Př: $x = \cos x$

$g(x) = \cos x, \; x \geq 0$

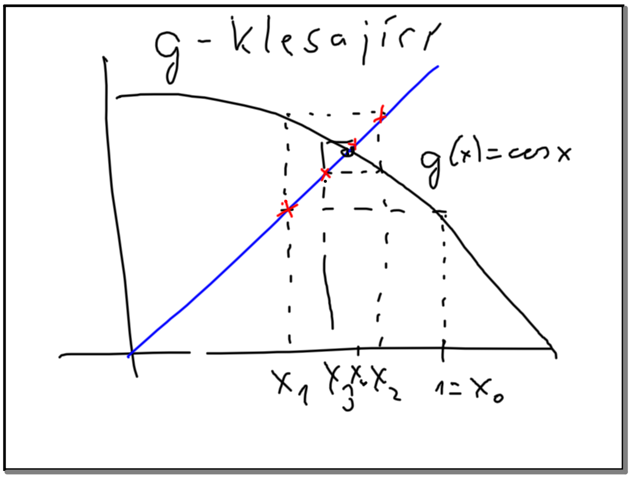
$x \in [0, \frac{\pi}{2}] = I$?

$g(0) = 1 \in I, \; g(\frac{\pi}{2}) = 0 \in I$

$|g'(x)| = \sin x, \; \sin\frac{\pi}{2} = 1$ – nelze

$I = [0,1], \; g(I) \subseteq I, \; L = \sin 1 < 1$

2 26-16:04



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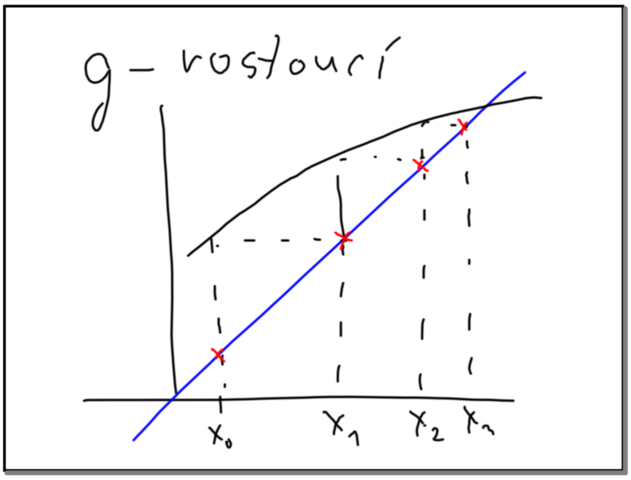

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Řád metody půlení intervalu

$|e_{k+1}| \approx \frac{1}{2}|e_k|$

$\left|\frac{e_{k+1}}{e_k}\right| \approx \frac{1}{2}, \; \lim_{k\to\infty} \frac{|e_{k+1}|}{|e_k|^1} = \frac{1}{2}$

$p = 1$ – metoda řádu 1

2 26-16:49

$p = 2$ $\quad \frac{|e_{k+1}|}{|e_k|^2} \approx C$ $\quad |e_{k+1}| \approx C \cdot |e_k|^2$

$|e_k| \approx \frac{1}{10}$

$|e_{k+1}| \approx \frac{1}{100}, \; |e_{k+2}| \approx 10^{-4}$

$|e_{k+3}| \approx 10^{-8}$

2 26-16:53

Důkaz řádu metody

$x_{k+1} = g(x_k)$

$g(\xi + h) = \underbrace{g(\xi)}_{\xi} + \frac{1}{1!}\overset{=0}{g'(\xi)} \cdot h + \frac{1}{2!}\underbrace{g''(\xi)}_{=0} \cdot h^2 + \dots + \frac{1}{(p-1)!}\overset{=0}{g^{(p-1)}(\xi)} \cdot h^{p-1} +$

$+ \frac{1}{p!} g^{(p)}(\xi) \cdot h^p + o(h^p)$

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$$g(\xi+h) = \xi + \frac{1}{p!} g^{(p)}(\xi)\cdot h^p + o(h^p)$$

$$g(\xi+h) - g(\xi) = g(\xi+h) - \xi =$$

$$= \frac{1}{p!} g^{(p)}(\xi)\cdot h^p + o(h^p)$$

$$x_k = \xi + h \;, \quad x_{k+1} = g(\xi+h)$$

chyba $\quad e_{k+1} = x_{k+1} - \xi$

$$e_k = h \qquad \frac{|e_{k+1}|}{|e_k|^p} = \frac{1}{p!}\,|g^{(p)}(\xi)|\cdot\cancel{\frac{|h|^p}{|h|^p}} + \underbrace{\frac{o(h^p)}{|h|^p}}_{\to 0}$$

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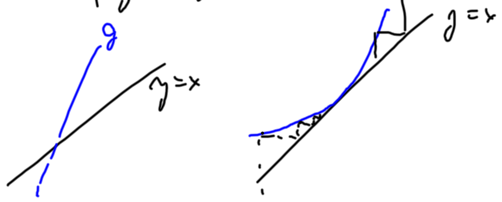
$$\left|\frac{g(x) - g(\xi)}{x-\xi}\right| < 1$$

$$|g(x) - g(\xi)| < 1\cdot|x-\xi|$$



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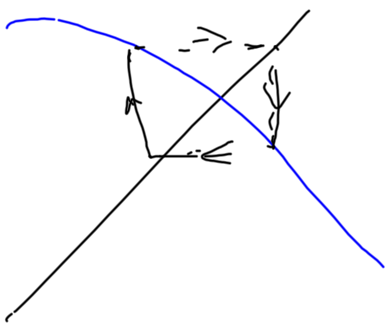

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