Monopolistic competition refers to an "industry" with a large number of firms, each facing a downward-sloping demand (for differentiated products) but making no profit because of fixed costs, and such that strategic interaction is absent (i.e., each firm can ignore its impact on other firms). As we will see, this latter property distinguishes the monopolistically competitive situations from the zero-profit equilibrium studied in the spatial-differentiation model. The supplementary section applies the familiar model of Dixit and Stiglitz (1977) and Spence (1976) to an analysis of product diversity in a monopolistically competitive economy.

In section 7.3, ideas developed in sections 7.1 and 7.2 are used to study informational differentiation, another type of product differentiation that results from consumers' uneven information about the characteristics (existence, price, quality, etc.) of various products. We focus on the link between advertising and differentiation. After reviewing the conventional wisdom on advertising, we will see how informative advertising can increase the elasticity of demand for a product and foster competition. We will also see that competition may yield (from a social standpoint) too much or too little informative advertising.

7.1 Spatial Competition

7.1.1 The Linear City

We first consider a model (originally due to Hotelling [1929]) in which a "linear city" of length 1 lies on the abscissa of a line and consumers are uniformly distributed with density 1 along this interval. There are two firms or stores, which sell the same physical good. For simplicity, and as a first step, these two stores are located at the extremes of the city; store 1 is at $x = 0$ and store 2 at $x = 1$. The unit cost of the good for each store is c. Consumers incur a transportation cost t per unit of length (this cost may include the value of time spent in travel). Thus, a consumer living at x incurs a cost of tx to go to store 1 and a cost of $t(1 - x)$ to go to store 2. The consumers have unit demands; i.e., each consumes one or zero unit of the good. Each consumer derives a surplus from consumption (gross of price and transportation costs) equal to $\bar{s}$.

We will also consider a variant of this model in which the transportation costs are quadratic instead of linear. In this case, a consumer at x incurs a cost of tx^2 to go to store 1 and a cost of $t(1 - x)^2$ to go to store 2. In this version, the marginal transportation cost increases with the distance to the store. As we will see, the quadratic model is sometimes more tractable than the linear one.

7.1.1.1 Price Competition

In this subsection we take the firms' locations as given and look for the Nash equilibrium in prices. Assuming that firms choose their prices p_1 and p_2 simultaneously,[3] we derive the demand function for *quadratic* transportation costs. Let us assume that the prices of the two firms do not differ so much that one firm faces no demand, and that the prices are not too high relative to $\bar{s}$ (so that all consumers buy—i.e., the market is covered). The first condition must clearly be satisfied in equilibrium, because a firm with no demand makes no profit and therefore has an incentive to lower its price to gain market share. The second condition is satisfied in equilibrium if the consumers' surplus from the good $\bar{s}$ is sufficiently large.

A consumer who is indifferent between the two firms is located at $x = D_1(p_1, p_2)$, where x is given by equating generalized costs; i.e.,

$$p_1 + tx^2 = p_2 + t(1 - x)^2.$$

The firms' respective demands are

$$D_1(p_1, p_2) = x = \frac{p_2 - p_1 + t}{2t}$$

and

$$D_2(p_1, p_2) = 1 - x = \frac{p_1 - p_2 + t}{2t}.$$

When the firms are located at the two extremes of the city, the demand functions are the same for linear cost as for quadratic cost. (This is not robust. It does not hold if the market is not covered, and, as we will see shortly, it is contingent on the locations' being the two

3. For the derivation of the demand functions for linear transportation costs, see section 2.1.

extremities of the city.) In both cases, firm i's profit is

$$\Pi^i(p_i, p_j) = (p_i - c)(p_j - p_i + t)/2t.$$

The goods produced by the two firms are strategic complements in prices ($\Pi^i_{ij} > 0$). This important property will hold for all the models in this chapter except that of monopolistic competition, in which interaction is absent. Its role will be clarified in the next chapter.

For either linear or quadratic transportation costs, firm i chooses p_i so as to maximize its profit given the price p_j charged by its rival; i.e.,

$$\Pi^i = \max_{p_i} [\Pi^i(p_i, p_j)].$$

The first-order condition for firm i is

$$p_j + c + t - 2p_i = 0,$$

and the second-order condition is satisfied. Using the symmetry of the problem, we obtain the competitive prices and profits under product differentiation:

$$p_1^c = p_2^c = c + t \tag{7.1}$$

and

$$\Pi^1 = \Pi^2 = t/2. \tag{7.2}$$

We speak of differentiated products even though they are physically identical. The products are differentiated more for the consumer when the transportation cost is higher. When t increases, both stores compete less strenuously for "the same consumers"; indeed, the neighboring clientele of a store becomes more captive, giving the store "monopoly power" (which, in turn, allows it to increase its price). On the other hand, when $t = 0$ all the consumers can go to either store for the same cost (0). The absence of product differentiation leads to the Bertrand result.

Because we are also interested in the firms' choice of product differentiation, we would like to know how the equilibrium prices vary with the firms' locations. We have looked at one polar case—the one in which firms are located as far as possible from each other (maximal differentiation). The other polar case is that in which they produce the same product—i.e., they are located at the same point (say x_0) and their goods are perfect substitutes. Comparing the generalized costs $p_i + t|x - x_0|$ (or, in the quadratic case, $p_i + t(x - x_0)^2$) for a consumer located at any point x amounts to just comparing prices p_1 and p_2. Hence, the Bertrand result holds for identical locations:

$$p_1^c = p_2^c = 0 \tag{7.3}$$

and

$$\Pi^1 = \Pi^2 = 0. \tag{7.4}$$

More generally, let us assume that firm 1 is located at point $a \geqslant 0$ and firm 2 at point $1 - b$, where $b \geqslant 0$ and, without loss of generality, $1 - a - b \geqslant 0$ (firm 1 is to the "left" of firm 2; $a = b = 0$ corresponds to maximal differentiation and $a + b = 1$ corresponds to minimal differentiation, i.e., perfect substitutes). The linear-cost model is not very tractable if firms are located inside the interval, because when a firm lowers its price to the point that it just attracts the consumers located between the two firms it also attracts all consumers located on the other side of the rival.[4] The firms' demand functions are discontinuous. Their profit functions are discontinuous and nonconcave. Consequently, the price-competition problem is not well behaved. Indeed, d'Aspremont, Gabszewicz, and Thisse (1979) show that if the firms are located close to the center of the segment (but not at the same location), no pure-strategy price equilibrium exists.[5]

The quadratic-cost model allows us to sidestep these technical issues. The demand and profit functions are well behaved (continuous and concave). We obtain

$$D_1(p_1, p_2) = x = a + \frac{1 - a - b}{2} + \frac{p_2 - p_1}{2t(1 - a - b)} \tag{7.5}$$

4. Suppose that a consumer is located at $x \geqslant 1 - b > a$. This consumer belongs to firm 2's "turf" or "back yard." His choice between the two firms is determined by the comparison between

$$p_1 + t(x - a)$$

and

$$p_2 + t[x - (1 - b)],$$

i.e., between p_1 and $p_2 - t(1 - a - b)$. Thus, all consumers located to the right of firm 2 always make the same brand choice as the consumer located at firm 2's location. This means that at $p_1 = p_2 - t(1 - a - b)$ the demand functions are discontinuous; all consumers on firm 2's turf switch to firm 1 for a small reduction in p_1.

5. A mixed-strategy price equilibrium does exist. See Dasgupta and Maskin 1986.

and

$$D_2(p_1, p_2) = 1 - x = b + \frac{1 - a - b}{2} + \frac{p_1 - p_2}{2t(1 - a - b)} \tag{7.6}$$

(as long as these are non-negative and do not exceed 1 and as long as $\bar{s}$ is sufficiently large that the market is covered).

To interpret equation 7.5, notice that for equal prices firm 1 controls its own turf (of size a) and receives the half of the consumers located between the two firms who are closer to firm 1 (numerically, $(1 - b - a)/2$). The third term of equation 7.5 expresses the sensitivity of demand to the price differential.

The Nash equilibrium in prices, which always exists, is

$$p_1^c(a, b) = c + t(1 - a - b)\left(1 + \frac{a - b}{3}\right), \tag{7.7}$$

$$p_2^c(a, b) = c + t(1 - a - b)\left(1 + \frac{b - a}{3}\right). \tag{7.8}$$

*Exercise 7.1** Check equations 7.5 through 7.8.

7.1.1.2 Product Choice

Suppose now that there are two firms and that each firm is allowed to choose only one product (that is, only one location). This defines a two-stage game in which (1) the firms choose their locations simultaneously and (2) given the locations, they choose prices simultaneously. As was mentioned earlier, each firm must anticipate how its choice of location affects not only its demand function but also the intensity of price competition. Therefore, to study location (product) competition, we use the reduced-form profit functions, e.g.,

$$\Pi^1(a, b) = [p_1^c(a, b) - c] D_1[a, b, p_1^c(a, b), p_2^c(a, b)], \tag{7.9}$$

where D_1 is given by equation 7.5. An equilibrium in location is such that firm 1 maximizes $\Pi^1(a, b)$ with respect to a, taking b as given, and similarly for firm 2. (This procedure is similar to the two-stage, capacity-and-then-price competition studied in chapter 5.)

D'Aspremont et al. (1979) show that for quadratic transportation costs, the equilibrium has the two firms locating at the two extremes of the city (*maximal differentiation*). Each firm locates far from its rival in order not to trigger a low price from the rival, and thus price competition is softened. To show this, we could compute the reduced-form profit functions $\Pi^i(a, b)$ explicitly using equations 7.5 through 7.8 and solve for a Nash equilibrium; however, it is more elegant and instructive to proceed otherwise. Suppose, without loss of generality, that in equilibrium

$$0 \leqslant a \leqslant 1 - b \leqslant 1.$$

We know that to maximize $\Pi^1(a, b)$ (given by equation 7.9) with respect to a we need not take the derivative

$$\frac{\partial \Pi^1}{\partial p_1} \frac{\partial p_1^c}{\partial a}.$$

This is due to the envelope theorem: Firm 1 maximizes with respect to price in the second period, so $\partial \Pi^1 / \partial p_1 = 0$. Thus, we need only look at the direct effect of a on Π^1 (the *demand effect*) and the indirect effect through the change in firm 2's price (the *strategic effect*). That is,

$$\frac{d\Pi^1}{da} = (p_1^c - c)\left(\frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_2} \frac{dp_2^c}{da}\right).$$

Using equations 7.5, 7.7, and 7.8 we get

$$\frac{\partial D_1}{\partial a} = \frac{1}{2} + \frac{p_2^c - p_1^c}{2t(1 - a - b)^2} = \frac{3 - 5a - b}{6(1 - a - b)}, \tag{7.10}$$

and using equations 7.5 and 7.8 we get

$$\frac{\partial D_1}{\partial p_2} \frac{dp_2^c}{da} = \left(\frac{1}{2t(1 - a - b)}\right)\left[t\left(-\frac{4}{3} + \frac{2a}{3}\right)\right]$$

$$= \frac{-2 + a}{3(1 - a - b)}. \tag{7.11}$$

Adding equations 7.10 and 7.11 and using the fact that the mark-up $(p_1^c - c)$ is positive, we can easily show that $d\Pi^1/da < 0$. Hence, firm 1 always wants to move leftward if it is to the left of firm 2, and similarly for firm 2. Therefore, the equilibrium in locations exhibits maximal differentiation.

Use of the envelope theorem (which will be reiterated in the next chapter) is also instructive. It exhibits the conflict between two effects. First, equation 7.10 shows that if a is not too big (in particular, if it does not exceed $\frac{1}{2}$, using $1 - b \geqslant a$), firm 1 will want to move toward the center to increase its market share given the price structure. This is part of a more general result that, for given prices, the two firms want to locate at or near the center (see subsection 7.1.3). However, firm 1 also acknowledges that the associated decrease in product differentiation forces firm 2 to lower its price. The computations show that this strategic effect dominates the market-share effect.

It is interesting to compare the market-determined locations to the socially optimal ones. Suppose that the social planner chooses locations for the two firms. Because consumption is fixed, the social planner minimizes the consumers' average transportation cost (this holds whether the firms exercise their market power as above or are forced to price at marginal cost; for given locations, and as long as the market is covered, the pricing structure does not affect the sum of consumer surplus and profits in this inelastic-demand model). By symmetry of the problem, the social planner chooses to locate the two firms equidistantly on either side of the middle of the segment, so that for equal prices a firm serves the left or the right half of the market. Hence, the location that minimizes the average transportation cost on a market segment is the middle of this market segment when the density of consumers is uniform. Thus, the socially optimal locations are $\frac{1}{4}$ and $\frac{3}{4}$. In this example, the market outcome yields socially too much product differentiation.

*Exercise 7.2*** Consider the model of differentiation on the line. The two firms' locations are fixed, and they are the two extremities of the segment. Transportation costs are linear in distance, and the distribution of consumers is uniform along the segment. The firms have constant marginal costs, c_1 and c_2, which are not necessarily equal (but, for simplicity, assume that they do not differ too much, so that each firm has a positive market share in equilibrium).

(i) Compute the reaction functions $p_i = R_i(p_j)$. Infer the Nash-equilibrium prices $p_i(c_i, c_j)$ and the reduced-form profits $\Pi^i(c_i, c_j)$ as functions of the two marginal costs.

(ii) Show that $\partial^2\Pi^i/\partial c_i \partial c_j < 0$.

(iii) Suppose that, before competing in price, the firms play a first-period game in which they simultaneously choose their marginal cost. (Think of an investment cost $\phi(c)$ of choosing marginal cost c, with $\phi' < 0$ and $\phi'' > 0$.) Show that, as in the previous choice-of-location game, this investment game gives rise to a direct effect and a strategic effect.

7.1.2 The Circular City

7.1.2.1 The Model

The above consideration of a linear city allowed us to examine price competition with differentiated products, as well as the choice of product in duopoly. Now let us study entry and location when there are no "barriers to entry" other than fixed costs or entry costs. Assuming that there exist a large number of identical potential firms, we will look at the number of firms entering the market. To do so, it is actually more convenient to consider a circular city with a uniform distribution of consumers. In this case, the product space is completely homogeneous (no location is *a priori* better than another), which makes the study of the issue at hand more tractable.

The following model is due to Salop (1979). Consumers are located uniformly on a circle with a perimeter equal to 1. Density is unitary around this circle. Firms are also located around the circle, and all travel occurs along the circle (like the linear city, this is a bit contrived in order to simplify the analysis, but one may think of a city around a lake, with boats being an inefficient transportation technology; or of supermarkets in a circular suburbia with a costly-to-cross city at its center; or else of aircraft departure times).

As before, consumers wish to buy one unit of the good, have a unit transport cost t (for simplicity, we will consider only linear transportation costs), and are willing to buy at the smallest generalized cost so long as the latter does not exceed the gross surplus they obtain from the good ($\bar{s}$). Each firm is allowed to locate in only one location (we will discuss this assumption below, and especially in the next chapter, where we will examine the possibility of entry deterrence through brand proliferation). In order to address the issue of the number of firms, we introduce a fixed cost of entry, f. Once a firm is in and is located at a point on the product space, it faces a marginal cost c (smaller than $\bar{s}$). Thus, firm i's profit is $(p_i - c)D_i - f$ if it enters (where D_i is the demand it faces), and 0 otherwise.

Salop considers the following two-stage game: In the first stage, potential entrants simultaneously choose whether or not to enter. Let n denote the number of entering firms. Those firms do not choose their location, but rather are automatically located equidistant from one