Bi7740: Scientific computing

Optimization: a brief summary - Exercise

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The beer can problem

- variables: x_1 the diameter, x_2 the height of the can
- Problem: let $\mathbf{x} = [x_1, x_2]^T$

$$\text{minimize} \quad f(\mathbf{x}) = \frac{\pi x_1^2}{2} + \pi x_1 x_2 \quad \text{with respect to } x_1, x_2$$

subject to

$$-2x_1 + x_2 \leq 0$$

$$c_{eq}(\mathbf{x}) = 333 - \frac{\pi x_1^2}{4} x_2 = 0$$

The differentials:

- the gradient of the objective function:

$$\nabla f(\mathbf{x}) = \left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}\right]^T = [\pi(x_1 + x_2), \pi x_1]^T$$

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- the Jacobian of the nonlinear constraint function (which is a scalar function, so it's a gradient finally):

$$\mathbf{J}_{c_{eq}} = \left[\frac{\partial c_{eq}}{\partial x_1}, \frac{\partial c_{eq}}{\partial x_2}\right] = \left[-\frac{\pi}{2} x_1 x_2, -\frac{\pi}{4} x_1^2\right]$$

MATLAB implementation

Constraint function implementation (save it as `beer_constraint.m` file):

```
function [c ceq c_j ceq_j] = beer_constraint(x, vol)
% Evaluate the constraint for the beer can.
%
% x(1) - diameter
% x(2) - height

c = [];  % no nonlinear inequalities
ceq = vol - 0.25*pi*x(1)^2*x(2);

% Jacobian:
if nargout > 2
    c_j = [];
    ceq_j = [-0.5*pi*x(1)*x(2), -0.25*pi*x(1)^2];
end

return
```

Objective function implementation (save it as `beer_area.m` file):

```
function [f nabla_f] = beer_area(x)
% Compute the area of a beer can.
% x(1) - diameter
% x(2) - height

 f = pi*x(1)*x(2) + 0.5*pi*x(1)^2;

 % gradient:
 if nargout > 1
   nabla_f = [pi*x(1)+pi*x(2); pi*x(1)]; % column ...
       vector!
 end
return
```

MATLAB session: try also `optimtool`!

```
vol = 333; % cm^3
% to bound the parameter:
h_constraint = @(x) (beer_constraint(x, vol));

A = [-2 1]; b = 0;
lb = [4;  5];
ub = [8; 15];

x0 = [6; 10];

% set the options:
options = optimoptions('fmincon');
options = optimoptions(options,'Display', 'off');
options = optimoptions(options,'Algorithm', ...
    'active-set');

x = fmincon(@beer_area, x0, A, b, [], [], lb, ub, ...
    h_constraint, options);
```