Homework assignment #1

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Due date: March 26, 2013 (before midterm)

Problem 1

The utility function is $U(x_1, x_2) = x_1 x_2$, prices are $(p_1, p_2) = (1, 1)$ and the budget is $w = 8$.

1. Derive the Hicksian compensated demand for both goods. Depict your solution.
2. Suppose that p_1 increases to $p_1^{'} = 2$. Compute the substitution effect and show it graphically.
3. Derive the Marshallian demands for both goods. Compute the income effect and depict it in the graph. Show the total effect?
4. Compute the compensation variation (CV) and equivalence variation (EV) for the price increase. Present the solution graphically.

Problem 2

You know that $U(x_1, x_2) = x_1 - x_2^{-1}$, prices of goods x_1 and x_2 are p_1 and p_2, and w is the available budget.

1. Find the Marshallian demand functions for both goods and check whether they are normal or inferior, Giffen or non-Giffen. Are both goods complements or substitutes?
2. Compute elasticities of each good with respect to p_1 and p_2. Is any of the goods luxury?
3. Obtain the indirect utility function, derive the cost function and the Hicksian demand.
4. Setup the cost minimization problem and verify that the Hicksian demand in the point above is correct. Compute the income and substitution effects for both goods separately and show that the Slutsky equation holds. How is x_1 different from x_2. Draw IE and SE for both goods in separate graphs.

Problem 3

Assume two - period setup with $U(c_t) = log(c_t)$. The government levies a tax rate τ on young workers income y and invests τy on behalf of the young workers, realizes the common real rate of return R on it, and returns it to the agents when they are retired. In addition, young workers may save an amount b in the bond market on their own. Setup the consumer maximization problem and derive conditions for the consumer to participate in the government funded pension scheme.

Problem 4

From Macroeconomics (Doepke): Exercise 3.3. on p. 31 and exercise 13.3. on p. 150.

Problem 5

Suppose that the economy is populated by two consumers: A and B with respective utility functions: $U^A(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$ and $U^B(x_1, x_2) = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$. Endowments are $(e_1^A, e_2^A) = (1, 1)$ and $(e_1^B, e_2^B) = (3, 2)$.

1. Setup the maximization problem of each consumer and solve it.
2. Setup the market clearing conditions.
3. Define the competitive equilibrium. Is the price vector unique in this equilibrium? Given the price vector, is the allocation for both consumers unique?
4. Draw your findings in the Edgeworth box and provide respective analysis.

Problem 6

The consumer seeks to solve the following maximization problem:

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t (log(c_t) + \gamma log(c_{t-1}))$$

$$\text{s.t.: } k_{t+1} + c_t \leq A k_t^{\alpha},$$

$$A > 0,\ \alpha,\ \gamma \in (0,1),\ c_t > 0,\ k_t > 0,\ k_0, \text{ and } c_{-1} \text{ are given.}$$

1. Clearly identify the state and control variables. Setup the Bellman equation and take the first - order conditions.
2. Guess and verify that the value function is $V(k_t, c_{t-1}) = E + Flog(k_t) + Glog(c_{t-1})$. Find the constants E, F and G.