Důkaz Chol. rozkladu

$A = L\cdot R$, L - dolní △, R - horní △

$l_{ii} = 1$, $D = \begin{pmatrix} r_{11} & & 0 \\ & \ddots & \\ 0 & & r_{nn} \end{pmatrix}$, $r_{ii} \neq 0$

$R = D\cdot\tilde{R}$, $\tilde{R} = (\tilde{r}_{ij})$, $\tilde{r}_{ij} = \frac{r_{ij}}{r_{ii}}$

$D_1 = \sqrt{D} = \begin{pmatrix} \sqrt{r_{11}} & & 0 \\ & \ddots & \\ 0 & & \sqrt{r_{nn}} \end{pmatrix}$ — může být komplexní

$D = D_1^2$

$A = L\cdot D_1\cdot D_1\cdot\tilde{R}$ $L, \tilde{R}$ - jedničky na hl. d.

$A = A^T = \tilde{R}^T D_1\cdot D_1 L^T \Rightarrow D_1\tilde{R} = D_1 L^T = T$, $A = T^T\cdot T$

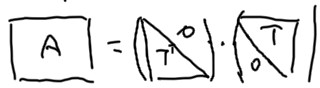
5 7-16:08

Praktický výpočet Chol. rozkladu

$A = (a_{ij})$ $a_{ij} = a_{ji}$

$T = (t_{ij})$, $t_{ij} = 0$ pro $i > j$

$A = T^T\cdot T$



$a_{11} = t_{11}\cdot t_{11} \Rightarrow t_{11} = \sqrt{a_{11}}$

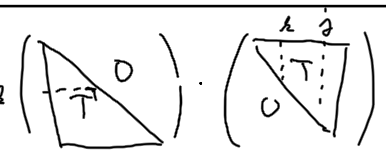
1. řádek: $a_{1j} = t_{11}\cdot t_{1j} \Rightarrow t_{1j} = \frac{a_{1j}}{t_{11}}$

2. —"— : $a_{22} = t_{12}\cdot t_{12} + t_{22}\cdot t_{22} \Rightarrow t_{22} = \sqrt{a_{22} - t_{12}^2}$

$a_{2j} = t_{12}\cdot t_{1j} + t_{22}\cdot t_{2j}$ $j = 3, \dots, n$

$t_{2j} = \frac{1}{t_{22}}(a_{2j} - t_{12}t_{1j})$

5 7-16:23



k-tý řádek:

$a_{kk} = t_{1k}\cdot t_{1k} + t_{2k}t_{2k} + \dots + t_{kk}t_{kk}$

$t_{kk} = \sqrt{a_{kk} - \sum_{i=1}^{k-1} t_{ik}^2}$

$j > k$: $a_{kj} = t_{1k}t_{1j} + t_{2k}t_{2j} + \dots + t_{kk}t_{kj}$

$t_{kj} = \frac{1}{t_{kk}}\left(a_{kj} - \sum_{i=1}^{k-1} t_{ik}t_{ij}\right)$

$k = 2, \dots, n$

5 7-16:33

Příklad

$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 2 & 4 \\ -1 & 4 & 8 \end{pmatrix}$

$T = \begin{pmatrix} 1 & 2 & -1 \\ 0 & i\sqrt{2} & -i3\sqrt{2} \\ 0 & 0 & 5 \end{pmatrix}$

$t_{kk} = \sqrt{a_{kk} - \sum_{i=1}^{k-1} t_{ik}^2}$

$t_{kj} = \frac{1}{t_{kk}}\left(a_{kj} - \sum_{i=1}^{k-1} t_{ik}t_{ij}\right)$

$t_{11} = \sqrt{1} = 1$

$t_{12} = \frac{1}{1}\cdot 2 = 2$

$t_{13} = -1$

$t_{22} = \sqrt{2 - 2^2} = \sqrt{-2} = i\sqrt{2}$

$t_{23} = \frac{1}{i\sqrt{2}}(4 - 2\cdot(-1)) = -i3\sqrt{2}$

$t_{33} = \sqrt{8 - (-1)^2 - (-i3\sqrt{2})^2} = \sqrt{8 - 1 + 18} = 5$

5 7-16:40

$Ax = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix}$, $T^T \underbrace{T\cdot x}_{y} = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix}$, $T^T\cdot y = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix}$

$T = \begin{pmatrix} 1 & 2 & -1 \\ 0 & i\sqrt{2} & -i3\sqrt{2} \\ 0 & 0 & 5 \end{pmatrix}$

1. ř: $1\cdot y_1 = 1 \Rightarrow y_1 = 1$

2. ř: $2y_1 + i\sqrt{2}y_2 = 3$

$i\sqrt{2}y_2 = 1$

$y_2 = \frac{1}{i\sqrt{2}} = -i\frac{\sqrt{2}}{2}$

3. ř: $-y_1 - i3\sqrt{2}y_2 + 5y_3 = 6$

$-1 - i3\sqrt{2}(-i)\frac{\sqrt{2}}{2} + 5y_3 = 6$

$-4 + 5y_3 = 6$, $y_3 = 2$

$Tx = y$

3. ř: $5x_3 = 2$, $x_3 = \frac{2}{5}$

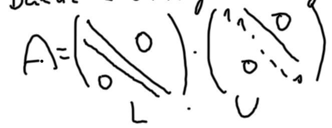
2. ř: $i\sqrt{2}x_2 - i\cdot 3\sqrt{2}\cdot\frac{2}{5} = -i\frac{\sqrt{2}}{2}$

$x_2 - \frac{6}{5} = -\frac{1}{2} \Rightarrow x_2 = \frac{7}{10}$

1. ř: $x_1 + 2\cdot\frac{7}{10} - \frac{2}{5} = 1 \Rightarrow x_1 = 0$

5 7-16:48

Důkaz Croutovy metody



$a_{11} = l_{11}\cdot 1 \Rightarrow l_{11} = a_{11}$

pod hl. diagonálou:

$a_{i+1,i} = l_{i+1,i}\cdot \underset{1}{u_{i,i}} + l_{i+1,i+1}\cdot \underset{0}{u_{i+1,i}} \Rightarrow l_{i+1,i} = a_{i+1,i}$

hl. diag: $a_{ii} = l_{i,i-1}u_{i-1,i} + l_{ii}\cdot \underset{1}{u_{ii}}$

nad hl. diag: $a_{i,i+1} = l_{i,i-1}\cdot \underset{0}{u_{i-1,i+1}} + l_{ii}\cdot u_{i,i+1}$

5 7-17:00

Pořadí výpočtu:

$l_{11} = a_{11}$

$a_{12} = l_{11} \cdot u_{12} \Rightarrow u_{12} = \frac{a_{12}}{l_{11}}$ } 1. řádek

2. řádek: $a_{22} = l_{2,1} \cdot u_{12} + l_{22} \Rightarrow l_{22} = a_{22} - l_{21} u_{12}$

$a_{23} = l_{22} \cdot u_{23} \Rightarrow u_{23} = \frac{a_{23}}{l_{22}}$

k-tý řádek – známe $u_{k-1,k}$

$a_{kk} = l_{k,k-1} \cdot u_{k-1,k} + l_{kk} \Rightarrow l_{kk} = a_{kk} - l_{k,k-1} u_{k-1,k}$

$a_{k,k+1} = l_{kk} \cdot u_{k,k+1} \Rightarrow u_{k,k+1} = \frac{a_{k,k+1}}{l_{kk}}$

5 7-17:09

Croutova metoda – př.:

$$A = \begin{pmatrix} 4 & 1 & & & & \\ 1 & 4 & 1 & & 0 & \\ & 1 & 4 & 1 & & \\ & & 1 & 4 & 1 & \\ & 0 & & 1 & 4 & 1 \\ & & & & 1 & 4 \end{pmatrix}, \quad U = \begin{pmatrix} 1 & \frac{1}{4} & & & & \\ & 1 & \frac{4}{15} & & 0 & \\ & & 1 & \frac{15}{56} & & \\ & 0 & & 1 & \frac{209}{56} & \\ & & & & 1 & \frac{209}{780} \\ & & & & & 1 \end{pmatrix}$$

$a_{i,i+1} = 1, \; a_{ii} = 4, \; l_{i,i-1} = 1$

i=1: $1 = 4 \cdot u_{12} \Rightarrow u_{12} = \frac{1}{4}$

i=2: $l_{22} = 4 - \frac{1}{4} = \frac{15}{4}$

$u_{23} = \frac{1}{\frac{15}{4}} = \frac{4}{15}$

i=3: $l_{33} = 4 - \frac{4}{15} = \frac{56}{15}$

$u_{34} = \frac{15}{56}$

i=4: $l_{44} = 4 - \frac{15}{56} = \frac{209}{56}$

$u_{45} = \frac{56}{209}$

i=5: $l_{55} = 4 - \frac{56}{209} = \frac{780}{209}$

$$L = \begin{pmatrix} 4 & & & & & \\ 1 & \frac{15}{4} & & & 0 & \\ & 1 & \frac{56}{15} & & & \\ & & 1 & \frac{56}{209} & & \\ & 0 & & 1 & \frac{780}{209} & \\ & & & & 1 & l_{66} \end{pmatrix}$$

5 7-17:17

$l_{66} = 4 - \frac{209}{780} = \dots \frac{2911}{780}$

5 7-17:29