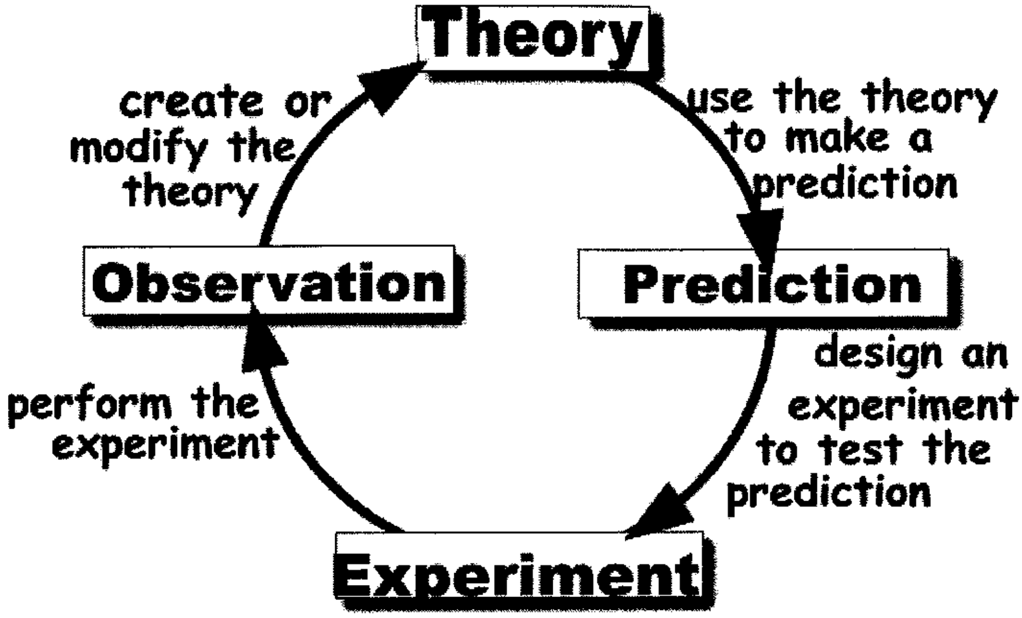
Scientific Method Steps

http://www.youtube.com/watch?v=bNc9vWLDSCA



Pre-listening.

1. The speaker mentions five steps you go through in the scientific method.
   Try to order these steps

   Experiment   Research   Conclusion   Question   Hypothesis

2. These are the verbs that combine with the steps – nouns. Match nouns and verbs.

   come up with   do   form   draw   design   ask   pose   test

   conduct   define   run   reject   prove

Listening. Listen to and watch the video and answer Qs.

1. In the speaker´s opinion, what is important for doing science? ....
2. What is typical for the scientist regarding his attitude to research? ....
3. What kind of a question does the speaker pose? ....
4. What is the difference between a scientific and non-scientific hypothesis? ....
5. What sort of an experiment would the speaker do? ....
6. What is drawing conclusions a tricky step? ....
7. What is the difference between a hypothesis and a theory? ....
8. Which theory does the speaker mention which was rejected by scientists in the past?
   ....

Listen to the beginning of the talk again and try to correct mistakes in these automatic subtitles.

depending on your teacher your textbook you'll see many different ways of
breaking down the scientific method but no matter how many were versions i've
seen the all seem to share some way
these five steps the first thing that whenever you're doing science is ask a
question
do some research
come up with a hypothesis
tested with a hypothesis it started with an experiment
and then finally draw conclusion
now some people have this idea that in order to do sign do you have to have
been wearing a lab coat and have a p_h_d_ that's not it at all
and its heart
decides on his views beatrix
instructions can questions about the world
for example of this could range anywhere from isn't important indeed as how much
your cancer too
as simple as what's the fastest way for me to get from san jose to san francisco
now that i've asked a question it's time for me to find out what the people who
know more thinking up
and this is where some people wonder running into trouble because
a scientist or somebody who's thinking like a scientist when they do their
research that always skeptical about
but they're reading

Section 2 Development

4. Read this:

Axioms

Some mathematical *laws* are accepted without *proof*. These fundamental laws are known as *axioms*. In algebra, for example, there are the following axioms.

1) $x+y = y+x$
2) $xy = yx$
3) $x+(y+z) = (x+y)+z$
4) $x(yz) = (xy)z$
5) $x(y+z) = xy+xz$
6) $x+0 = x$
7) $1x = x$
8) For every real number x, there is a real number y such that $x+y = 0$.
9) For every non-zero real number x, there is a real number y such that $xy = 1$.

- It is axiomatic that if $x+y = a$, then $y+x = a$.

Write similar sentences about the other axioms.

5. Read this:

- Axiom 1 is illustrated by the following example: $2+3 = 5; 3+2 = 5$.

Write similar sentences using these examples:

a) $1 \times 22 = 22$
b) 5 is a non-zero real number; 1/5 is a real number
c) $5 \times 7 = 35; 7 \times 5 = 35$
d) $3 \times (11+19) = 3 \times 30 = 90; (3 \times 11)+(3 \times 19) = 33+57 = 90$
e) $3+0 = 3$
f) $3+(4+5) = 3+9 = 12; (3+4)+5 = 7+5 = 12$
g) 5 is a real number: -5 is a real number
h) $2 \times (3 \times 4) = 2 \times 12 = 24; (2 \times 3) \times 4 = 6 \times 4 = 24$

Section 3 Reading

6. Read this:

Theorems

From the axioms given in the last section we can prove all the laws of algebra. Laws which are not axioms are called *theorems*. There are various methods of proving theorems. Some examples are given here.

1 *Proof by deduction*
Theorem (1) To prove: If $a+b = a+c$, then $b = c$.

Proof
By axiom (8), there is a number y such that $y+a = 0$.
Adding y to both sides gives $y+(a+b) = y+(a+c)$.
By axiom (3), we have $(y+a)+b = (y+a)+c$.
But $y+a = 0$, therefore $0+b = 0+c$.
By axiom (6), $0+b = b$ and $0+c = c$.
Therefore $b = c$.

2 and 3 *Proofs by elimination and contradiction*
Thereom (2) To prove: Given a and b, there is one and only one x such that $a+x = b$.

Proof:
There are three possibilities: more than one x, less than one x, exactly one x. If we eliminate two of these possibilities, then the third must be true. We can divide the proof into three parts.

a) To prove that there is not less than one x such that $a+x = b$.
By axiom (8), there is a number y such that $a+y = 0$.

Let $x = y+b$.
Then $a+x = a+(y+b)$.
By axiom (3) $a+x = (a+y)+b$.
But $a+y = 0$, therefore $a+x = 0+b$.
By axiom (6), $a+x = b$.
Hence there is not less than one x such that $a+x = b$.

b) To prove there is not more than one x such that $a+x = b$.
We can prove this by using proof by contradiction, i.e. we assume the opposite of what we are trying to prove and show that this leads to a contradiction.
Assume that there are several different x, such that $x_1 \neq x_2 \neq x_3$, etc.
and $a+x_1 = b, a+x_2 = b, a+x_3 = b$, etc.
i.e. $a+x_1 = a+x_2 = a+x_3$, etc.
By theorem (1), we have $x_1 = x_2 = x_3$, etc., but this contradicts our assumption that $x_1 \neq x_2 \neq x_3$, etc.
Therefore there is not more than one x such that $a+x = b$.

c) Two possibilities have been eliminated, therefore the only remaining possibility is that there is one and only one x such that $a+x = b$.

Say whether the following statements are true or false. Correct the false statements.

a) Theorems are fundamental laws.
b) There are not less than three ways of proving theorems.
c) All of the symbols a, b, x, y used in the above proofs refer to real numbers.
d) All of the symbols a, b, x, y used in the above proofs refer to non-negative numbers.
e) The three kinds of proof shown are mutually exclusive.

7. Look and read:

Here are the proofs of two more theorems. Complete them by inserting the correct words and say what kinds of proof are used.

a) Theorem (3): If $ab = ac$ and $a \neq 0$, then $b = c$.
By ...... there is a number y ...... $ya = 1$.
________ both sides by y, ... $y(ab) = y(ac)$.
By ...... $(ya)b = (ya)c$.
But $ya = 1$, ________ $1b = 1c$.
By ......, $1b = b$ and $1c = c$.
________ $b = c$.

b) Theorem (4): Given $a \neq 0$ and b, there is one and only one x such that $ax = b$.

There are three possibilities, ... one x, ... one x, ________ one x.

i) By ...... there is a number y ...... $ya = 1$.

________ $x = yb$.
________ $ax = a(yb)$.
By ......, we have $ax = (ay)b$.
________ $ay = 1$, ________ $ax = 1b$.
By ...... $ax = b$.
________ there is ......

ii) ________ $ax_1 = b, ax_2 = b, ax_3 = b$ etc.
and $x_1 \neq x_2 \neq x_3$ etc.
________ $ax_1 = ax_2 = ax_3$ etc.
By ......, we have $x_1 = x_2 = x_3$ etc.
But this ________ our ________ that $x_1 \neq x_2 \neq x_3$ etc.
________ there is ......

iii) ________ the only ________ possibility is that there is ......

Adapted from Nucleus - Maths

Proof: Beginnings (Trzeciak Jerzy,Writing Mathematical Papers in English. European Mathematical Society, 1995)

We prove (show, recall, observe) that……….Let us first outline (give the main ideas of) the proof. We claim that…. Our proof starts with the observation that……. Suppose ……. Assuming……… The proof falls naturally into three parts. To deduce (3) from (2), take………….. We have divided the proof into a sequence of lemmas. To see that ……..

Proof: Arguments

By definition….By assumption… By a similar argument…. Define …. Theorem 4 now shows that…. Since *f* is compact…… Therefore *Lf= 0* by Theorem 6. From this we conclude……….. We now prove that……..From what has already been proven……According to…From the above it follows that……. We conclude from (5) that ………………..Changing the formula we obtain….

Proof: Conclusions

This proves the theorem. This completes the proof. This establishes the formula. This is the desired conclusion. Q.E.D. This contradicts our assumption that …. The same proof works for…. Note that we have actually proven that….. The proof is strongly dependent on the assumption that …………Since ……we reached a contradiction…..The details are left to the reader. For more details we refer the reader to ….

Proof that *e* is irrational

From Wikipedia, the free encyclopedia

In mathematics, the series representation of Euler's number *e*

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$

can be used to prove that *e* is irrational. Of the many representations of e, this is the Taylor series for the exponential function e^y evaluated at $y = 1$.

Summary of the proof

This is Joseph Fourier's proof by contradiction. Initially *e* is assumed to be a rational number of the form a/b. We then analyze a blown-up difference *x* of the series representing *e* and its strictly smaller b^{th} partial sum, which approximates the limiting value *e*. By choosing the magnifying factor to be the factorial of *b*, the fraction a/b and the b^{th} partial sum are turned into integers, hence *x* must be a positive integer. However, the fast convergence of the series representation implies that the magnified approximation error *x* is still strictly smaller than 1. From this contradiction we deduce that *e* is irrational.

Now go through the proof and fill in the missing expressions using examples from proof writing.

Proof

Towards a contradiction, 1)………. e is a rational number. Then 2)………….. a and b such that $e = a/b$ where clearly $b > 1$.

3)…….. the number

$$x = b!\left(e - \sum_{n=0}^{b} \frac{1}{n!}\right)$$

To see that if e is rational, then x is an integer, 4)………e $e = a/b$ into this definition to 5)……

$$x = b!\left(\frac{a}{b} - \sum_{n=0}^{b} \frac{1}{n!}\right) = a(b-1)! - \sum_{n=0}^{b} \frac{b!}{n!}.$$

The first term is an integer, and every fraction in the sum is actually an integer because $n \le b$ for each term. 6)………. x is an integer.

7)………….. $0 < x < 1$. First, to prove that x is strictly positive, we 8)……. the above series representation of e into the definition of x and obtain

$$x = b!\left(\sum_{n=0}^{\infty} \frac{1}{n!} - \sum_{n=0}^{b} \frac{1}{n!}\right) = \sum_{n=b+1}^{\infty} \frac{b!}{n!} > 0,$$

because all the terms with $n \le b$ cancel and the remaining ones are strictly positive.

We now prove that $x < 1$. For 9)…….. with $n \ge b + 1$ we have the upper estimate

$$\frac{b!}{n!} = \frac{1}{(b+1)(b+2)\cdots(b+(n-b))} \le \frac{1}{(b+1)^{n-b}}.$$

This inequality is strict for every $n \ge b + 2$. 10)……… the index of summation to $k = n - b$ and using the formula for the infinite geometric series, we obtain

$$x = \sum_{n=b+1}^{\infty} \frac{b!}{n!} < \sum_{n=b+1}^{\infty} \frac{1}{(b+1)^{n-b}} = \sum_{k=1}^{\infty} \frac{1}{(b+1)^k} = \frac{1}{b+1}\left(\frac{1}{1-\frac{1}{b+1}}\right) = \frac{1}{b} <$$

Since there is no integer strictly between 0 and 1, we have 11)………., and so e must be irrational. 12)…………