Metoda sečen - důkaz

Poměrné diference

f - funkce x, y, z

$$f[x,y] = \frac{f(x) - f(y)}{x-y} = f'(\xi_1)$$

$$f[x,y,z] = \frac{f[x,y] - f[y,z]}{x-z} = \frac{1}{2} f''(\xi_2)$$

$$x_{i+1} - \xi = x_i - \xi - \frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})} (f(x_i) - f(\xi))$$

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$$x_i - \xi - \frac{x_i - \xi - (x_{i-1} - \xi)}{f(x_i) - f(\xi) - (f(x_{i-1}) - f(\xi))} (f(x_i) - f(\xi))$$

$$= x_i - \xi - \frac{f(x_i) - f(\xi)}{f[x_i, x_{i-1}]} =$$

$$x_{i+1} - \xi = (x_i - \xi)(x_{i-1} - \xi) \frac{f[x_i, x_{i-1}, \xi]}{f[x_i, x_{i-1}]} =$$

$$= (x_i - \xi)(x_{i-1} - \xi) \frac{\frac{1}{2} f''(\alpha_i)}{f'(\beta_i)}$$

$|x_{i+1} - \xi| \le |x_i - \xi| \cdot |x_{i-1} - \xi| \cdot M$ na okolí ξ

Volíme $|x_1 - \xi| < 1$, $|x_0 - \xi| \cdot M < 1$

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Vzdálenost x_2 od ξ: $|x_2 - \xi|$

je menší než $|x_1 - \xi|$ $\quad x_i \to \xi$

$$x_{i+1} - \xi = (x_i - \xi)(x_{i-1} - \xi) \cdot C_i, \quad C_i \to \frac{\frac{1}{2} f''(\xi)}{f'(\xi)}$$

$$\frac{|x_{i+1} - \xi|}{|x_i - \xi|^p} \to L$$

$$\frac{|x_{i+1} - \xi|}{|x_i - \xi| |x_{i-1} - \xi|} \to \left| \frac{\frac{1}{2} f''(\xi)}{f'(\xi)} \right| = C$$

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$$\frac{|x_{i+1} - \xi|}{|x_i - \xi| |x_{i-1} - \xi|} = \boxed{\frac{|x_{i+1} - \xi|}{|x_i - \xi|^p}} \frac{|x_i - \xi|^{1-p}}{|x_{i-1} - \xi|^1} =$$

$$\downarrow$$

$$L$$

$$= \frac{|x_{i+1} - \xi|}{|x_i - \xi|^p} \cdot \left(\frac{|x_i - \xi|}{|x_{i-1} - \xi|^p} \right)^{1-p} = \frac{|x_{i+1} - \xi|}{|x_i - \xi|^p} \frac{|x_i - \xi|^{1-p}}{|x_{i-1} - \xi|^{p(1-p)}}$$

Rovnost platí pro $1 = p(1-p) = p - p^2$

$$p^2 - p + 1 = 0 \qquad p_{1,2} = \frac{1 + \sqrt{5}}{2}$$

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Nás. kořen polynomu

$$P(x) = (x - \xi)^M \cdot Q(x)$$

$$(f \cdot g)^{(i)} = \sum_{j=0}^{i} \binom{i}{j} f^{(j)} g^{(i-j)}$$

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Násobné kořeny - řád konvergence

$$x_{k+1} = x_k - M \frac{f(x_k)}{f'(x_k)}$$

$$x_{k+1} - \xi = x_k - \xi - M \frac{f(x_k)}{f'(x_k)}$$

$$(x_{k+1} - \xi) f'(x_k) = (x_k - \xi) \cdot f'(x_k) - M \cdot f(x_k) = \sigma(x_k)$$

$$\sigma(x) = (x - \xi) f'(x) - M \cdot f(x)$$

$$\sigma^{(j)}(x) = (x - \xi) f^{(j+1)}(x) + j f^{(j)}(x) - M \cdot f^{(j)}(x)$$

$\sigma^{(j)}(\xi) = 0$ pro $j = 0, \dots, M$ $\qquad \xi$ - $M+1$násobný kořen σ

3 12-17:00

J. R.:

$$\sigma(x) = \underset{\overset{\shortparallel}{0}}{\sigma(\xi)} + \frac{1}{1!}\underset{\overset{\shortparallel}{0}}{\sigma'(\xi)}(x-\xi) + \dots + \frac{\sigma^{(n+1)}(\alpha)}{(n+1)!}(x-\xi)^{n+1}$$

$$f(x) = \underset{\overset{\shortparallel}{0}}{f(\xi)} + \frac{1}{1!}\underset{\overset{\shortparallel}{0}}{f'(\xi)}\cdot(x-\xi) + \dots + \frac{1}{(n-1)!}f^{(n)}(\beta)(x-\xi)^{n-1}$$

$$(x_{k+1}-\xi)\cdot\frac{1}{(n-1)!}f^{(n)}(\beta_k)(x_k-\xi)^{n-1} = \frac{\sigma^{(n+1)}(\alpha_k)}{(n+1)!}(x_k-\xi)^{n+1}$$

$$\frac{x_{k+1}-\xi}{(x_k-\xi)^2} = \frac{1}{n(n+1)}\,\frac{\sigma^{(n+1)}(\alpha_k)}{f^{(n)}(\beta_k)} \longrightarrow \frac{1}{n(n+1)}\,\frac{\sigma^{(n+1)}(\xi)}{f^{(n)}(\xi)}$$

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Obyčejné diference

$\{x_k\}$ - posl.

$\Delta x_k = x_{k+1} - x_k$ 1. ř.

$\Delta^2 x_k = \Delta(\Delta x_k) = \Delta x_{k+1} - \Delta x_k =$

$= x_{k+2} - x_{k+1} - (x_{k+1} - x_k) = x_{k+2} - 2x_{k+1} + x_k$

$$\hat{x}_k = x_k - \frac{(\Delta x_k)^2}{\Delta^2 x_k}$$

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