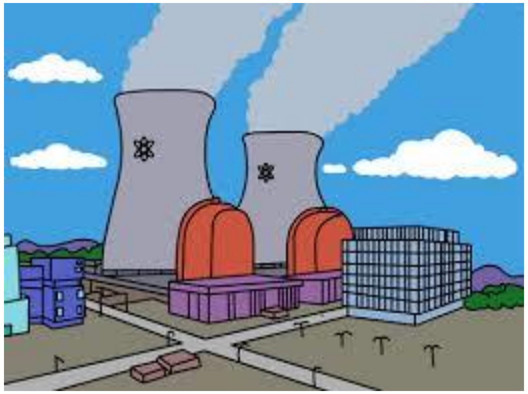
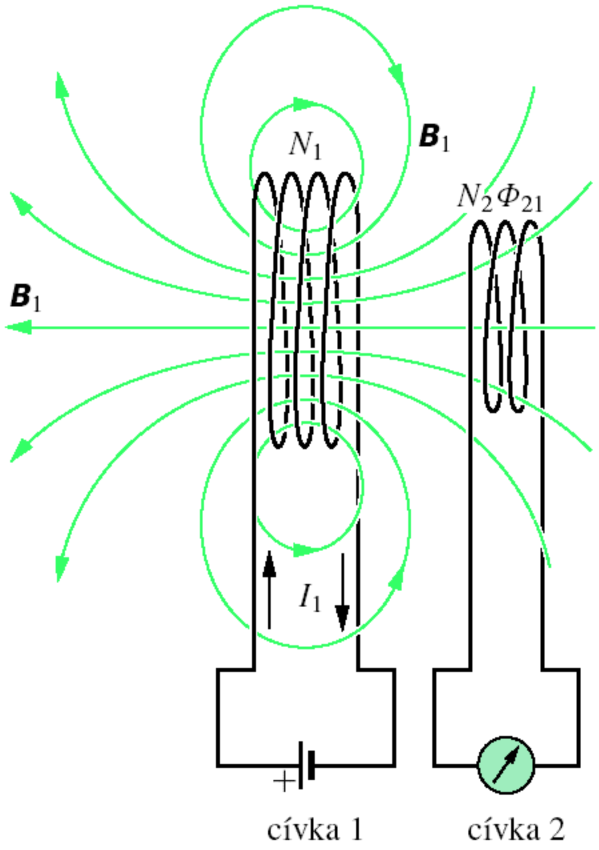
indukčnost
obvody se střídavým proudem



vzájemná indukce



$$\mathcal{E}_2^{ind} = -\frac{\mathrm{d}[N_2\Phi_{21}(t)]}{\mathrm{d}t}$$

$$N_2\Phi_{21}(t) = N_2 \iint_{S_2} \vec{B}_1(2)\cdot \mathrm{d}\vec{S}$$

$$\oint_C \vec{B}_1 \cdot \mathrm{d}\vec{s} = \mu_0 N_1 I_1(t)$$

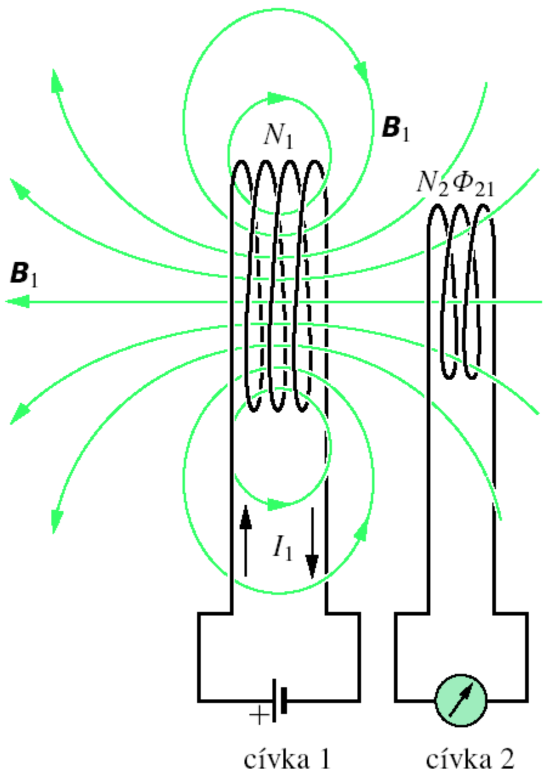
$$N_2\Phi_{21}(t) \propto B_1(t) \propto I_1(t)$$

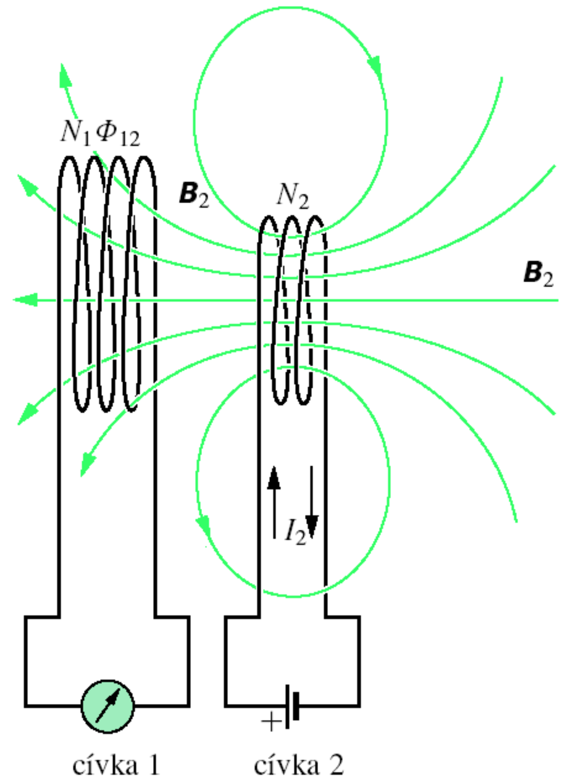
$$N_2\Phi_{21}(t) = M_{21} I_1(t)$$

vzájemná indukčnost

$$\mathcal{E}_2^{ind} = -M_{21}\frac{\mathrm{d}I_1(t)}{\mathrm{d}t}$$

vzájemná indukce





$$\mathcal{E}_2^{ind} = -M_{21}\frac{\mathrm{d}I_1(t)}{\mathrm{d}t} \qquad \mathcal{E}_1^{ind} = -M_{12}\frac{\mathrm{d}I_2(t)}{\mathrm{d}t}$$

$$M_{21} = M_{12} = M$$

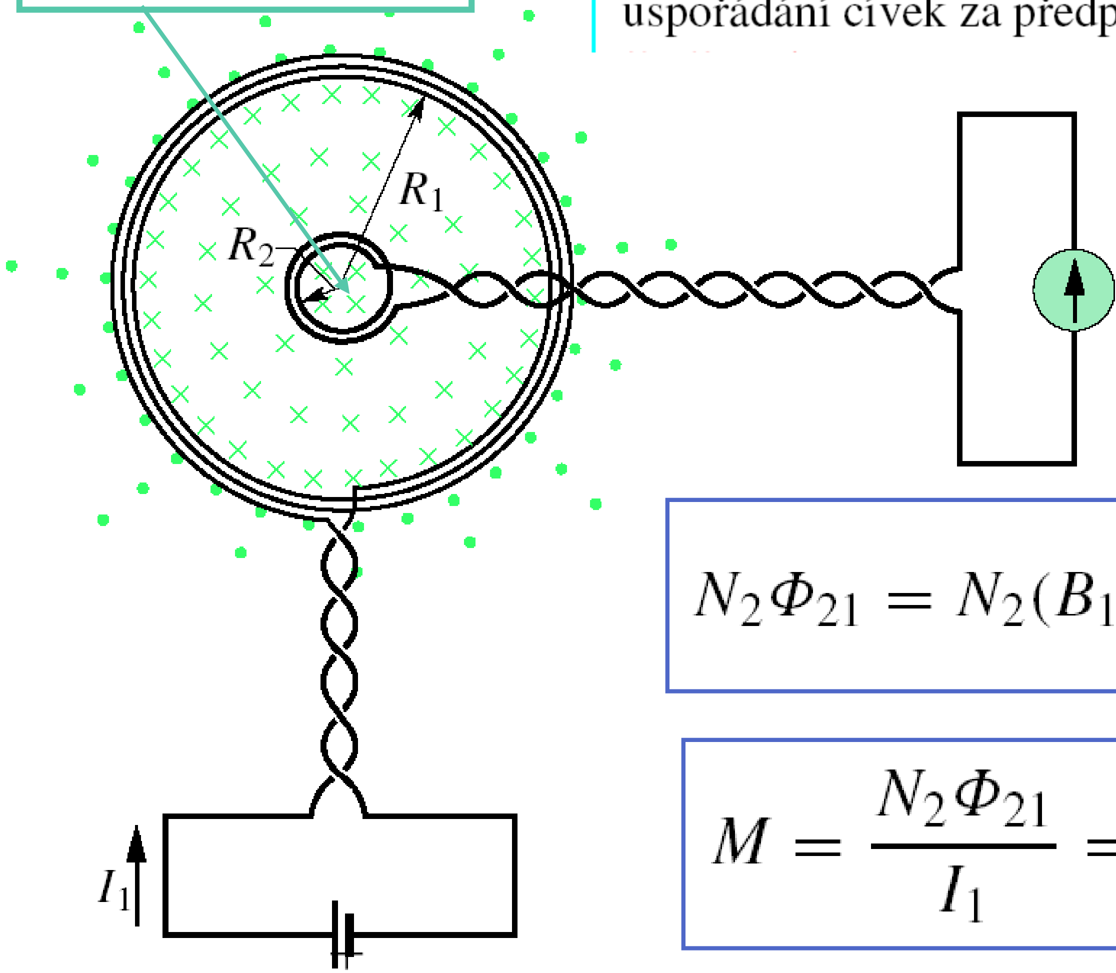
vzájemná indukce

PŘÍKLAD 31.12

Na obr. 31.25 jsou dvě kruhové hustě vinuté souosé cívky ležící ve stejné rovině. Menší má poloměr R_2 a počet závitů N_2, větší má poloměr R_1 a počet závitů N_1.

(a) Odvoďte výraz pro vzájemnou indukčnost M pro toto uspořádání cívek za předpokladu, že $R_1 \gg R_2$.

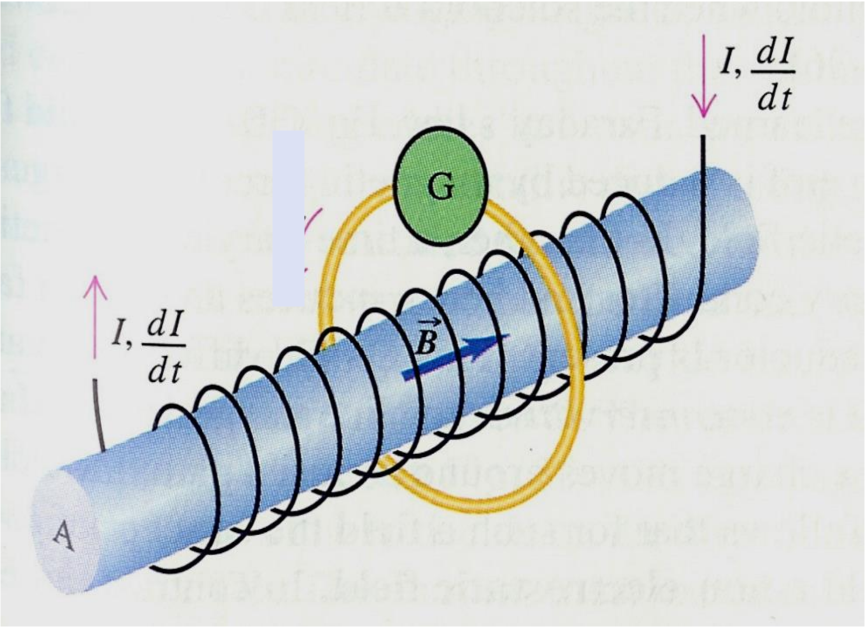
$$B_1 = \frac{\mu_0 I_1 N_1}{2R_1}$$



$$N_2 \Phi_{21} = N_2 (B_1)(\pi R_2^2) = \frac{\pi \mu_0 N_1 N_2 R_2^2 I_1}{2R_1}$$

$$M = \frac{N_2 \Phi_{21}}{I_1} = \frac{\pi \mu_0 N_1 N_2 R_2^2}{2R_1}$$

vlastní indukce



$$\mathcal{E}^{ind} = -\frac{\mathrm{d}[N\Phi(t)]}{\mathrm{d}t}$$

$$N\Phi(t) = N \iint_{S_2} \vec{B} \cdot \mathrm{d}\vec{S}$$

$$\oint_C \vec{B} \cdot \mathrm{d}\vec{s} = \mu_0 N I(t)$$

$$N\Phi(t) \propto B(t) \propto I(t)$$

$$N\Phi(t) = LI(t)$$

vlastní indukčnost

$$\mathcal{E}^{ind} = -L\frac{\mathrm{d}I(t)}{\mathrm{d}t}$$

jednotka vlastní i vzájemné indukčnosti

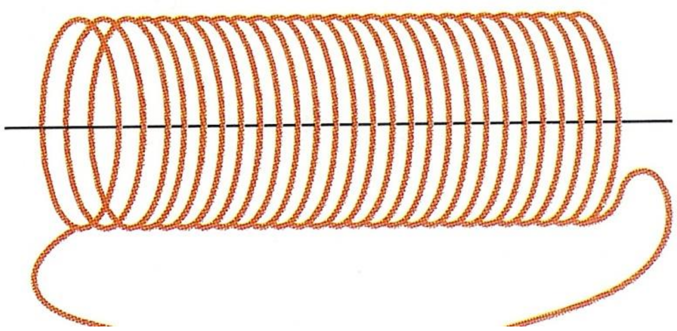
1 H (henry) = $1\ T \cdot m^2 \cdot A^{-1}$

Joseph Henry
(vitráž v Prvním presbyteriánském kostele v Albany, NY, kde byl pokřtěn)

indukčnost solenoidu

$$L = \frac{N\Phi_B}{I} = \mu_0 n^2 l S.$$

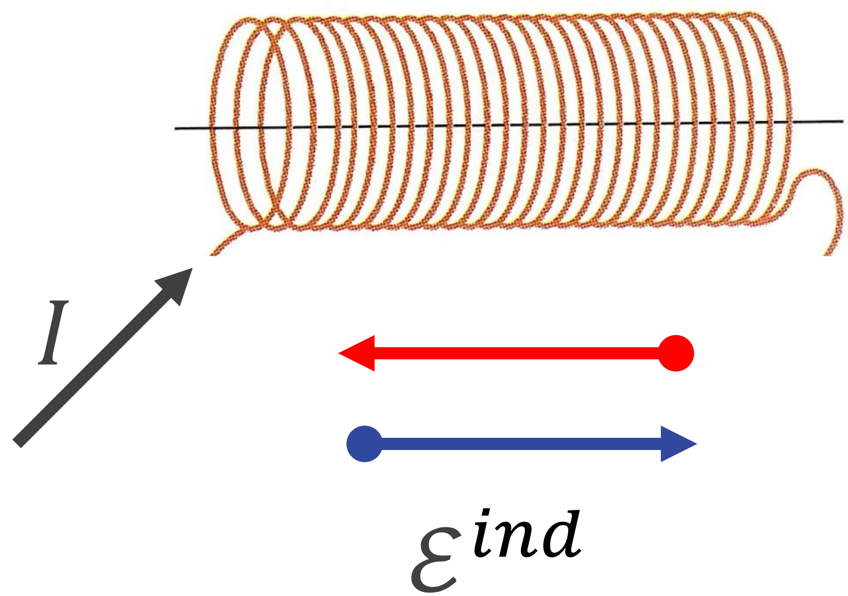
$$N\Phi_B = (nl)(BS), \longleftarrow B = \mu_0 I n$$



s magnetikem (μ_r):

$$L = \mu_r \mu_0 n^2 l S$$

směr indukovaného emn

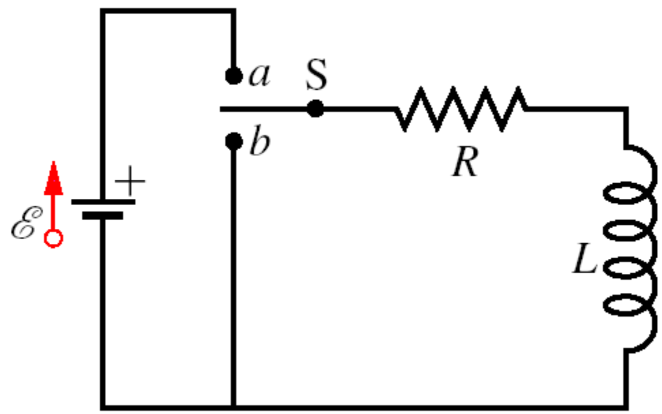


proud roste

proud klesá

RL obvod

$\mathcal{E}^{ind}$



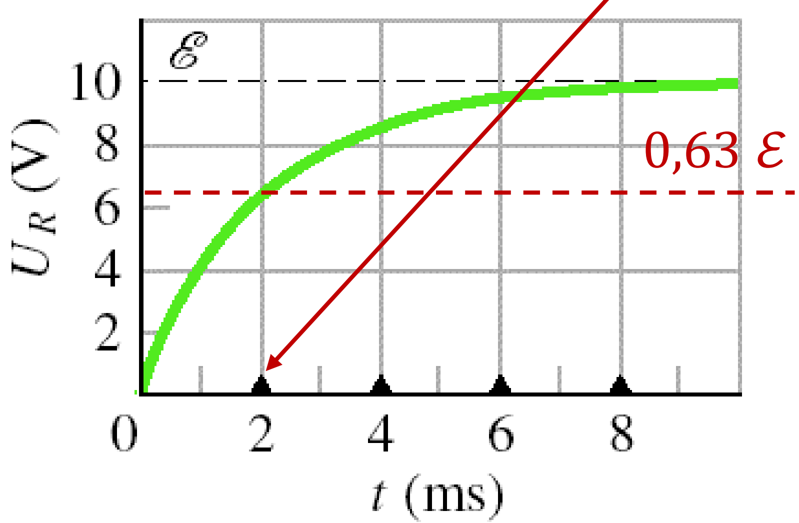
$$\mathcal{E} - RI - L\frac{\mathrm{d}I}{\mathrm{d}t} = 0$$

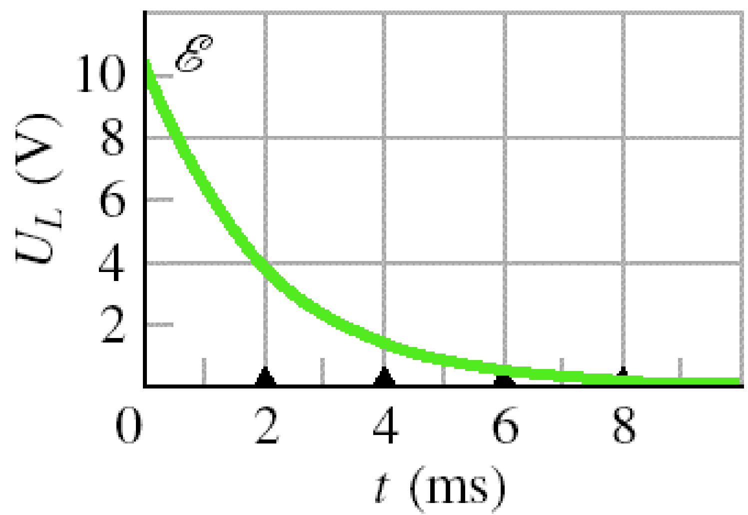
$$L\frac{\mathrm{d}I}{\mathrm{d}t} + RI = \mathcal{E}$$

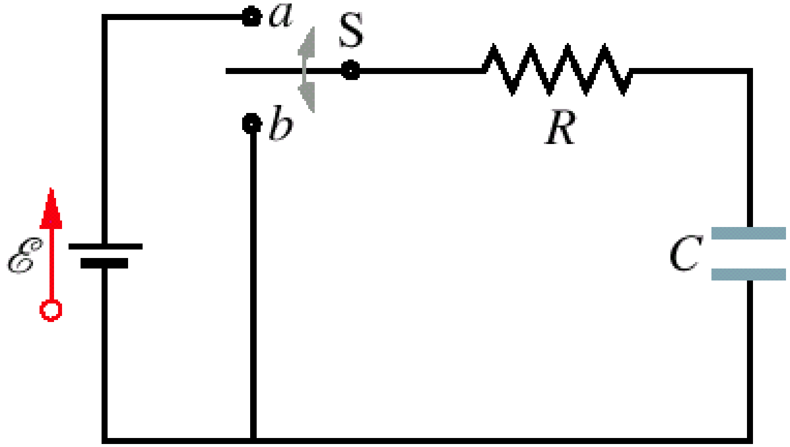
$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)$$

$$\tau_L = \frac{L}{R}$$

$$U_L = L\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{\mathcal{E}}{R}e^{-t/RC}$$







$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}$$

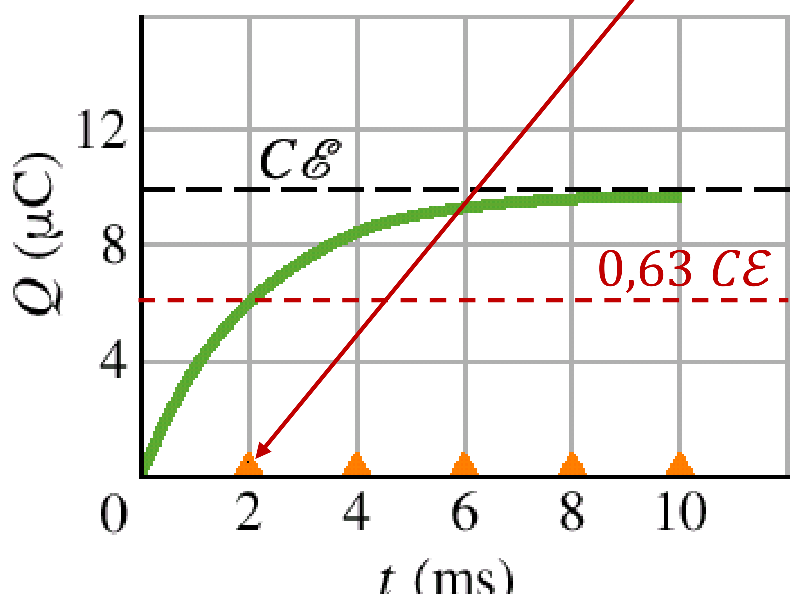
$$\mathcal{E} - RI - \frac{Q}{C} = 0$$

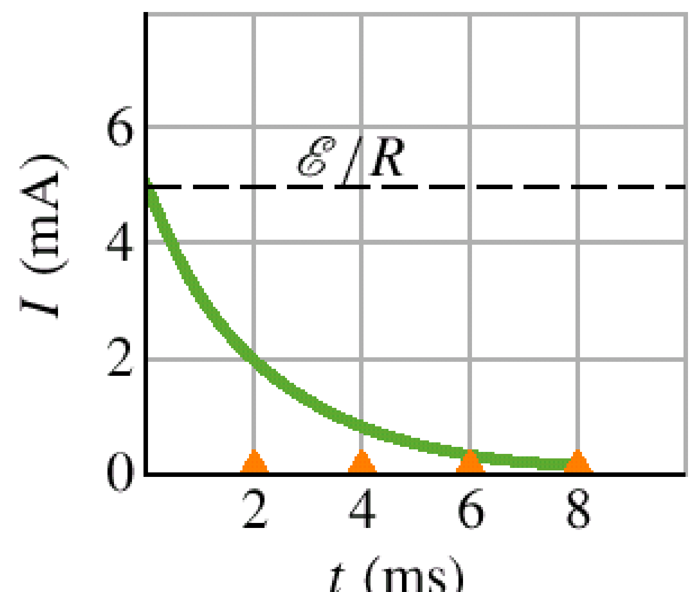
$$R\frac{\mathrm{d}Q}{\mathrm{d}t} + \frac{Q}{C} = \mathcal{E}$$

$$Q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

τ_C

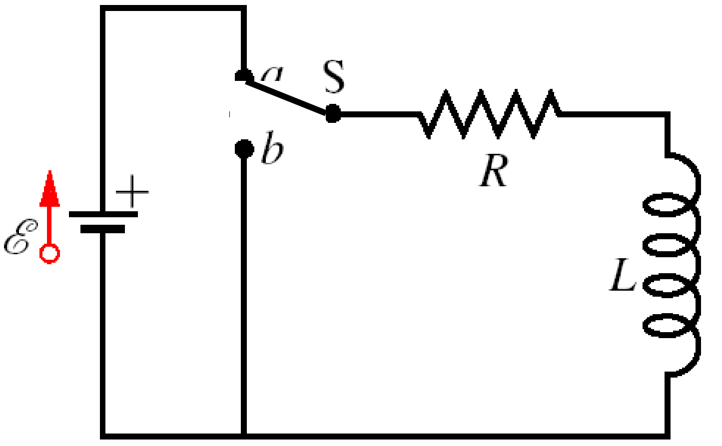
$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{\mathcal{E}}{R}e^{-t/RC}$$





energie magnetického pole

$$\mathscr{E} = L\frac{dI}{dt} + IR$$



$$\mathscr{E}I = LI\frac{dI}{dt} + I^2R,$$

$$dE_{mg} = LI\,dI$$

$$\int_0^{E_{mg}} dE'_{mg} = \int_0^I LI'\,dI'$$

$$E_{mg} = \frac{1}{2}LI^2$$

Kde je energie?

V magnetickém poli

BS

solenoid

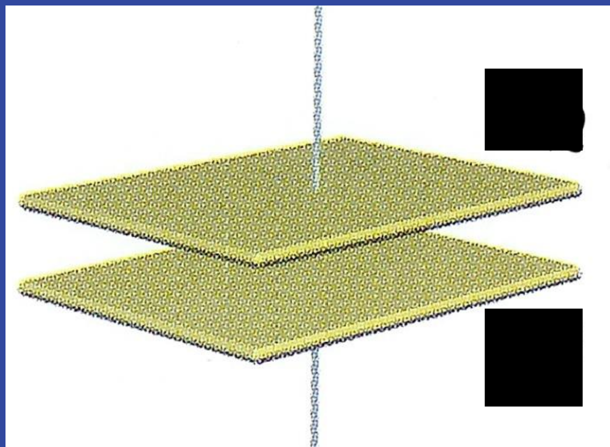
$\frac{Bl}{\mu_0 N}$

V

$$E_{mg} = \frac{1}{2}(L\,I)\,I = \frac{1}{2}N\Phi\,I = \frac{1}{2}(NBS)\left(\frac{Bl}{\mu_0 N}\right) = \left(\frac{B^2}{2\mu_0}\right)(Sl)$$

s hustotou

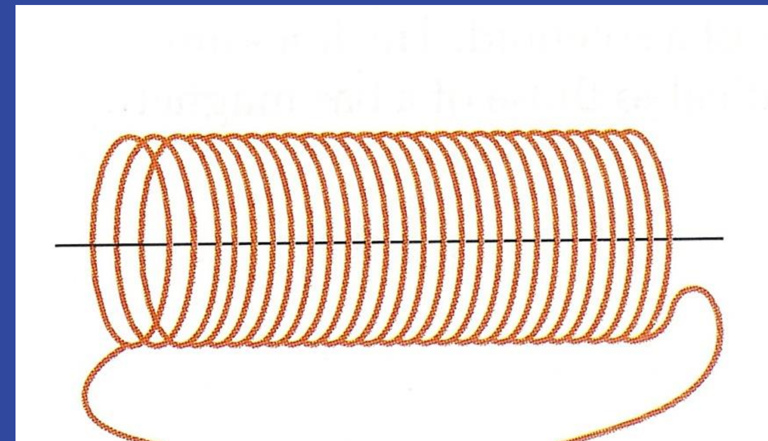
$$w_{mg} = \frac{B^2}{2\mu_0}$$



$$C = \frac{\varepsilon_r \varepsilon_0 S}{d}$$

$$E_C = \frac{1}{2} C U^2$$

$$w_{el} = \frac{1}{2} \varepsilon_0 E^2$$



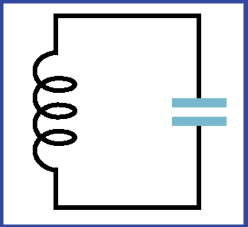
$$L = \frac{\mu_r \mu_0 N^2 S}{l}$$

$$E_L = \frac{1}{2} L I^2$$

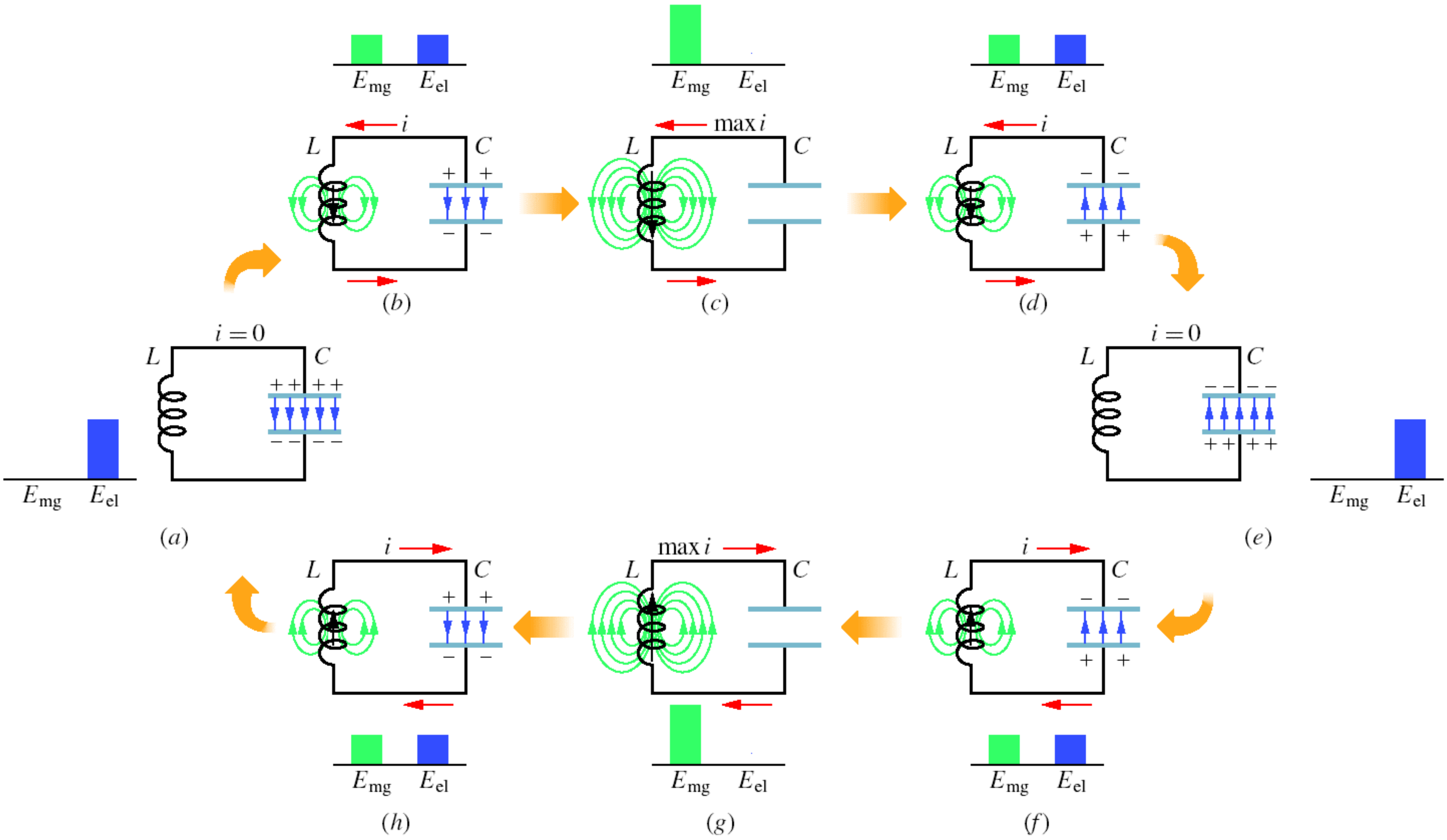
$$w_{mg} = \frac{B^2}{2\mu_0}$$

Obvod LC

$$E_{mg} = \frac{Li^2(t)}{2}$$



$$E_{el} = \frac{q^2(t)}{2C}$$



$$E = E_{\mathrm{mg}} + E_{\mathrm{el}} = \frac{Li^2}{2} + \frac{q^2}{2C}$$

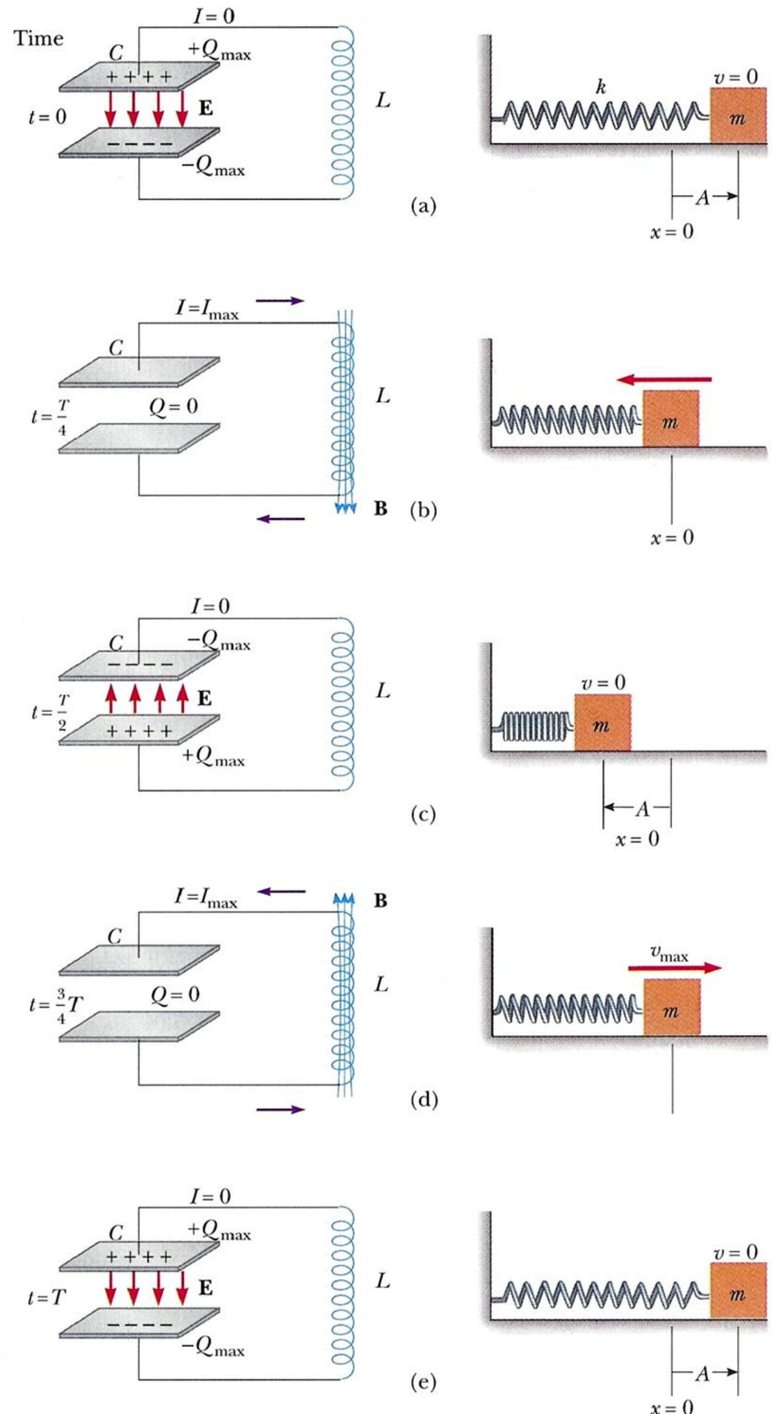
$$\frac{\mathrm{d}E}{\mathrm{d}t} = 0 = Li\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{q}{C}\frac{\mathrm{d}q}{\mathrm{d}t}$$

$\ddot{q}$ i

$$\frac{\mathrm{d}^2 q}{\mathrm{d}t^2} + \frac{1}{LC} q = 0$$

ω^2

$$q(t) = Q\cos(\omega t + \varphi)$$



$$E = E_{\mathrm{k}} + E_{\mathrm{p}} = \frac{mv^2}{2} + \frac{kx^2}{2}$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = 0 = mv\frac{\mathrm{d}v}{\mathrm{d}t} + kx\frac{\mathrm{d}x}{\mathrm{d}t}$$

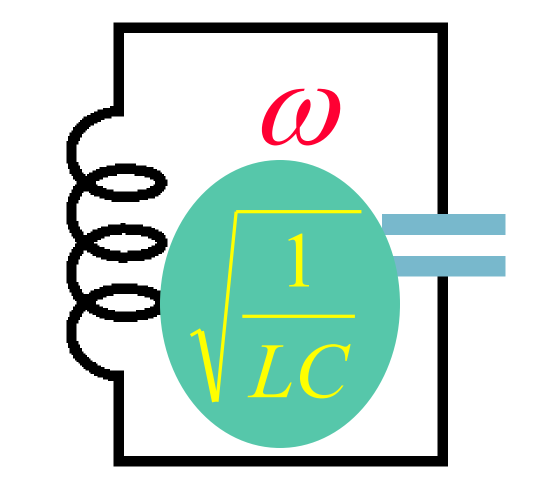
$\ddot{x}$ v

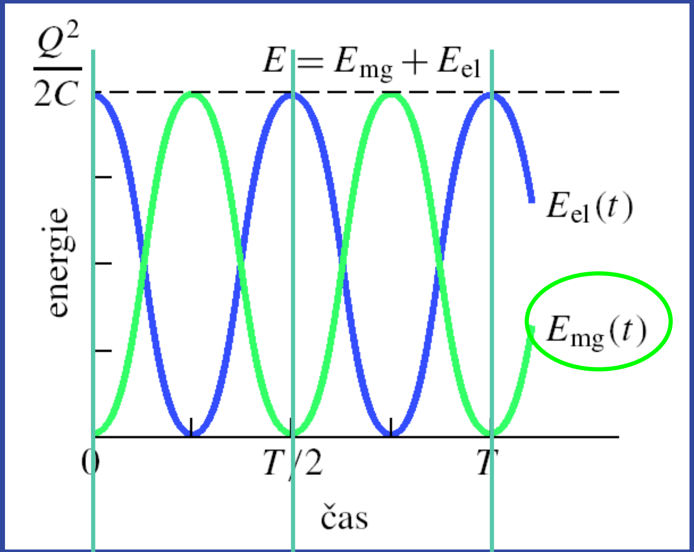
$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \frac{k}{m} x = 0$$

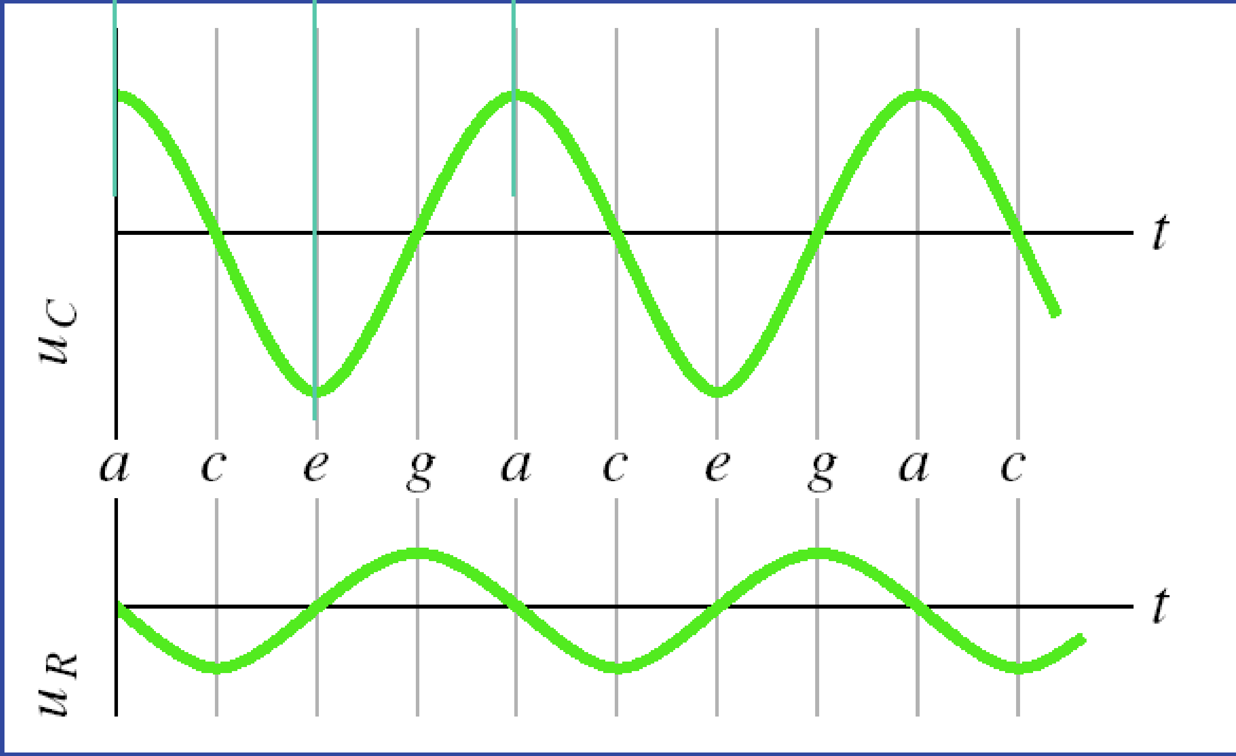
ω^2

$$x(t) = X\cos(\omega t + \varphi)$$

Oscilátor LC







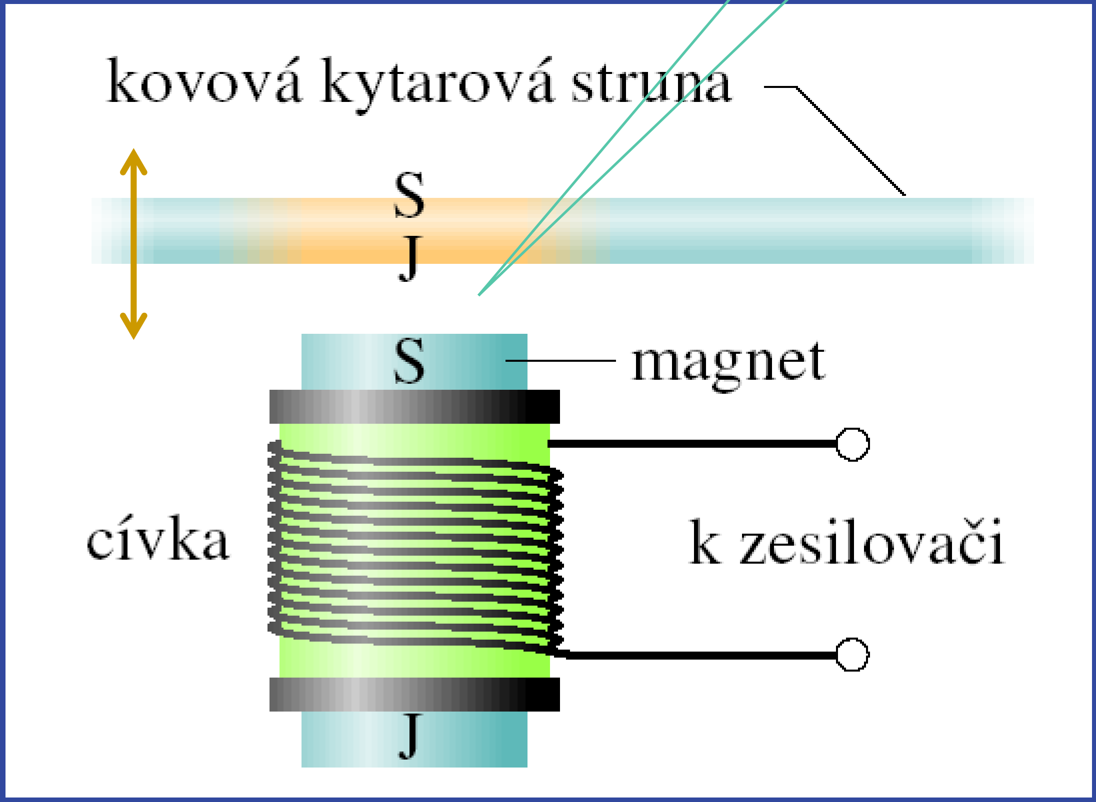
$$q(t) = Q\cos\omega t$$

$$i(t) = -I\sin\omega t$$

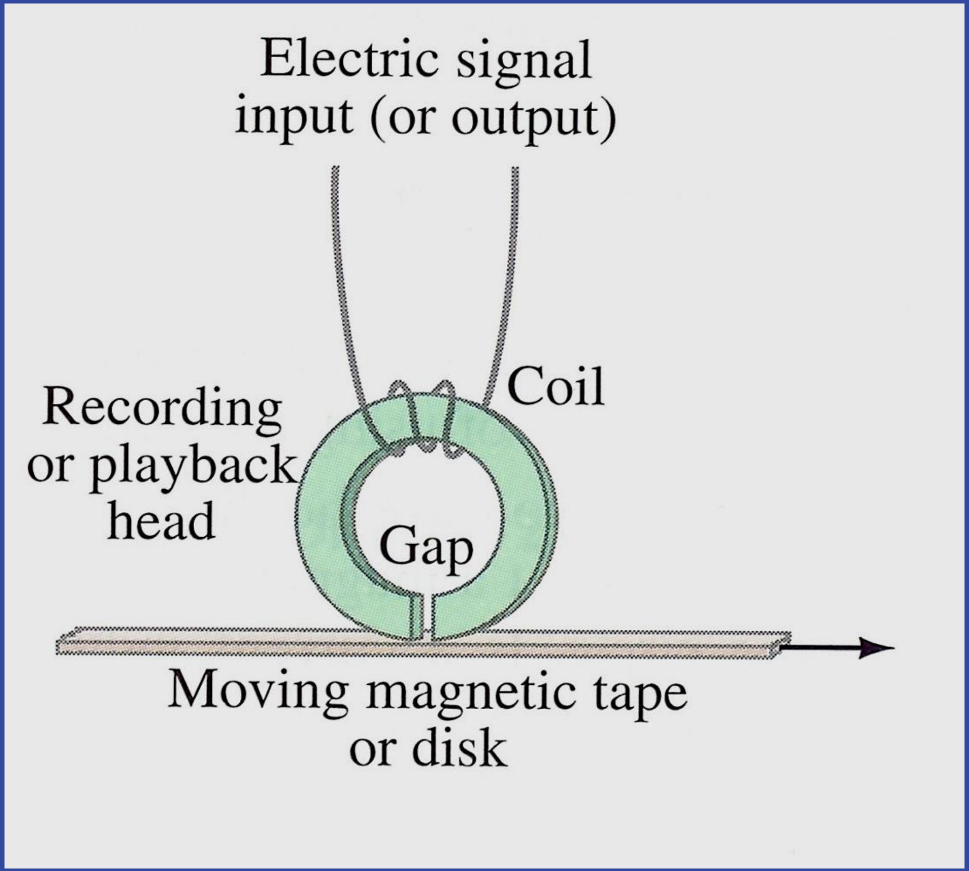
Aplikace

Čidla, telefon …

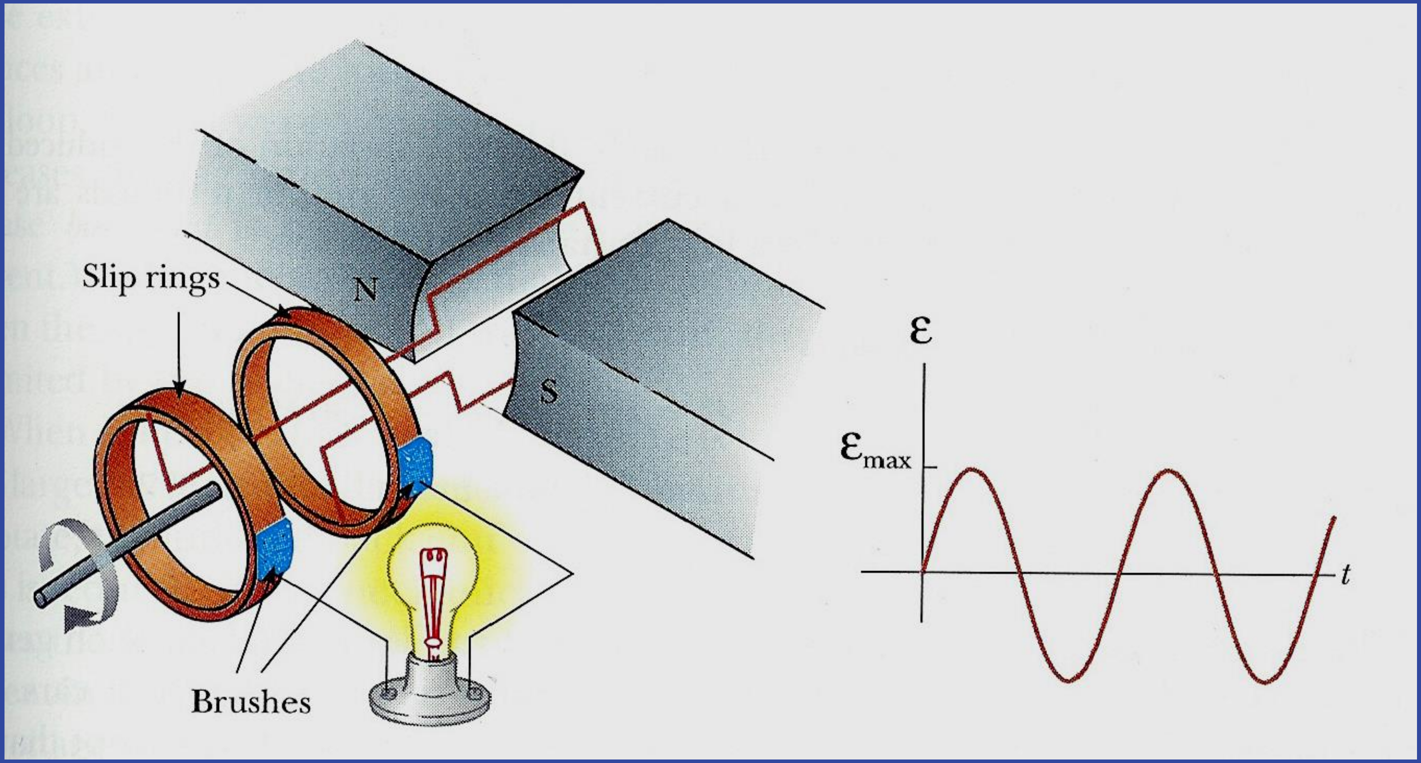
$\Phi(t)$



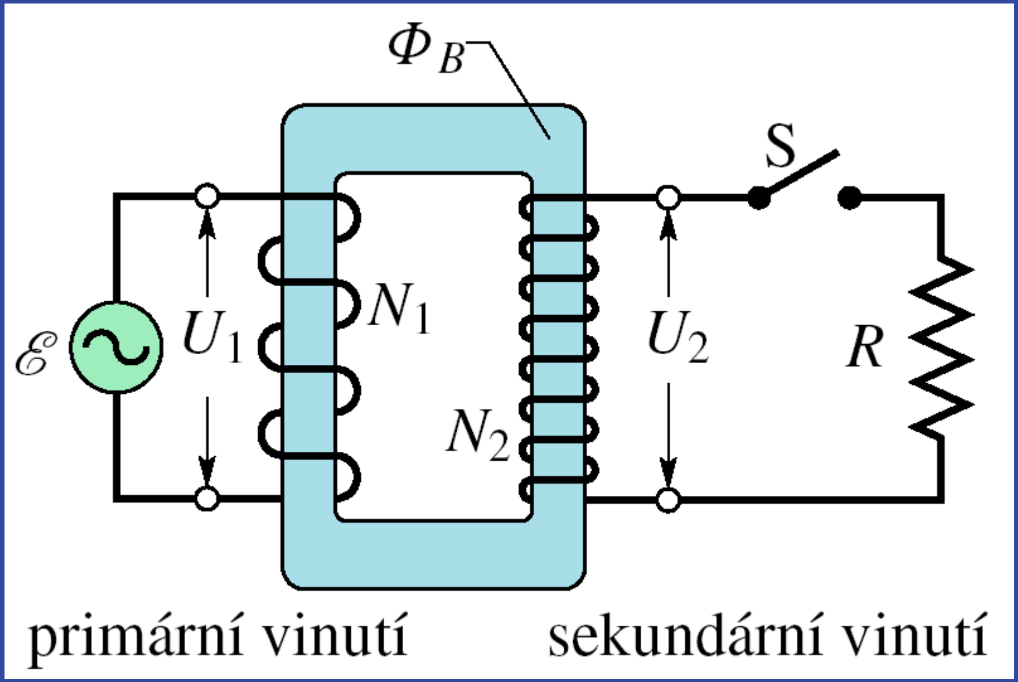
Záznam signálu na magnetické medium



Výroba elektrického proudu



Transformátor

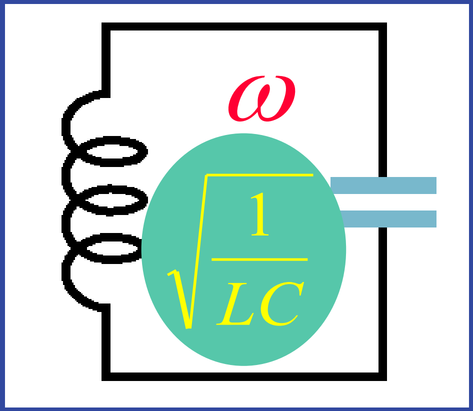


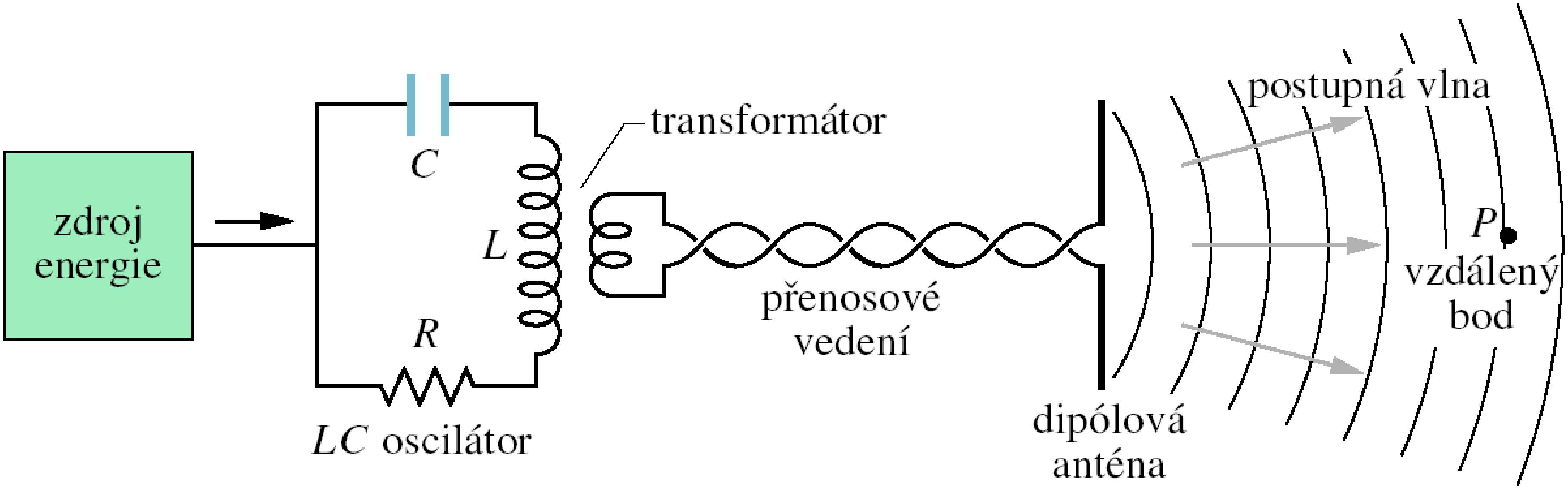
$$\mathcal{E}(t) - \frac{\mathrm{d}(N_1\Phi)}{\mathrm{d}t} = 0$$

$$U_2 = -\frac{\mathrm{d}(N_2\Phi)}{\mathrm{d}t}$$

$$\boxed{\frac{U_2}{U_1} = \frac{N_2}{N_1}}$$

Generátor oscilací





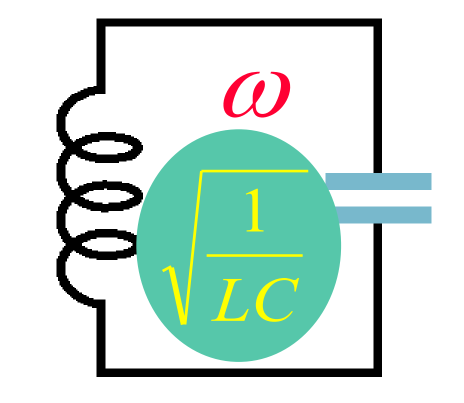
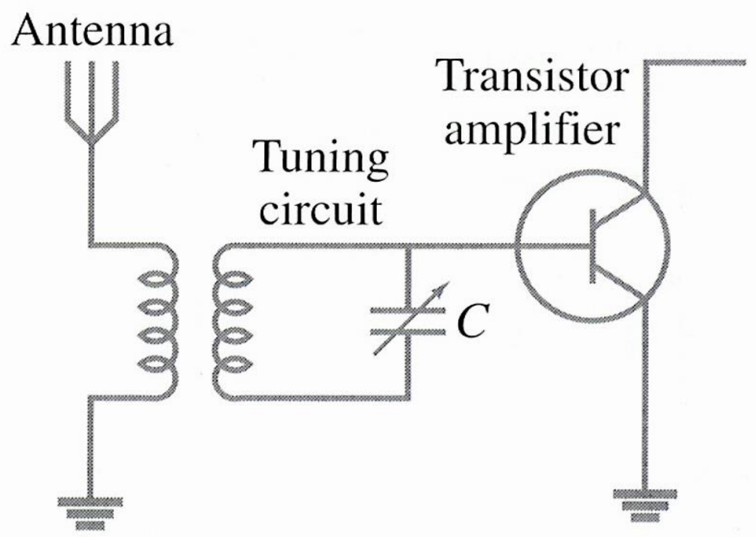


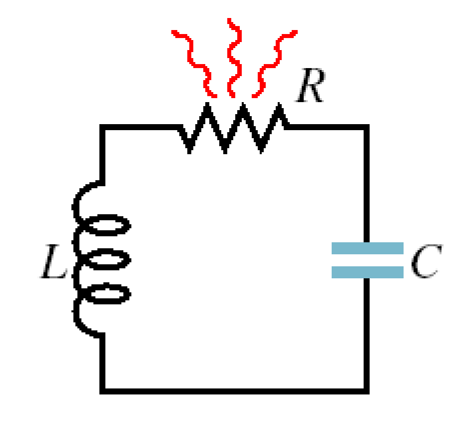
FIGURE 32–20 Simple tuning stage of a radio.



Tlumené kmity v obvodu RLC

energie v obvodu:

$$E = E_{\mathrm{mg}} + E_{\mathrm{el}} = \frac{Li^2}{2} + \frac{q^2}{2C}.$$



její úbytek:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -Ri^2$$

po úpravě:

$$L\frac{\mathrm{d}^2 q}{\mathrm{d}t^2} + R\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{1}{C}q = 0$$

řešení:

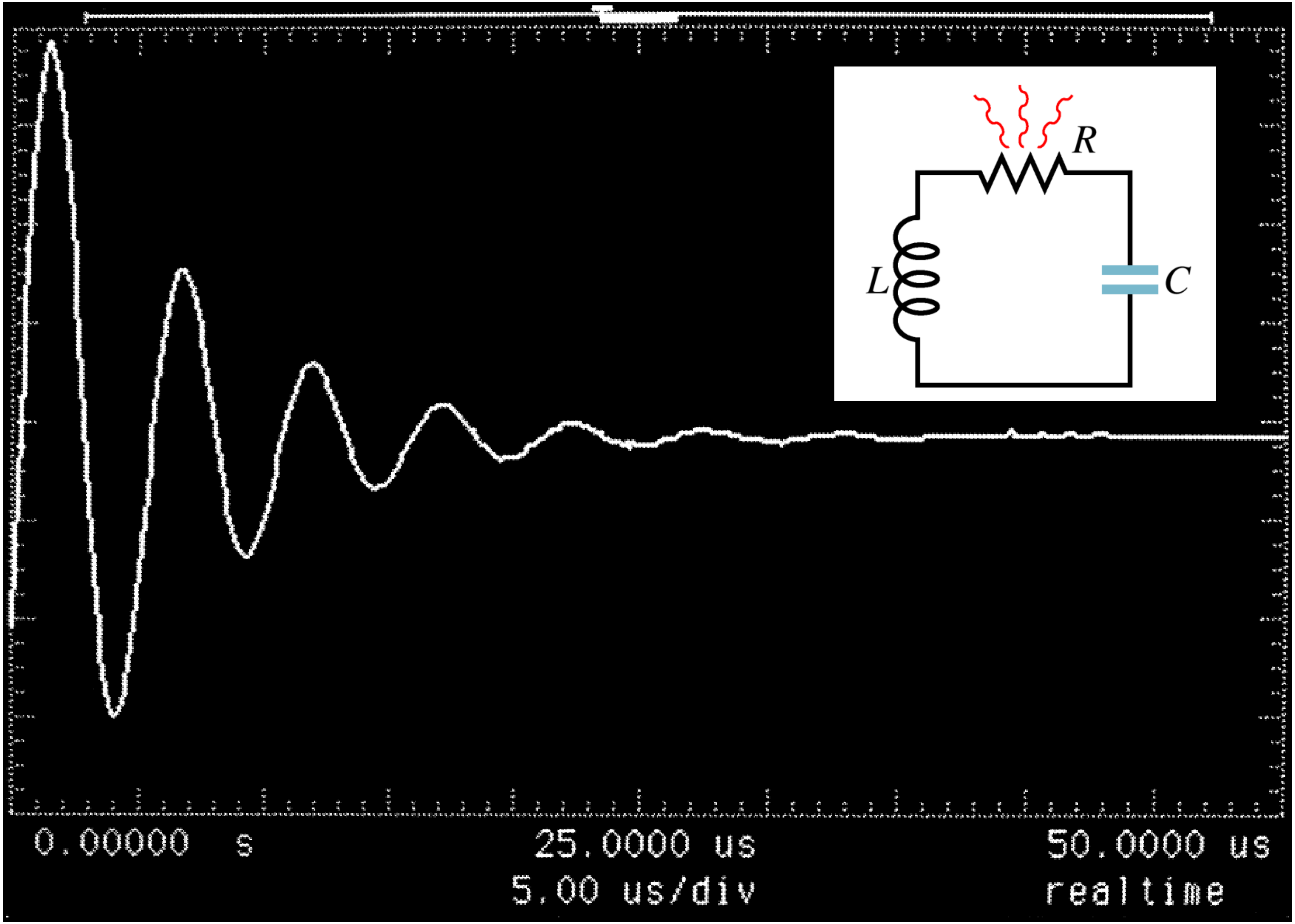
$$q = Q\mathrm{e}^{-Rt/(2L)}\cos(\omega' t + \varphi)$$

$$\omega' = \sqrt{\omega^2 - (R/2L)^2}$$

$$\omega = 1/\sqrt{LC}$$

střední hodnota energie v obvodu:

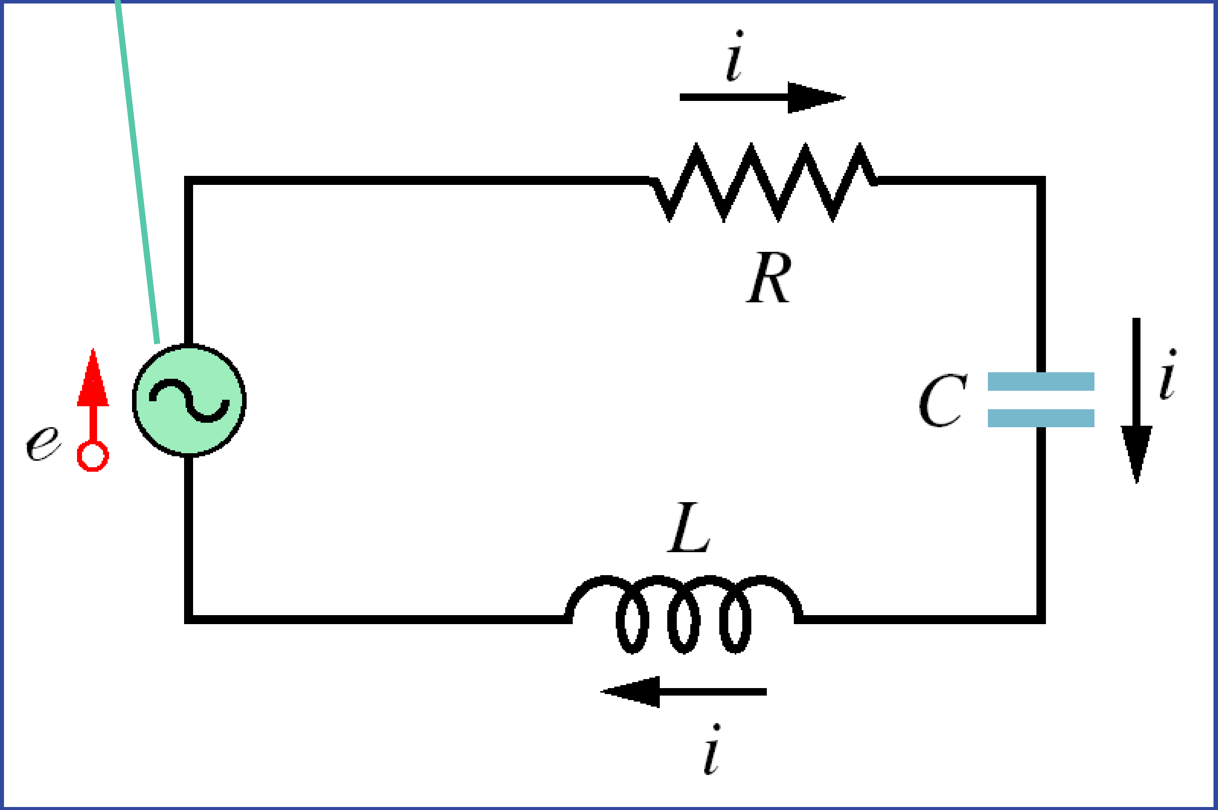
$$E = \frac{Q^2}{2C}\mathrm{e}^{-Rt/L}.$$


R
L
C
0.00000 s
25.0000 us
50.0000 us
5.00 us/div
realtime

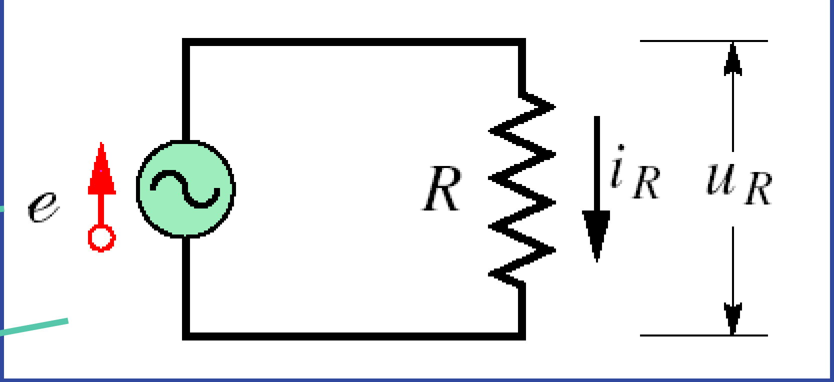
Nucené kmity v obvodech RLC

$$e = \mathscr{E} \sin \omega_b t \,.$$

$$i = I \sin(\omega_b t - \varphi),$$



Odporová zátěž



$$e = \mathscr{E} \sin \omega_b t .$$

$$e - u_R = 0.$$

$$u_R = \mathscr{E} \sin \omega_b t .$$

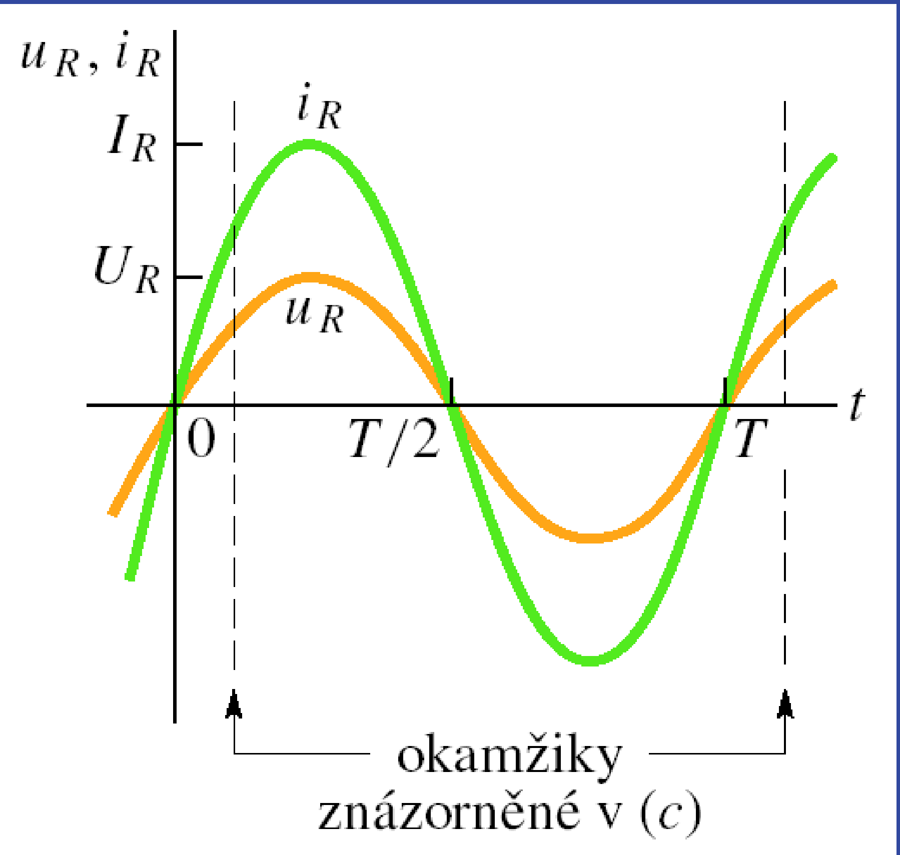
$$u_R = U_R \sin \omega_b t .$$

$$i_R = \frac{u_R}{R} = \frac{U_R}{R} \sin \omega_b t .$$

$$\varphi = 0^\circ$$

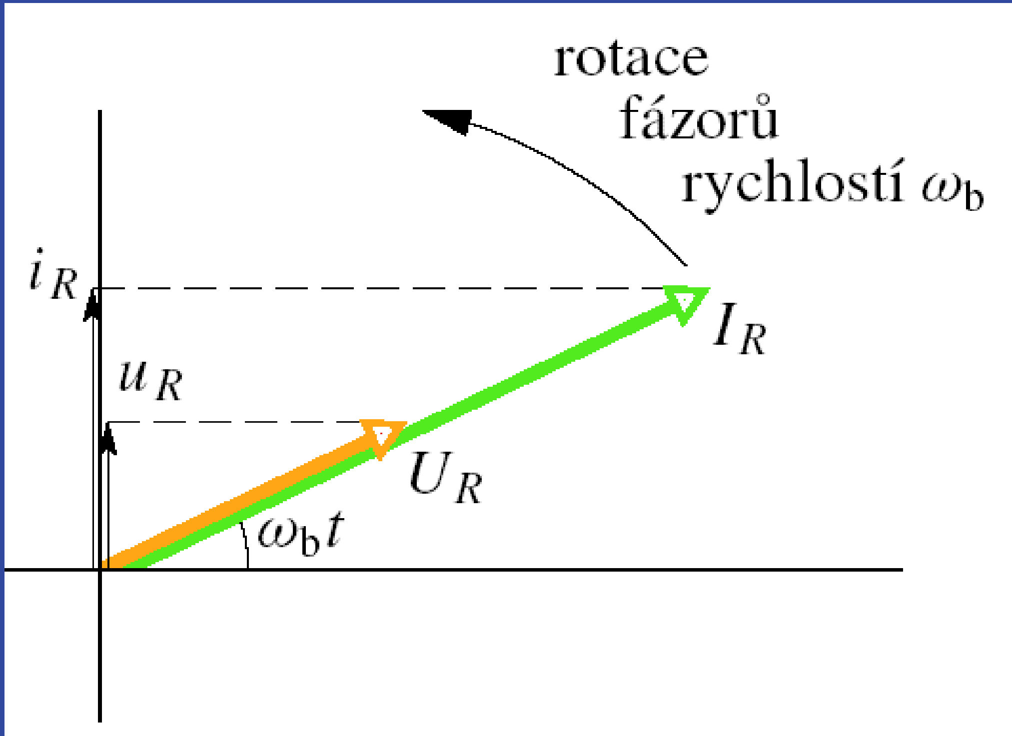
$$U_R = I_R R \qquad \text{(rezistor)}.$$

Odporová zátěž



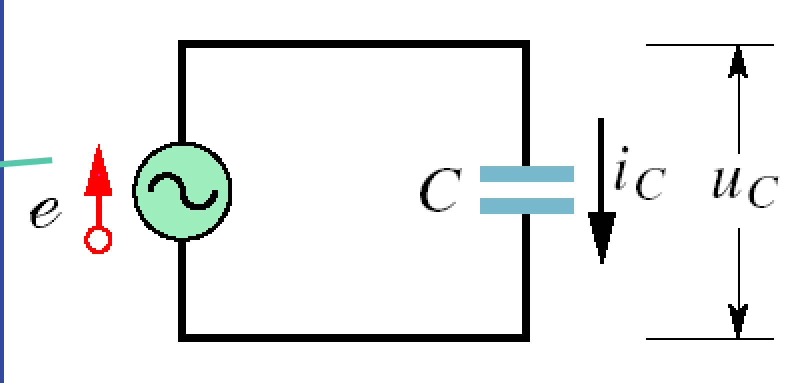
$$i_R = \frac{u_R}{R} = \frac{U_R}{R}\sin\omega_b t.$$

$$u_R = U_R \sin\omega_b t.$$



$$\varphi = 0^\circ$$

Kapacitní zátěž



$$u_C = U_C \sin \omega_b t$$

$$q_C = C u_C = C U_C \sin \omega_b t$$

$$i_C = \frac{dq_C}{dt} = \omega_b C U_C \cos \omega_b t$$

$$X_C = \frac{1}{\omega_b C} \qquad \text{(kapacitní reaktance).}$$

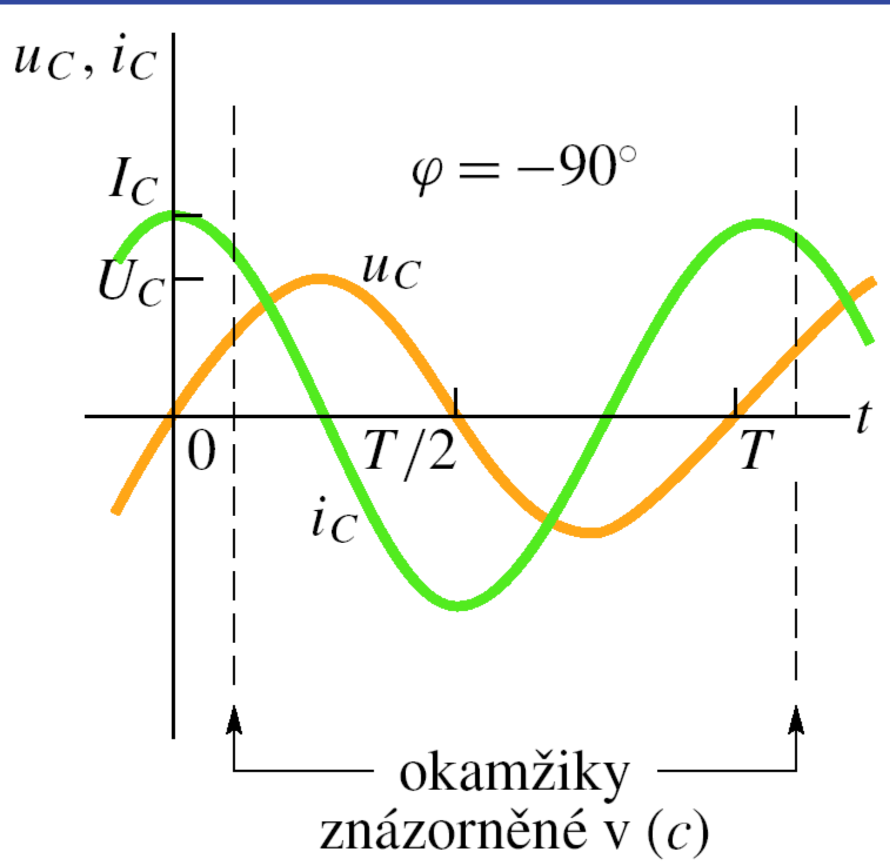
$$\cos \omega_b t = \sin(\omega_b t + 90^\circ)$$

$$i_C = \left(\frac{U_C}{X_C}\right) \sin(\omega_b t + 90^\circ)$$

$$U_C = I_C X_C$$

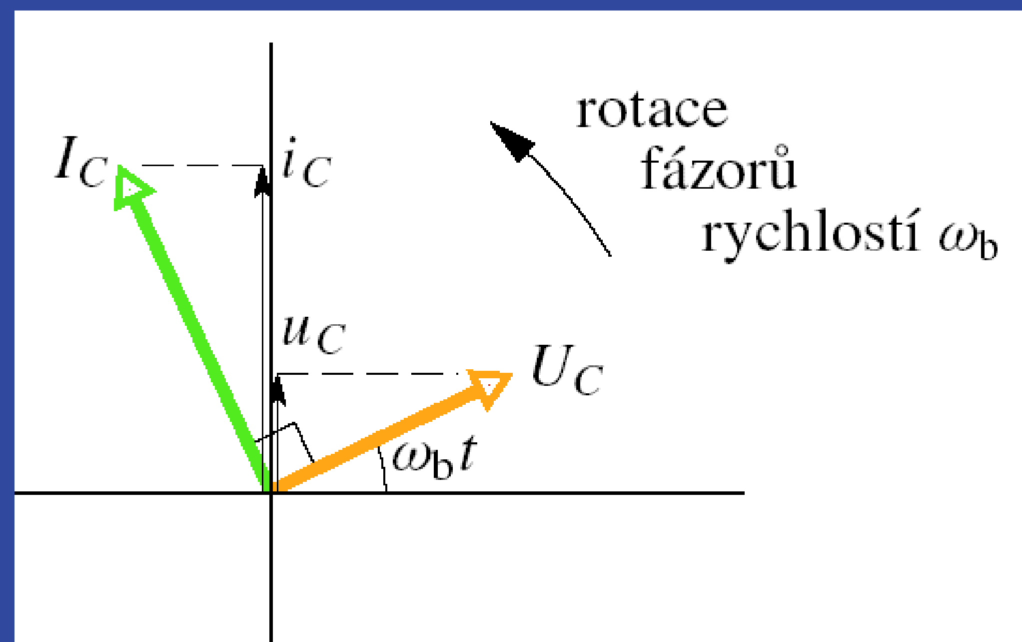
$$\varphi = -90^\circ$$

Kapacitní zátěž



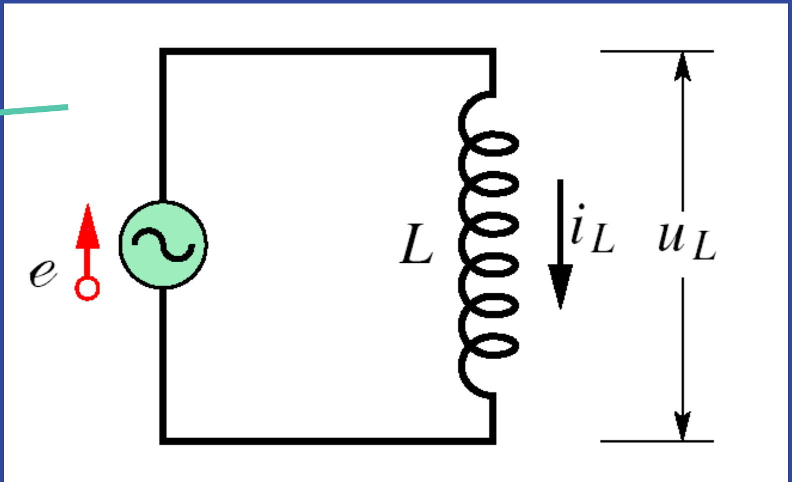
$$u_C = U_C \sin \omega_b t$$

$$i_C = \left(\frac{U_C}{X_C}\right) \sin(\omega_b t + 90^\circ)$$



$$\varphi = -90^\circ$$

Induktivní zátěž



$$u_L = U_L \sin \omega_b t$$

$$u_L = L \frac{di_L}{dt}.$$

$$\frac{di_L}{dt} = \frac{U_L}{L} \sin \omega_b t.$$

$$i_L = -\left(\frac{U_L}{\omega_b L}\right) \cos \omega_b t$$

$$X_L = \omega_b L \qquad \text{(induktivní reaktance).}$$

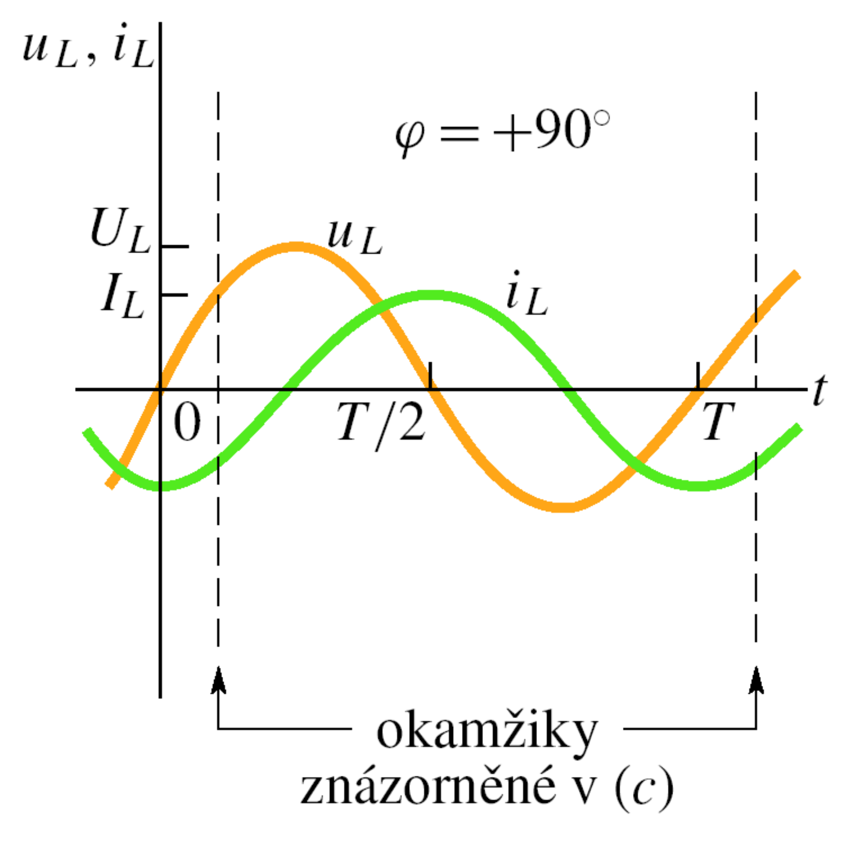
$$-\cos \omega_b t = \sin(\omega_b t - 90^\circ).$$

$$i_L = \frac{U_L}{X_L} \sin(\omega_b t - 90^\circ).$$

$$U_L = I_L X_L$$

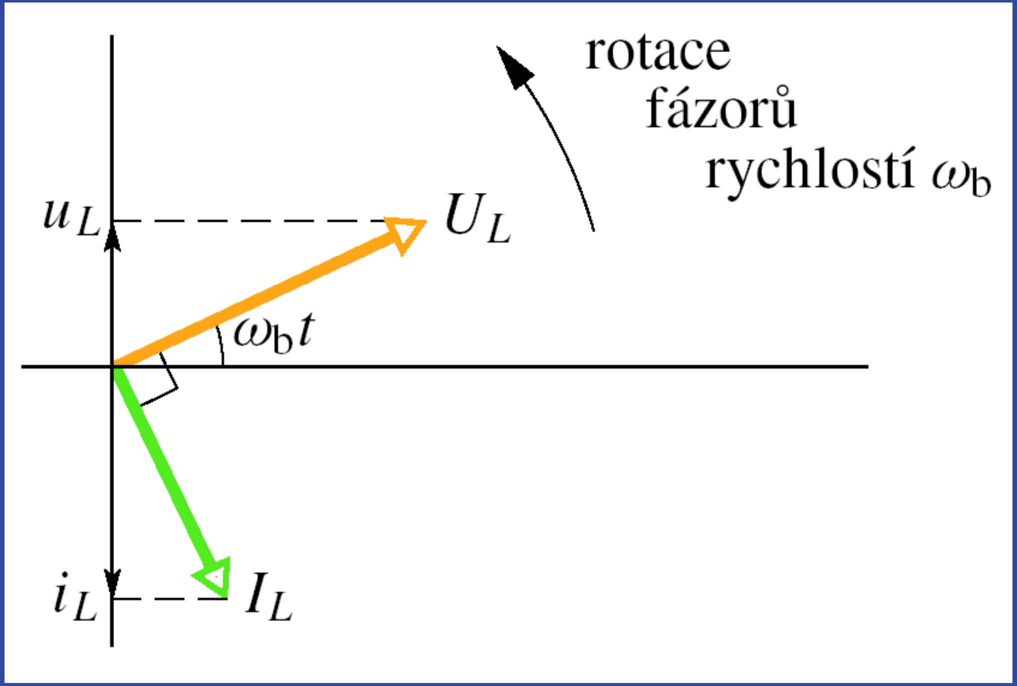
$$\varphi = +90^\circ$$

Induktivní zátěž



$$u_L = U_L \sin \omega_b t$$

$$i_L = \frac{U_L}{X_L} \sin(\omega_b t - 90^\circ).$$



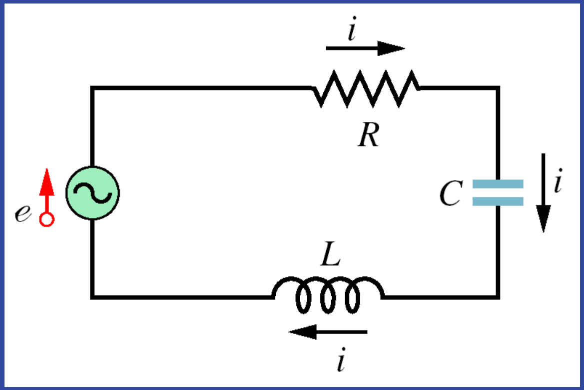
$$\varphi = +90^\circ$$

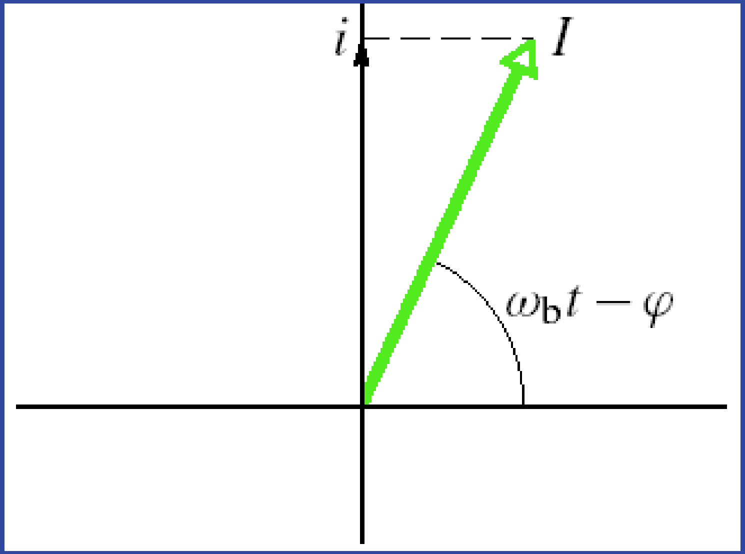
Tabulka 33.2 Vztahy mezi amplitudou a fází pro střídavé proudy a napětí

Obvodový prvek	Symbol	Rezistance nebo reaktance*	Fáze proudu	Fázový posun φ	Vztah mezi amplitudami
Rezistor	R	R	ve fázi s u_R	$0°$	$U_R = I_R R$
Kondenzátor	C	$X_C = 1/(\omega_b C)$	předbíhá u_C o $90°$	$-90°$	$U_C = I_C X_C$
Cívka	L	$X_L = \omega_b L$	zpožděna za u_L o $90°$	$+90°$	$U_L = I_L X_L$

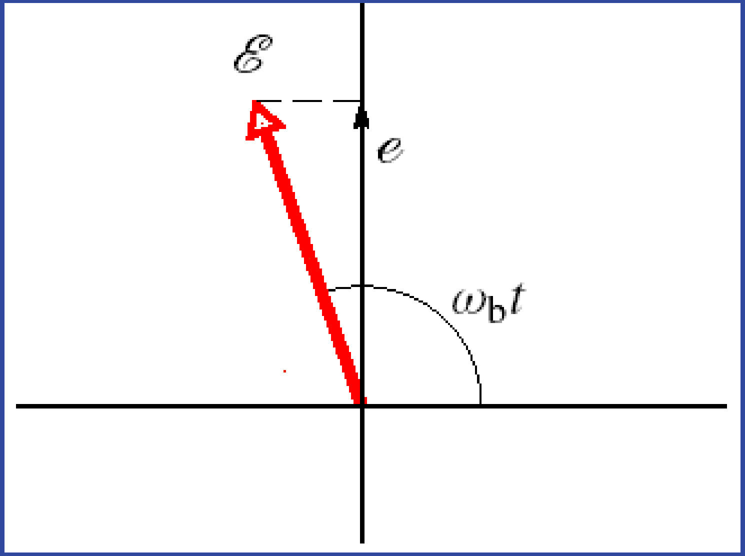
* Někdy se pro kapacitní reaktanci užívá název **kapacitance** a pro induktivní reaktanci název **induktance**.

Sériový obvod RLC

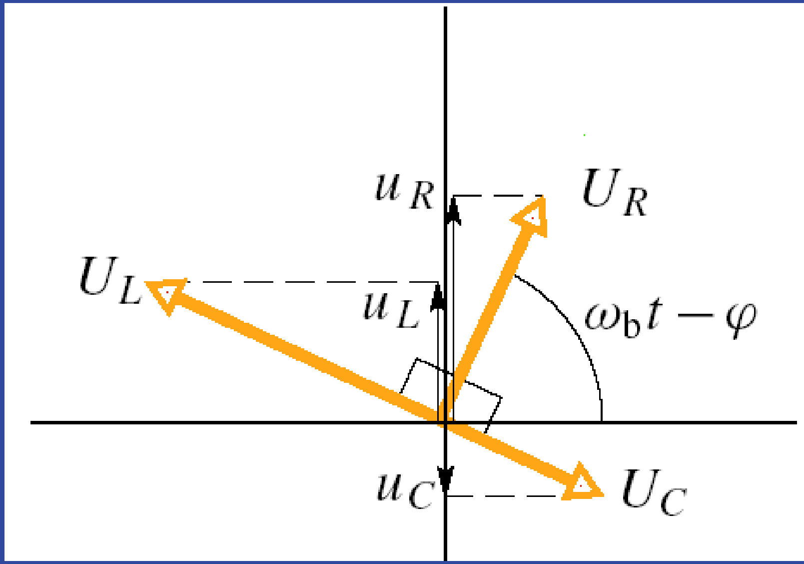




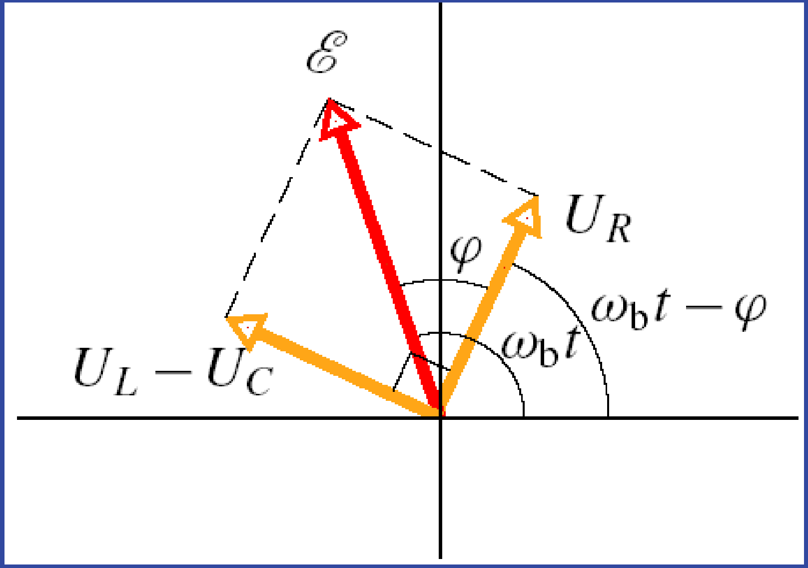
$$i = I \sin(\omega_b t - \varphi)$$



$$e = \mathscr{E} \sin \omega_b t$$



$$e = u_R + u_C + u_L.$$



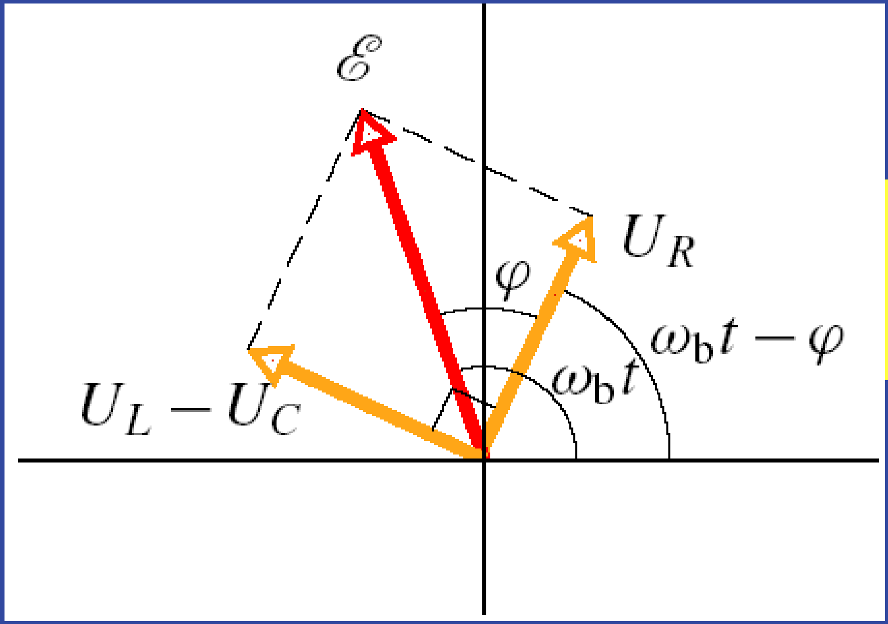
$$\mathcal{E}^2 = U_R^2 + (U_L - U_C)^2.$$

$$\mathcal{E}^2 = (IR)^2 + (IX_L - IX_C)^2$$

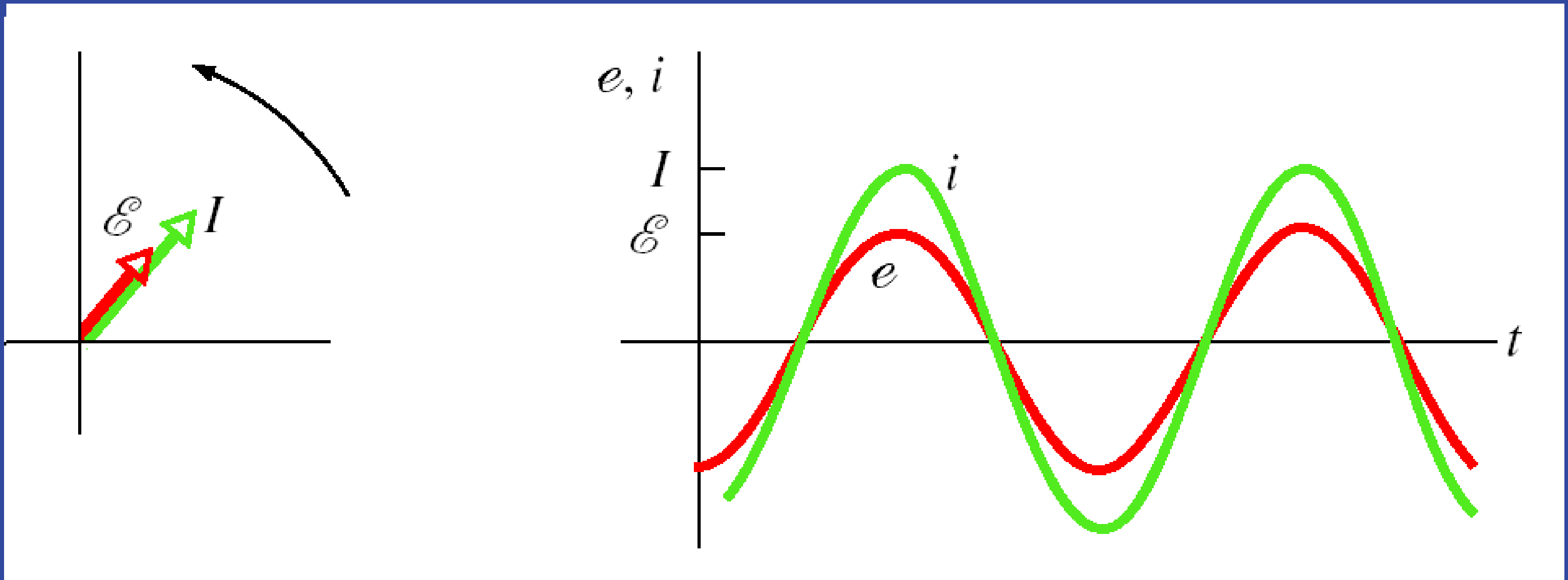
amplituda proudu:

$$I = \frac{\mathcal{E}}{\sqrt{R^2 + (X_L - X_C)^2}}.$$

impedance



$$\operatorname{tg}\varphi = \frac{X_L - X_C}{R} \qquad \text{(fázový posun).}$$



nulová φ

Rezonance

$$\omega_b = \omega = \frac{1}{\sqrt{LC}} \qquad \text{(rezonanční frekvence).}$$

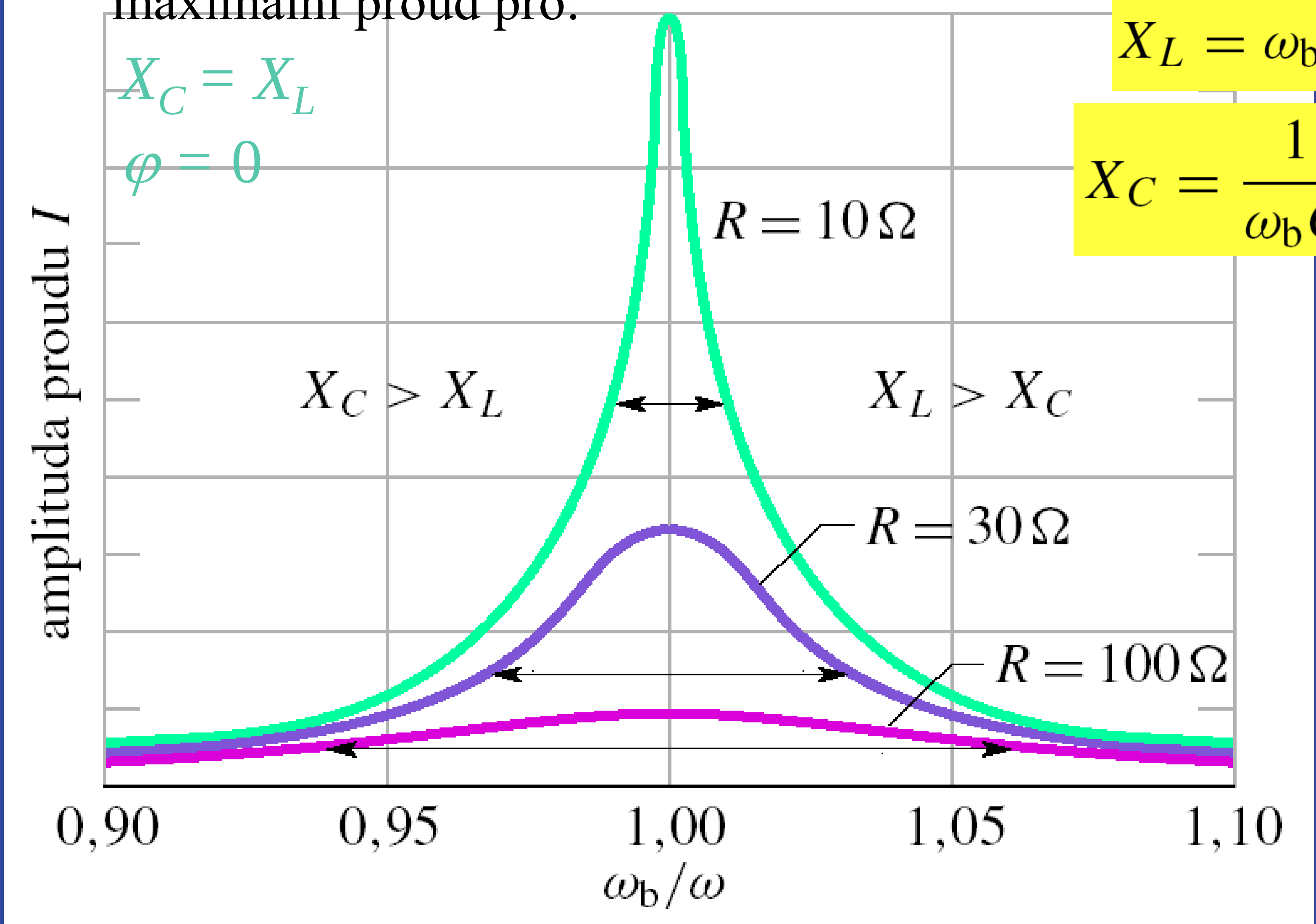
maximální proud pro:

$X_C = X_L$

$\varphi = 0$

$$X_L = \omega_b L$$

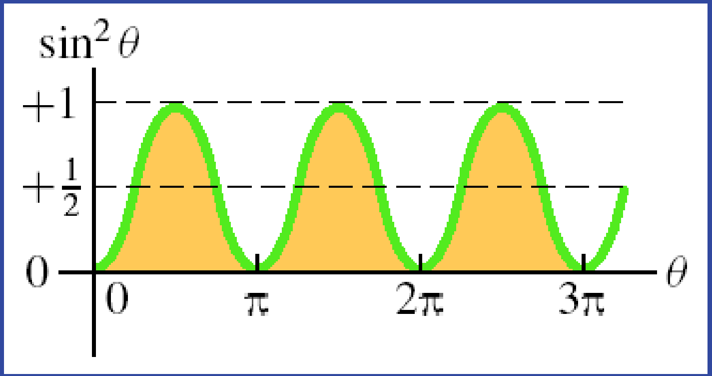
$$X_C = \frac{1}{\omega_b C}$$



Výkon v obvodech se střídavým proudem

okamžitý výkon v rezistoru:

$$P = i^2 R = I^2 R \sin^2(\omega_b t - \varphi).$$



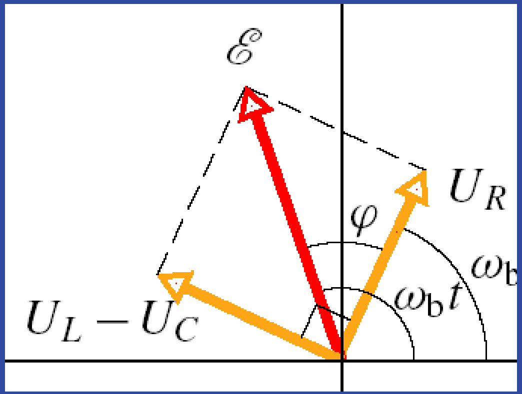
střední výkon:

$$\overline{P} = \frac{I^2 R}{2} = \left(\frac{I}{\sqrt{2}}\right)^2 R.$$

I_{ef}

$$U_{ef} = \frac{U}{\sqrt{2}} \quad \text{a} \quad \mathscr{E}_{ef} = \frac{\mathscr{E}}{\sqrt{2}}$$

(efektivní napětí a emn)



$$\cos\varphi = \frac{U_R}{\mathscr{E}} = \frac{IR}{IZ} = \frac{R}{Z}.$$

$$\overline{P} = \frac{\mathscr{E}_{ef}}{Z} I_{ef} R = \mathscr{E}_{ef} I_{ef} \frac{R}{Z}. = \mathscr{E}_{ef} I_{ef} \cos\varphi$$