Sequences 3: Fibonacci

http://www.youtube.com/watch?v=_NmSEEhtc1U

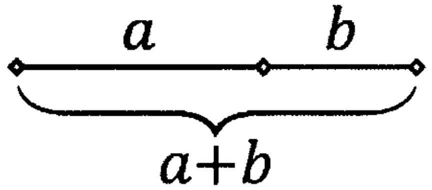
1) What is a sequence, how is it different from a series?
2) Do you know who Fibonacci was?

Answer questions.

1. Which problems were dealt with in Fibonacci´ s book called the Book of Squares?
2. What is inductive reasoning and why is it a very pleasant experience?
3. Describe the pattern followed in the Fibonacci sequence.
4. What is the connection between the Da Vinci Code and the Fibonacci sequence?
5. What is interesting about male honeybees?
6. How are they different from female honeybees?
7. When was Fibonacci´ s book written?

Golden ratio

From Wikipedia, the free encyclopedia



$a+b$ is to a as a is to b

The **golden section** is a line segment divided according to the golden ratio: The total length ***a* + *b*** is to the length of the longer segment ***a*** as the length of ***a*** is to the length of the shorter segment ***b***.

a) Answer these questions.

1. **What does it mean when two quantities are in the golden ratio?**
2. **How many synonyms are there of the golden ratio? How would you translate it into Czech (Slovak)?**
3. **How do you distinguish (in notation) the golden ratio and its reciprocal?**
4. **Why was the golden ratio interesting for architects?**
5. **Why were mathematicians interested in it?**

In mathematics and the arts, two quantities are in the **golden ratio** if the ratio of the sum of the quantities to the larger quantity is equal to the ratio of the larger quantity to the smaller one. The golden ratio is an irrational mathematical constant, approximately 1.6180339887. Other names frequently used for the golden ratio are the **golden section** (Latin: *sectio aurea*) and **golden mean**. Other terms encountered include **extreme and mean ratio, medial section, divine proportion, divine section** (Latin: *sectio divina*), **golden proportion, golden cut, golden number**, and **mean of Phidias.** In this article the golden ratio is denoted by the Greek lowercase letter phi (φ) , while its reciprocal, $1/\varphi$ or φ^{-1}, is denoted by the uppercase variant Phi (Φ).

The figure on the right illustrates the geometric relationship that defines this constant. Expressed algebraically:

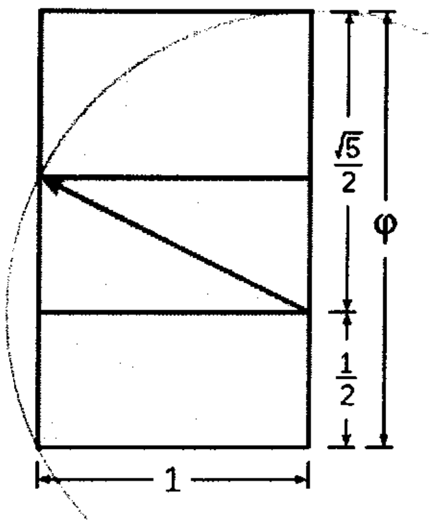
$$\frac{a+b}{a} = \frac{a}{b} \equiv \varphi.$$

This equation has one positive solution in the set of algebraic irrational numbers:

$$\varphi = \frac{1+\sqrt{5}}{2} = 1.6180339887\ldots$$

At least since the Renaissance, many artists and architects have proportioned their works to approximate the golden ratio—especially in the form of the golden rectangle, in which the ratio of the longer side to the shorter is the golden ratio—believing this proportion to be aesthetically pleasing. Mathematicians have studied the golden ratio because of its unique and interesting properties.

b) Study the drawing of the golden rectangle and try to write instructions for its construction.

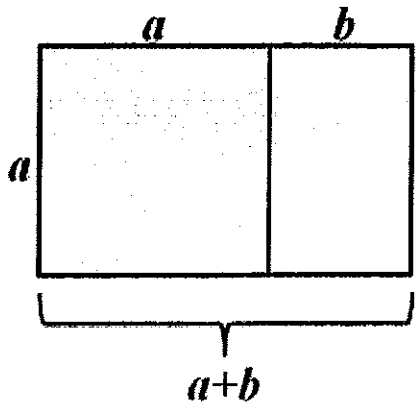


Construction of a golden rectangle:

1. ....

2. ....

3. ....



A golden rectangle with longer side ***a*** and shorter side ***b***, when placed adjacent to a square with sides of length ***a***, will produce a similar golden rectangle with longer side ***a* + *b*** and shorter side ***a***. This illustrates the relationship $\frac{a+b}{a} = \frac{a}{b} \equiv \varphi$.

Geometry

c) Try to explain the meaning of these words.

diagonal **regular icosahedron** **orthogonal**

apothem **geometric progression** **arithmetic progression** **perimeter**

irrational number **pyramid** **tangent** **coincidental relationship**

d) Read mathematical notation in the part Relationship to the Fibonacci sequence.

The number φ turns up frequently in geometry, particularly in figures with pentagonal symmetry. The length of a regular pentagon's diagonal is φ times its side. The vertices of a regular icosahedron are those of three mutually orthogonal golden rectangles.

Relationship to the Fibonacci sequence

The mathematics of the golden ratio and of the Fibonacci sequence are intimately interconnected. The Fibonacci sequence is:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, ...

The closed-form expression for the Fibonacci sequence involves the golden ratio:

$$F(n) = \frac{\varphi^n - (1-\varphi)^n}{\sqrt{5}} = \frac{\varphi^n - (-\varphi)^{-n}}{\sqrt{5}}.$$

The golden ratio is the limit of the ratios of successive terms of the Fibonacci sequence (or any Fibonacci-like sequence), as originally shown by Kepler:

$$\lim_{n\to\infty} \frac{F(n+1)}{F(n)} = \varphi.$$