LR rozklad – př:

$$A=\begin{pmatrix}2&4&-1\\1&1&-3\\4&1&2\end{pmatrix}\begin{matrix}(-\frac12)(-2)\end{matrix}\longrightarrow\begin{pmatrix}2&4&-1\\0&-1&-\frac52\\0&-7&4\end{pmatrix}(-7)\longrightarrow$$

$$\longrightarrow\begin{pmatrix}2&4&-1\\0&-1&-\frac52\\0&0&\frac{43}{2}\end{pmatrix}=R$$

$$L=\begin{pmatrix}1&0&0\\\frac12&1&0\\2&7&1\end{pmatrix},\quad L\cdot R=\begin{pmatrix}2&4&-1\\1&1&-3\\4&1&2\end{pmatrix}=A$$

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$$Ax=\begin{pmatrix}-5\\-9\\9\end{pmatrix},\quad L\underbrace{Rx}_{y}=\begin{pmatrix}-5\\-9\\9\end{pmatrix}$$

$$\begin{pmatrix}1&0&0\\\frac12&1&0\\2&7&1\end{pmatrix}\begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix}=\begin{pmatrix}-5\\-9\\9\end{pmatrix}$$

$y_1=-5$

$y_2=-9-\frac{y_1}{2}=-\frac{13}{2}$

$y_3=9-2y_1-7y_2$

$=19+\frac{91}{2}=\frac{129}{2}$

$$\begin{pmatrix}2&4&-1\\0&-1&-\frac52\\0&0&\frac{43}{2}\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}-5\\-\frac{13}{2}\\\frac{129}{2}\end{pmatrix}$$

$x_3=3$

$-x_2=-\frac{13}{2}+\frac{15}{2}=1$

$x_2=-1$

$2x_1=-5-4x_2+x_3=2$

$x_1=1$

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$$A=\begin{pmatrix}0{,}0001&1\\1&1\end{pmatrix}(-10^4)\longrightarrow\begin{pmatrix}0{,}0001&1\\0&-10^4\end{pmatrix}=R$$

zaokr. na 3 pl. číslice

$$L=\begin{pmatrix}1&0\\10^4&1\end{pmatrix}$$

$$L\cdot R=\begin{pmatrix}1&0\\10^4&1\end{pmatrix}\begin{pmatrix}10^{-4}&1\\0&-10^{4}\end{pmatrix}=\begin{pmatrix}10^{-4}&1\\1&0\end{pmatrix}\neq A$$

$$\frac{a_{21}}{a_{11}}=\frac{1}{10^{-4}}$$

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Př. s výběrem hl. prvku:

$$A=\begin{pmatrix}10^{-4}&1\\1&1\end{pmatrix}\longrightarrow\begin{pmatrix}1&1\\10^{-4}&1\end{pmatrix}(-10^{-4})\begin{pmatrix}1&1\\0&1\end{pmatrix}=R$$

$$L=\begin{pmatrix}1&0\\10^{-4}&1\end{pmatrix},\quad L\cdot R=\begin{pmatrix}1&0\\10^{-4}&1\end{pmatrix}\begin{pmatrix}1&1\\0&1\end{pmatrix}=$$

$$=\begin{pmatrix}1&1\\10^{-4}&1\end{pmatrix}=P\cdot A,\quad P=\begin{pmatrix}0&1\\1&0\end{pmatrix}$$

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LR rozklad s částečným výběrem pivota

$$A=\begin{pmatrix}2&4&-1\\1&1&-3\\4&1&2\end{pmatrix}\longrightarrow\begin{pmatrix}4&1&2\\1&1&-3\\2&4&-1\end{pmatrix}(-\tfrac14)(-\tfrac12)\longrightarrow$$

$$\longrightarrow\begin{pmatrix}4&1&2\\0&\frac34&-\frac72\\0&\frac72&-2\end{pmatrix}\longrightarrow\begin{pmatrix}4&1&2\\0&\frac72&-2\\0&\frac34&-\frac72\end{pmatrix}$$

$$-\frac34\cdot\frac27=-\frac{3}{14}$$

$$L=\begin{pmatrix}1&0&0\\\frac14&1&0\\\frac12&\frac{3}{14}&1\end{pmatrix}$$

$$p=\begin{pmatrix}1\\2\\3\end{pmatrix}\to\begin{pmatrix}3\\2\\1\end{pmatrix}\to\begin{pmatrix}3\\1\\2\end{pmatrix}\longrightarrow\begin{pmatrix}4&1&2\\0&\frac72&-2\\0&0&-\frac{43}{14}\end{pmatrix}=R$$

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$$L\cdot R=\begin{pmatrix}1&0&0\\\frac12&1&0\\\frac14&\frac{3}{14}&1\end{pmatrix}\begin{pmatrix}4&1&2\\0&\frac72&-2\\0&0&-\frac{43}{14}\end{pmatrix}=$$

$$=\begin{pmatrix}4&1&2\\2&4&-1\\1&1&-3\end{pmatrix}=\underbrace{\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}}_{P}\cdot\underbrace{\begin{pmatrix}2&4&-1\\1&1&-3\\4&1&2\end{pmatrix}}_{A}$$

$$\frac12-\frac{6}{14}-\frac{43}{14}=\frac{7-6-43}{14}=-\frac{42}{14}=-3$$

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Dk věty o hlavních minorech

- indukcí 1) $A = (a_{11})$, $L = (1)$, $R = (a_{11})$

$a_{11} \neq 0$

2) - platí pro $n-1$

$$A = \begin{pmatrix} & & & a_{1n} \\ & A_{n-1} & & a_{2n} \\ & & & \vdots \\ & & & a_{n-1,n} \\ a_{n1} & \cdots & a_{n,n-1} & a_{nn} \end{pmatrix} = \begin{pmatrix} A_{n-1} & u \\ v^T & a_{nn} \end{pmatrix}$$

$$L = \begin{pmatrix} L_{n-1} & 0 \\ x^T & 1 \end{pmatrix}, \quad R = \begin{pmatrix} R_{n-1} & y \\ 0 & r_{nn} \end{pmatrix}$$

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$$L \cdot R = \begin{pmatrix} L_{n-1} R_{n-1} & L_{n-1} y \\ x^T R_{n-1} & x^T y + r_{nn} \end{pmatrix} = \begin{pmatrix} A_{n-1} & u \\ v^T & a_{nn} \end{pmatrix}$$

podle induk. předp. ex. $L_{n-1}, R_{n-1}$

$A_{n-1} = L_{n-1} R_{n-1}$

$L_{n-1}$ - regul. dolní $\triangle$

$R_{n-1}$ - || - horní $\triangle$

dále $L_{n-1} y = u \rightarrow$ má řešení

$x^T R_{n-1} = v^T \Leftrightarrow R_n^T x = v \rightarrow$ má řešení

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$a_{nn} = x^T y + r_{nn} \Rightarrow r_{nn} = a_{nn} - x^T y$

Potřebujeme ukázat, že $r_{nn} \neq 0$, jinak by $R$ byla singulární.

$r_{nn} = 0 \Leftrightarrow a_{nn} = + x^T y$

$$A = \begin{pmatrix} A_{n-1} & u \\ v^T & a_{nn} \end{pmatrix}$$

$u$ - Lin. kombinace sloupců $A_{n-1}$

$\exists w : A_{n-1} w = u$

pak $a_{nn} \neq v^T \cdot w$, jinak by $A$ nebyla regulární

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$u = L_{n-1} y$, $v^T = x^T \cdot R_{n-1}$

$A_{n-1} \cdot w = u$, $L_{n-1} \underbrace{R_{n-1} \cdot w}_{y} = u$ $\quad v^T \cdot w \neq a_{nn}$

$y = R_{n-1} \cdot w$

$x^T \cdot y = x^T R_{n-1} \cdot w = v^T \cdot w \neq a_{nn}$

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