-1)\int_0^v xf(x)F^{N-2}(x)dx + \text{constant}.$$

Noting that a bidder with value zero must bid zero, we conclude that the constant above must be zero. Hence, it must be the case that

$$\hat{b}(v) = \frac{N-1}{F^{N-1}(v)} \int_0^v x f(x) F^{N-2}(x) dx,$$

which can be written more succinctly as

$$\hat{b}(v) = \frac{1}{F^{N-1}(v)} \int_0^v x dF^{N-1}(x). \tag{9.5}$$

There are two things to notice about the bidding function in (9.5). First, as we had assumed, it is strictly increasing in v (see Exercise 9.1). Second, it has been uniquely determined. Hence, in conclusion, we have proven the following.

THEOREM 9.1 ***First-Price Auction Symmetric Equilibrium***

If N bidders have independent private values drawn from the common distribution, F, then bidding

$$\hat{b}(v) = \frac{1}{F^{N-1}(v)} \int_0^v x dF^{N-1}(x)$$

whenever one's value is v constitutes a symmetric Nash equilibrium of a first-price, sealed-bid auction. Moreover, this is the only symmetric Nash equilibrium.[6]

EXAMPLE 9.1 Suppose that each bidder's value is uniformly distributed on $[0, 1]$. Then $F(v) = v$ and $f(v) = 1$. Consequently, if there are N bidders, then each employs the bidding function

$$\begin{aligned}
\hat{b}(v) &= \frac{1}{v^{N-1}} \int_0^v x dx^{N-1} \\
&= \frac{1}{v^{N-1}} \int_0^v x(N-1)x^{N-2} dx \\
&= \frac{N-1}{v^{N-1}} \int_0^v x^{N-1} dx \\
&= \frac{N-1}{v^{N-1}} \frac{1}{N} v^N \\
&= v - \frac{v}{N}.
\end{aligned}$$

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[6]Strictly speaking, we have not shown that this is an equilibrium. We have shown that *if* a symmetric equilibrium exists, then this must be it. You are asked to show that this is indeed an equilibrium in an exercise. You might also wonder about the existence of asymmetric equilibria. It can be shown that there are none, although we shall not do so here.

So, each bidder *shades* his bid, by bidding less than his value. Note that as the number of bidders increases, the bidders bid more aggressively. □

Because $F^{N-1}(\cdot)$ is the distribution function of the highest value among a bidder's $N-1$ competitors, the bidding strategy displayed in Theorem 9.1 says that each bidder bids the expectation of the second highest bidder's value conditional on his own value being highest. But, because the bidders use the *same strictly increasing* bidding function, having the highest value is equivalent to having the highest bid and so equivalent to winning the auction. So, we may say:

In the unique symmetric equilibrium of a first-price, sealed-bid auction, each bidder bids the expectation of the second-highest bidder's value conditional on winning the auction.

The idea that one ought to bid *conditional on winning* is very intuitive in a first-price auction because of the feature that one's bid matters only when one wins the auction. Because this feature is present in other auctions as well, this idea should be considered one of the basic insights of our strategic analysis.

Having analysed the first-price auction, it is an easy matter to describe behaviour in a Dutch auction.

9.2.2 BIDDING BEHAVIOUR IN A DUTCH AUCTION

In a Dutch auction, each bidder has a single decision to make, namely, 'At what price should I raise my hand to signal that I am willing to buy the good at that price?' Moreover, the bidder who chooses the highest price wins the auction and pays this price. Consequently, by replacing the word 'price' by 'bid' in the previous sentence we see that this auction is equivalent to a first-price auction! So, we can immediately conclude the following.

THEOREM 9.2 ***Dutch Auction Symmetric Equilibrium***

If N bidders have independent private values drawn from the common distribution, F, then raising one's hand when the price reaches

$$\frac{1}{F^{N-1}(v)} \int_0^v x dF^{N-1}(x)$$

whenever one's value is v constitutes a symmetric Nash equilibrium of a Dutch auction. Moreover, this is the only symmetric Nash equilibrium.

Clearly then, the first-price and Dutch auctions raise exactly the same revenue for the seller, ex post (i.e., for every realisation of bidder values $v_1, \ldots, v_N$).

We now turn to the second-price, sealed-bid auction.

9.2.3 BIDDING BEHAVIOUR IN A SECOND-PRICE, SEALED-BID AUCTION

One might wonder why we would bother considering a second-price auction at all. Is it not obvious that a first-price auction must yield higher revenue for the seller? After all, in a first-price auction the seller receives the *highest* bid, whereas in a second-price auction he receives only the *second-highest* bid.

While this might sound convincing, it neglects a crucial point: *The bidders will bid differently in the two auctions.* In a first-price auction, a bidder has an incentive to raise his bid to increase his chances of winning the auction, yet he has an incentive to reduce his bid to lower the price he pays when he does win. In a second-price auction, the second effect is absent because when a bidder wins, the amount he pays is independent of his bid. So, we should expect bidders to bid *more aggressively* in a second-price auction than they would in a first-price auction. Therefore, there is a chance that a second-price auction will generate higher expected revenues for the seller than will a first-price auction. When we recognise that bidding behaviour changes with the change in the auction format, the question of which auction raises more revenue is not quite so obvious, is it?

Happily, analysing bidding behaviour in a second-price, sealed-bid auction is remarkably straightforward. Unlike our analysis of the first-price auction, we need not restrict attention to the case involving symmetric bidders. That is, we shall allow the density functions $f_1, \ldots, f_N$, from which the bidders' values are independently drawn, to differ.[7]

Consider bidder i with value v_i, and let B denote the highest bid submitted by the other bidders. Of course, B is unknown to bidder i because the bids are sealed. Now, if bidder i were to win the auction, his bid would be highest and B would then be the second-highest bid. Consequently, bidder i would have to pay B for the object. In effect, then, the price that bidder i must pay for the object is the highest bid, B, submitted by the other bidders.

Now, because bidder i's value is v_i, he would strictly want to win the auction when his value exceeds the price he would have to pay, i.e., when $v_i > B$; and he would strictly want to lose when $v_i < B$. When $v_i = B$ he is indifferent between winning and losing. Can bidder i bid in a manner that guarantees that he will win when $v_i > B$ and that he will lose when $v_i < B$, even though he does not know B? The answer is yes. He can guarantee precisely this simply by bidding his value, v_i!

By bidding v_i, bidder i is the high bidder, and so wins, when $v_i > B$, and he is not the high bidder, and so loses, when $v_i < B$. Consequently, bidding his value is a payoff-maximising bid for bidder i *regardless of the bids submitted by the other bidders* (recall that B was the highest bid among any arbitrary bids submitted by the others). Moreover, because bidding below one's value runs the risk of losing the auction when one would have strictly preferred winning it, and bidding above one's value runs the risk of winning the auction for a price above one's value, bidding one's value is a weakly dominant bidding strategy. So, we can state the following.

[7] In fact, even the independence assumption can be dropped. (See Exercise 9.5.)

THEOREM 9.3 ***Second-Price Auction Equilibrium***

If N bidders have independent private values, then bidding one's value is the unique weakly dominant bidding strategy for each bidder in a second-price, sealed-bid auction.

This brings us to the English auction.

9.2.4 BIDDING BEHAVIOUR IN AN ENGLISH AUCTION

In contrast to the auctions we have considered so far, in an English auction there are potentially many decisions a bidder has to make. For example, when the price is very low, he must decide at which price he would drop out when no one has yet dropped out. But, if some other bidder drops out first, he must then decide at which price to drop out *given* the remaining active bidders, and so on. Despite this, there is a close connection between the English and second-price auctions.

In an English auction, as in a second-price auction, it turns out to be a dominant strategy for a bidder to drop out when the price reaches his value, regardless of which bidders remain active. The reason is rather straightforward. A bidder i with value v_i who, given the history of play and the current price $p < v_i$, considers dropping out can do no worse by planning to remain active a little longer and until the price reaches his value, v_i. By doing so, the worst that can happen is that he ends up dropping out when the price does indeed reach his value. His payoff would then be zero, just as it would be if he were to drop out now at price p. However, it might happen, were he to remain active, that all other bidders would drop out before the price reaches v_i. In this case, bidder i would be strictly better off by having remained active since he then wins the object at a price strictly less than his value v_i, obtaining a positive payoff. So, we have the following.

THEOREM 9.4 ***English Auction Equilibrium***

If N bidders have independent private values, then dropping out when the price reaches one's value is the unique weakly dominant bidding strategy for each bidder in an English auction.[8]

Given this result, it is easy to see that the bidder with the highest value will win in an English auction. But what price will he pay for the object? That, of course, depends on the price at which his *last remaining competitor* drops out of the auction. But his last remaining competitor will be the bidder with the *second-highest value*, and he will, like all bidders, drop out when the price reaches his value. Consequently, the bidder with highest value wins and pays a price equal to the second-highest value. Hence, we see that the outcome of the English auction is identical to that of the second-price auction. In particular, the English and second-price auctions earn exactly the same revenue for the seller, ex post.

[8] As in the second-price auction case, this weak dominance result does not rely on the independence of the bidder's values. It holds even if the values are correlated. However, it is important that the values are *private*.

9.2.5 REVENUE COMPARISONS

Because the first-price and Dutch auctions raise the same ex post revenue and the second-price and English auctions raise the same ex post revenue, it remains only to compare the revenues generated by the first- and second-price auctions. Clearly, these auctions need not raise the same revenue ex post. For example, when the highest value is quite high and the second-highest is quite low, running a first-price auction will yield more revenue than a second-price auction. On the other hand, when the first- and second-highest values are close together, a second-price auction will yield higher revenues than will a first-price auction.

Of course, when the seller must decide which of the two auction forms to employ, he does not know the bidders' values. However, knowing how the bidders bid as functions of their values, and knowing the distribution of bidder values, the seller can calculate the *expected revenue* associated with each auction. Thus, the question is, which auction yields the highest expected revenue, a first- or a second-price auction? Because our analysis of the first-price auction involved symmetric bidders, we must assume symmetry here to compare the expected revenue generated by a first-price versus a second-price auction. So, in what follows, $f(\cdot)$ will denote the common density of each bidder's value and $F(\cdot)$ will denote the associated distribution function.

Let us begin by considering the expected revenue, R_{FPA}, generated by a first-price auction (FPA). Because the highest bid wins a first-price auction and because the bidder with the highest value submits the highest bid, if v is the highest value among the N bidder values, then the seller's revenue is $\hat{b}(v)$. So, if the highest value is distributed according to the density $g(v)$, the seller's expected revenue can be written

$$R_{FPA} = \int_0^1 \hat{b}(v)g(v)dv.$$

Because the density, g, of the maximum of N independent random variables with common density f and distribution F is NfF^{N-1},[9] we have

$$R_{FPA} = N\int_0^1 \hat{b}(v)f(v)F^{N-1}(v)dv. \tag{9.6}$$

We have seen that in a second-price auction, because each bidder bids his value, the seller receives as price the second-highest value among the N bidder values. So, if $h(v)$ is the density of the second-highest value, the seller's expected revenue, R_{SPA}, in a second-price auction can be written

$$R_{SPA} = \int_0^1 vh(v)dv.$$

[9]To see this, note that the highest value is less than or equal to v if and only if all N values are, and that this occurs with probability $F^N(v)$. Hence, the distribution function of the highest value is F^N. Because the density function is the derivative of the distribution function the result follows.

Because the density, h, of the second-highest of N independent random variables with common density f and distribution function F is $N(N-1)F^{N-2}f(1-F)$,[10] we have

$$R_{SPA} = N(N-1)\int_0^1 vF^{N-2}(v)f(v)(1-F(v))dv. \tag{9.7}$$

We shall now compare the two. From (9.6) and (9.5) we have

$$\begin{aligned}
R_{FPA} &= N\int_0^1 \left[\frac{1}{F^{N-1}(v)}\int_0^v x dF^{N-1}(x)\right] f(v)F^{N-1}(v)dv \\
&= N(N-1)\int_0^1 \left[\int_0^v xF^{N-2}(x)f(x)dx\right] f(v)dv \\
&= N(N-1)\int_0^1 \int_0^v [xF^{N-2}(x)f(x)f(v)]dxdv \\
&= N(N-1)\int_0^1 \int_x^1 [xF^{N-2}(x)f(x)f(v)]dvdx \\
&= N(N-1)\int_0^1 xF^{N-2}(x)f(x)(1-F(x))dx \\
&= R_{SPA},
\end{aligned}$$

where the fourth equality follows from interchanging the order of integration (i.e., from *dxdv* to *dvdx*), and the final equality follows from (9.7).

EXAMPLE 9.2 Consider the case in which each bidder's value is uniform on [0, 1] so that $F(v) = v$ and $f(v) = 1$. The expected revenue generated in a first-price auction is

$$\begin{aligned}
R_{FPA} &= N\int_0^1 \hat{b}(v)f(v)F^{N-1}(v)dv \\
&= N\int_0^1 \left[v - \frac{v}{N}\right] v^{N-1} dv \\
&= (N-1)\int_0^1 v^N dv \\
&= \frac{N-1}{N+1}.
\end{aligned}$$

[10] One way to see this is to treat probability density like probability. Then the probability (density) that some particular bidder's value is v is $f(v)$ and the probability that exactly one of the remaining $N-1$ other bidders' values is above this is $(N-1)F^{N-2}(v)(1-F(v))$. Consequently, the probability that this particular bidder's value is v and it is second-highest is $(N-1)f(v)F^{N-2}(v)(1-F(v))$. Because there are N bidders, the probability (i.e., density) that the second-highest value is v is then $N(N-1)f(v)F^{N-2}(v)(1-F(v))$.

On the other hand, the expected revenue generated in a second-price auction is

$$\begin{aligned}R_{SPA} &= N(N-1)\int_0^1 vF^{N-2}(v)f(v)(1-F(v))dv \\ &= N(N-1)\int_0^1 v^{N-1}(1-v)dv \\ &= N(N-1)\left[\frac{1}{N}-\frac{1}{N+1}\right] \\ &= \frac{N-1}{N+1}.\end{aligned}$$

□

Remarkably, the first- and second-price auctions raise the *same* expected revenue, regardless of the common distribution of bidder values! So, we may state the following:

If N bidders have independent private values drawn from the common distribution, F, then all four standard auction forms (first-price, second-price, Dutch, and English) raise the same expected revenue for the seller.

This *revenue equivalence* result may go some way towards explaining why we see all four auction forms in practice. Were it the case that one of them raised more revenue than the others on average, then we would expect that one to be used rather than any of the others. But what is it that accounts for the coincidence of expected revenue in these auctions? Our next objective is to gain some insight into why this is so.

9.3 THE REVENUE EQUIVALENCE THEOREM

To explain the equivalence of revenue in the four standard auction forms, we must first find a way to fit all of these auctions into a single framework. With this in mind, we now define the notion of a *direct selling mechanism.*[11]

DEFINITION 9.1 *Direct Selling Mechanism*

A direct selling mechanism is a collection of N probability assignment functions,

$$p_1(v_1,\ldots,v_N),\ldots,p_N(v_1,\ldots,v_N),$$

and N cost functions

$$c_1(v_1,\ldots,v_N),\ldots,c_N(v_1,\ldots,v_N).$$

For every vector of values $(v_1,\ldots,v_N)$ *reported by the N bidders,* $p_i(v_1,\ldots,v_N)\in[0,1]$ *denotes the probability that bidder i receives the object and* $c_i(v_1,\ldots,v_N)\in\mathbb{R}$

[11] Our presentation is based upon Myerson (1981).