

Multinukleární NMR spektroskopie C6800

- Materiály v ISu
- Řešené úlohy ze spektroskopie nukleární magnetické resonance

<http://nmr.sci.muni.cz>

- Úlohy – vyřešit a odevzdat
- Prezentace (na konci semestru) 10-15 min na vybrané téma NMR
- Závěrečná písemná zkouška

NMR – Historical Perspective

- 1922 Electron spin observed (Stern-Gerlach)
- 1926 Nuclear spin - David Dennison (H_2)
- 1938 NMR was first observed in a molecular beam
- 1939 I. I. Rabi observes absorption of radio frequency (RF) energy by nuclei of H_2 gas
- Isidor I. Rabi Nobel prize in physics in 1944

"for his resonance method for recording the magnetic properties of atomic nuclei"

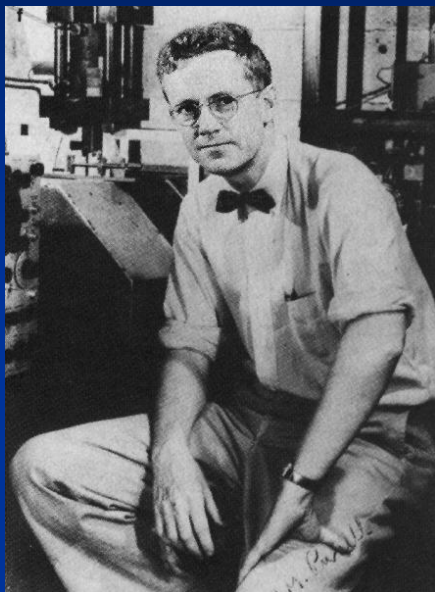


(1898 – 1988)

NMR – Historical Perspective

- 1945 Purcell, Torrey, Pound @ Harvard solid paraffin
- 1945 Bloch, Hansen, Packard @ Stanford liquid H₂O
- Varian Bros. & Russell klystron for radars (WWII)
- 1948 Pake, van Vleck solid state NMR
- 1950 W. G. Proctor, F. C. Yu @ Stanford δ - chemical shift in ¹⁴NH₄¹⁴NO₃
- 1950 W. C. Dickinson @ MIT δ - chemical shift in ¹⁹F
- 1952 Commercial NMR instruments used at DuPont, Shell, Humble Oil

NMR – Historical Perspective



Edward M. Purcell (1912-1997) & Felix Bloch (1905-1983)
NP in physics 1952

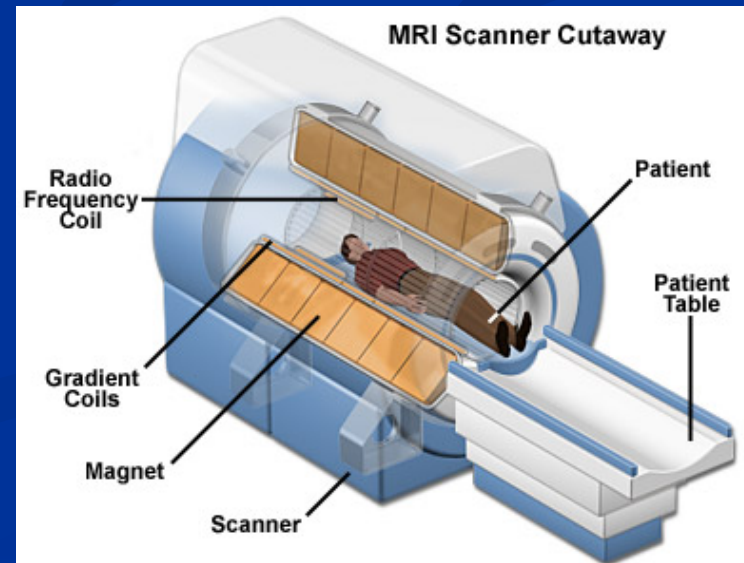
"for their development of new methods for nuclear
magnetic precision measurements and discoveries in
connection therewith"

NMR – Historical Perspective

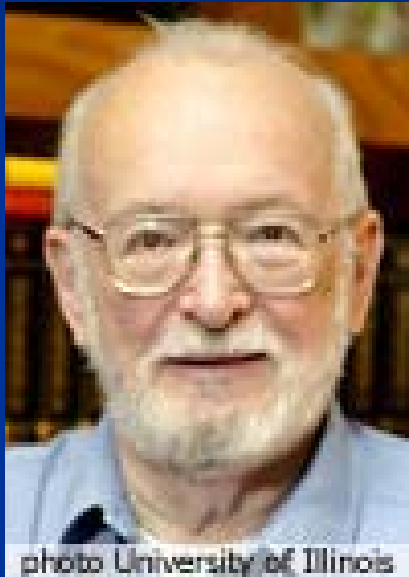
- 1952 Hahn, Maxwell @ Berkeley - J scalar coupling
- 1953 Gutowsky, McCall, Slichter @ U. of IL - J
- 1955 Bloom, Shoolery spin decoupling
- 1960 Shoolery integration
- 1966 Ernst, Anderson FT NMR at Varian
- 1968 Waugh @ MIT HR, multipulse NMR in solids

NMR – Historical Perspective

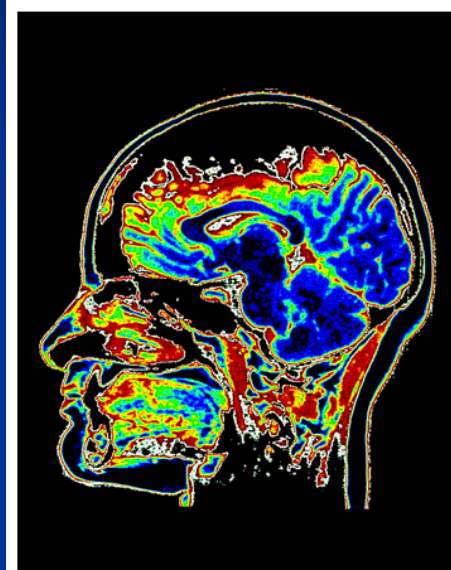
- 1971 Jeener 2D NMR
- 1971 Damadian different NMR relaxation times of tissues and tumors
- 1972 CP, HP decoupling
- 1972 The first routine ^{13}C NMR spectrometer (before mainly ^1H , ^{19}F , and ^{31}P NMR)
- 1973 Lauterbur MRI



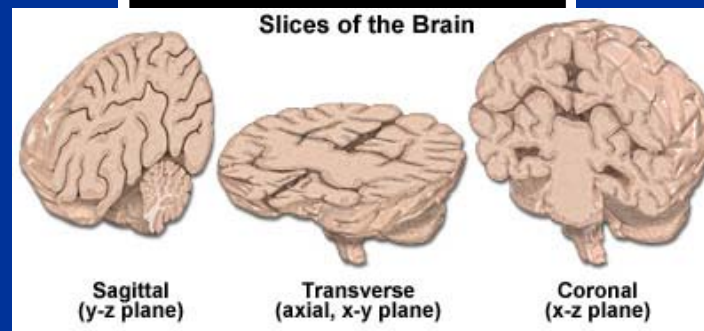
MRI-Magnetic Resonance Imaging



**Paul C. Lauterbur
(1929-)**



**Sir Peter Mansfield
(1933-)**

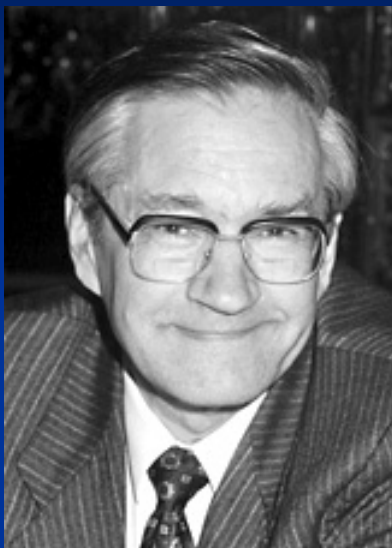


NP in physiology and medicine 2003

NMR – Historical Perspective

- 1974/1979 R. R. Ernst 2D COSY, NOESY
- 1977 MAS
- 1981 Bax, Freeman INADEQUATE
- 1982 APT
- 1983 Freeman BB decoupling, MLEV, WALTZ
- 1990 3D and $^1\text{H}/^{15}\text{N}/^{13}\text{C}$ Triple resonance
- 1991 R. R. Ernst NP in chemistry
- 2001 The first commercial 900 MHz instrument
- 2002 K. Wüthrich NP in chemistry

NMR – Historical Perspective



Richard R. Ernst
(1933-)
NP in chemistry 1991

"for his contributions to the development of the methodology of high resolution nuclear magnetic resonance (NMR) spectroscopy"



Kurt Wüthrich
(1938-)
NP in chemistry 2002

"for his development of nuclear magnetic resonance spectroscopy for determining the three-dimensional structure of biological macromolecules in solution"

NMR – Historical Perspective

Reviews

K. Wüthrich

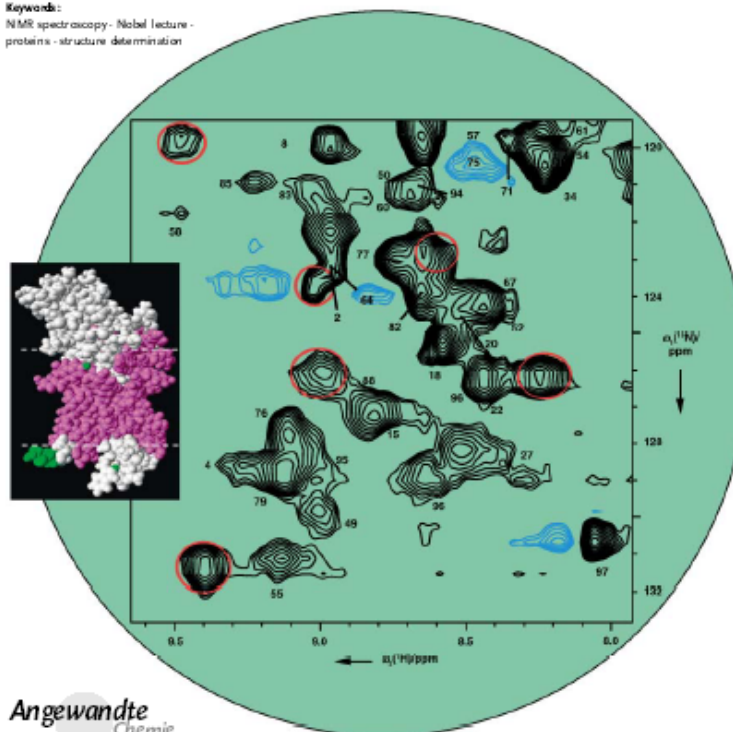
NMR of Biomacromolecules

NMR Studies of Structure and Function of Biological Macromolecules (Nobel Lecture)**

Kurt Wüthrich*

Keywords:

NMR spectroscopy - Nobel lecture - proteins - structure determination



Angewandte
Chemie

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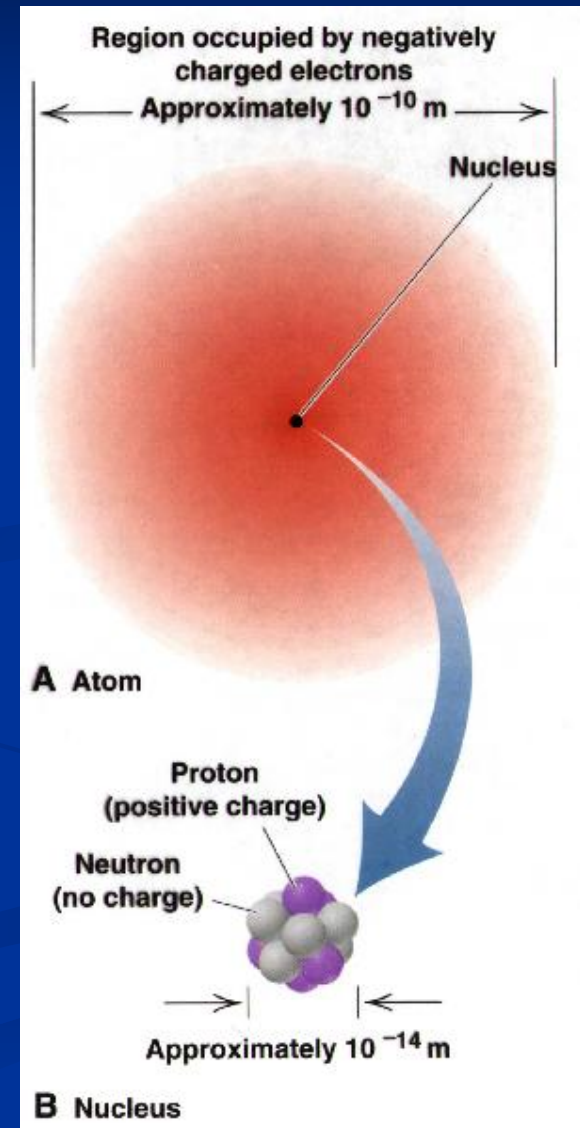
Angew. Chem. Int. Ed. 2003, 42, 3340–3360

Nuclear Magnetic Resonance

- High resolution liquid state NMR spectroscopy
- Solid state NMR spectroscopy
- High-pressure NMR
- NMR in the gas phase
- NMR spectroscopy in liquid crystalline media
- Magnetic resonance imaging (MRI)

Hyperfine Interactions

- Interactions of nuclei with the electric and magnetic fields
- Interactions between a nucleus and electrons
- Transfer of chemical (electronic) information from bonds and lone pairs to a nucleus
 - Indirect
 - Direct



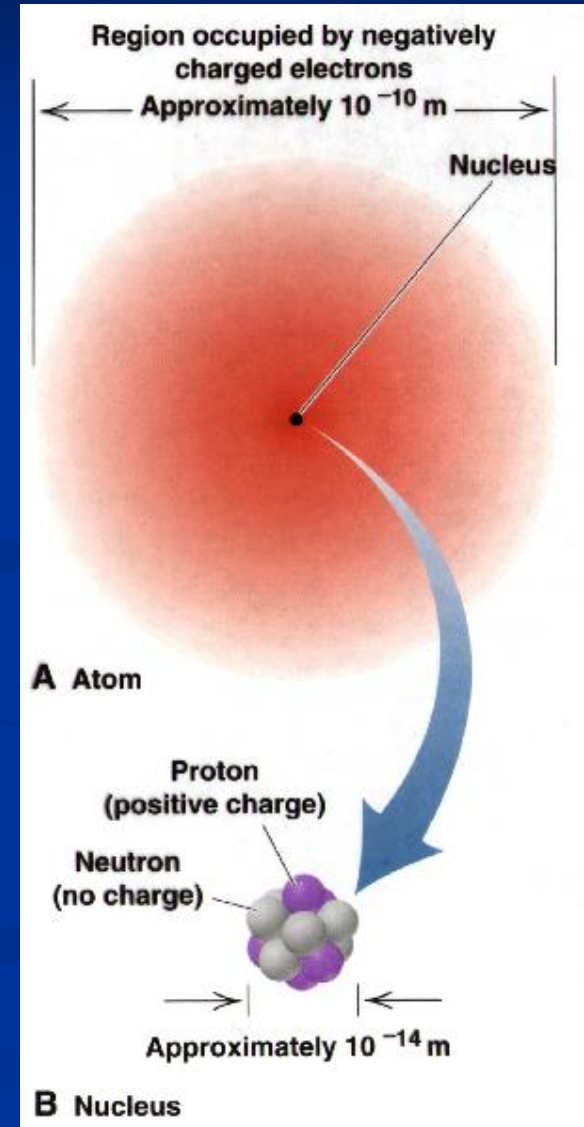
Hyperfine Interactions

Indirect

- Electric field gradient (EFG) with nuclear electric quadrupole
- Induced magnetic field with nuclear magnetic moments (shielding)

Direct

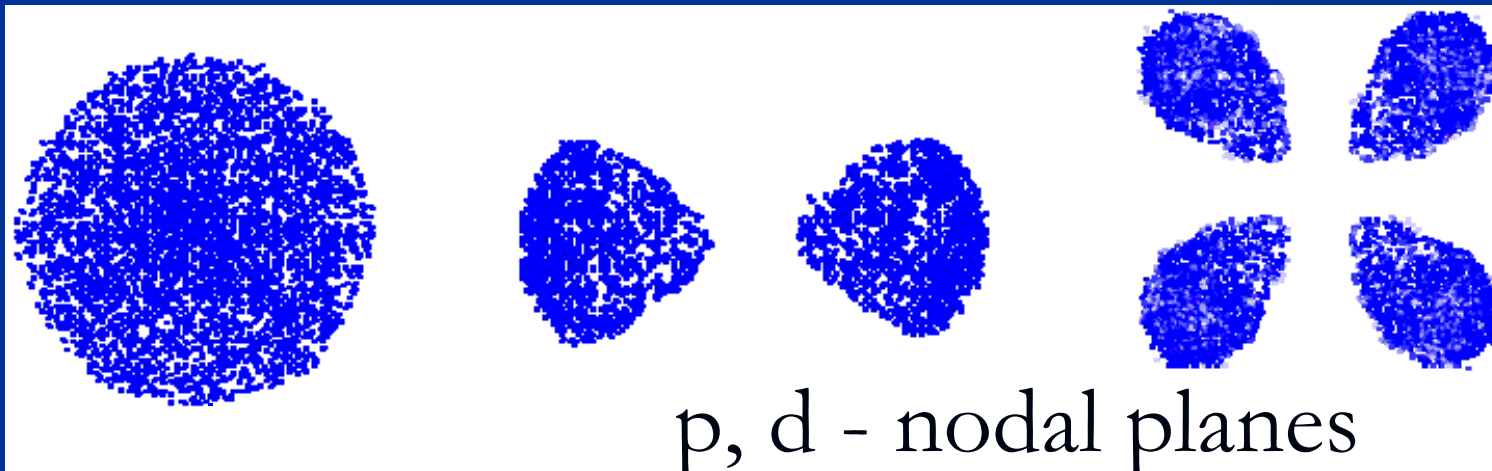
- s-electrons within nuclei, polarization of bonding spins (J-coupling)



Direct Interactions

ONLY s-electrons can interact with nuclei

ONLY s-electrons have non-zero electron density at a nucleus



Relationship Between Wavelength, Frequency and Energy

- Speed of light (c) is the same for all wavelengths

$$c = 2.9979 \times 10^8 \text{ m s}^{-1}$$

- Frequency (ν), the number of wavelengths per second, is inversely proportional to wavelength:

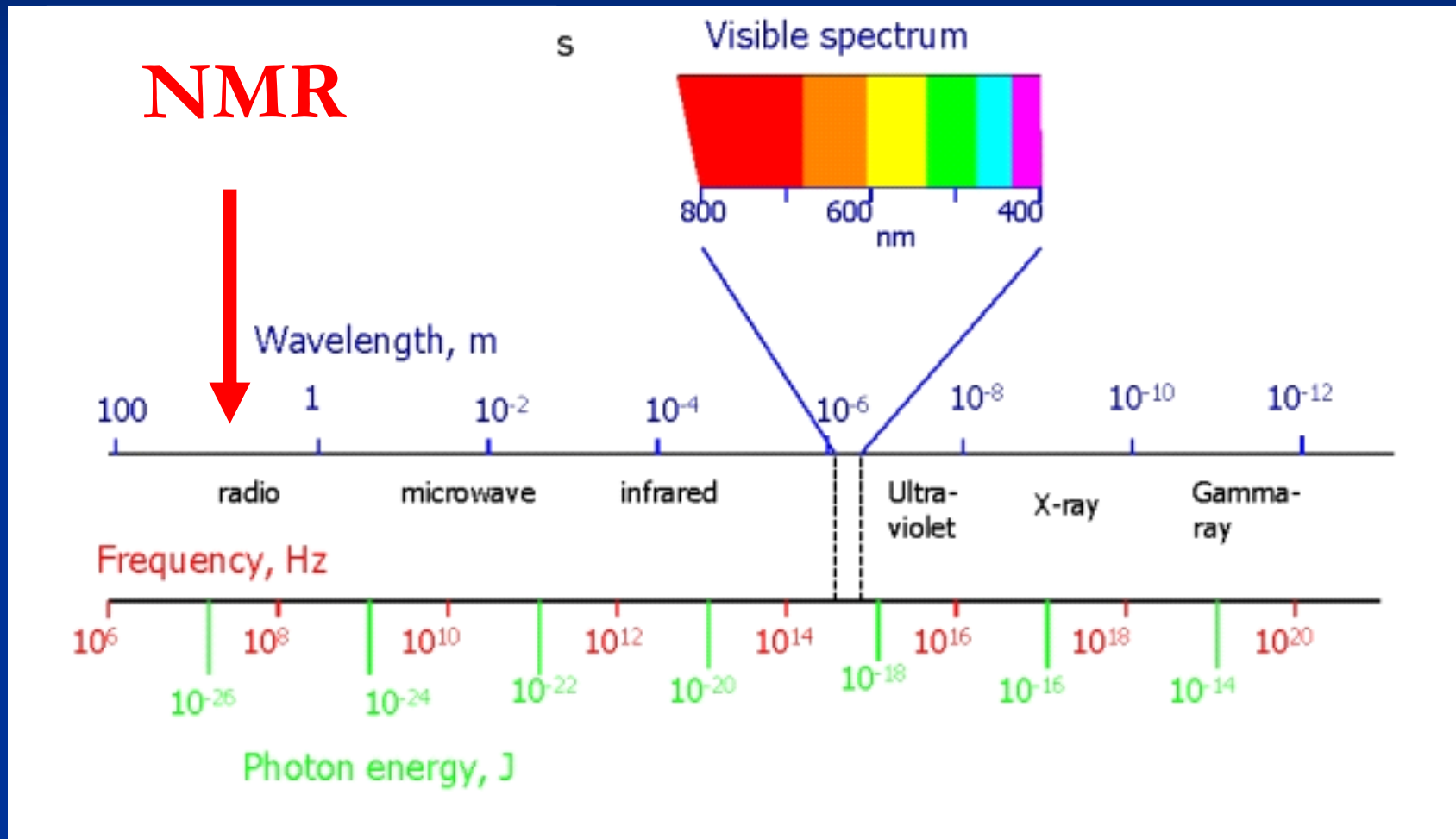
$$\nu = c/\lambda$$

- Energy of a photon is directly proportional to frequency and inversely proportional to wavelength:

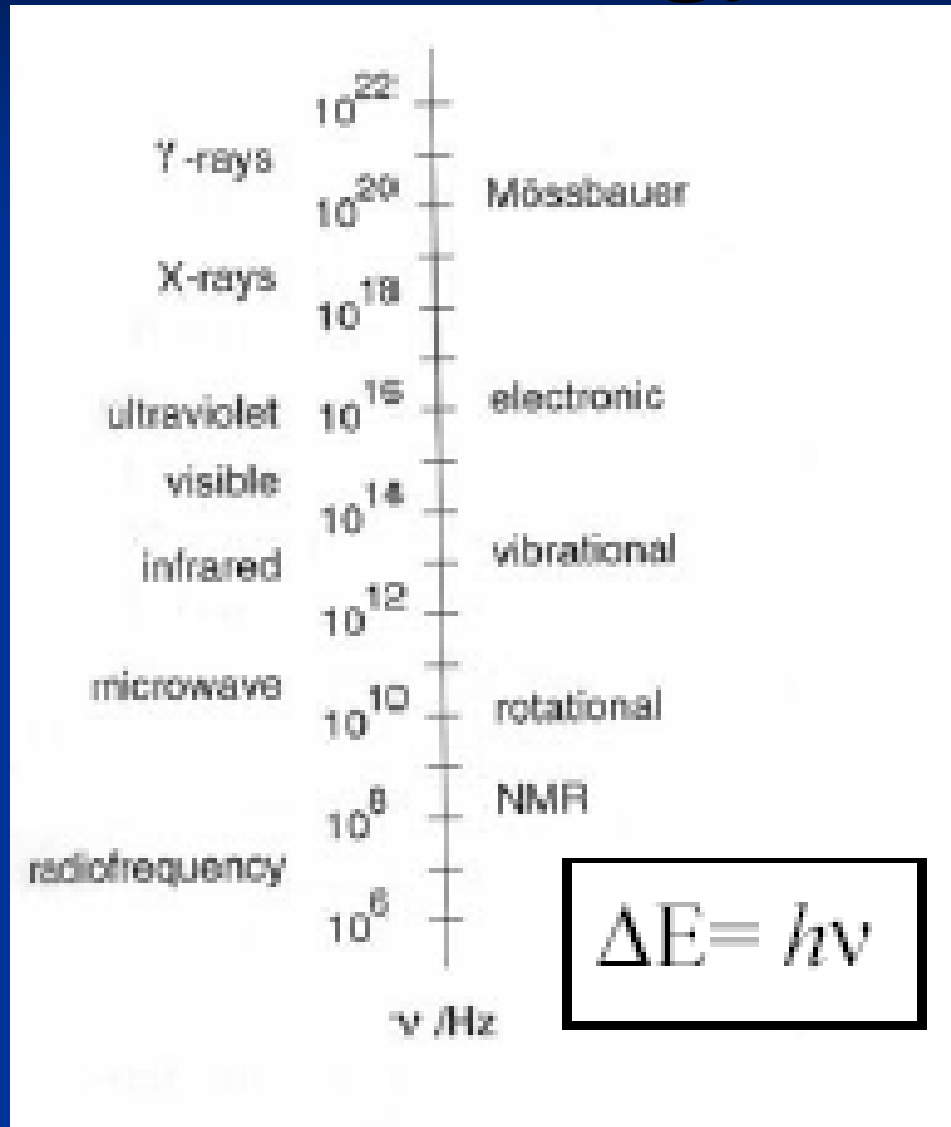
$$E = h\nu = hc/\lambda$$

$$h = \text{Plank's constant} = 6.626176 \times 10^{-34} \text{ J s}$$

Electromagnetic Radiation



Method Energy Scale



Energy Scale Conversion Factors

	Hz	eV	J mol ⁻¹
Hz	1	4.136×10^{-15}	3.990×10^{-10}
eV	2.418×10^{14}	1	9.649×10^4
J mol ⁻¹	2.506×10^9	1.036×10^{-5}	1

Isotopes

**Isotopes = a set of nuclides of an element, same Z, different A
there is about 2600 nuclides (stable and radioactive)**

340 nuclides found in nature

270 stable and 70 radioactive

Monoisotopic elements:

${}^9\text{Be}$, ${}^{19}\text{F}$, ${}^{23}\text{Na}$, ${}^{27}\text{Al}$, ${}^{31}\text{P}$, ${}^{59}\text{Co}$, ${}^{127}\text{I}$, ${}^{197}\text{Au}$

Polyisotopic elements:

${}^1\text{H}$, ${}^2\text{H}$ (D), ${}^3\text{H}$ (T)

${}^{10}\text{B}$, ${}^{11}\text{B}$

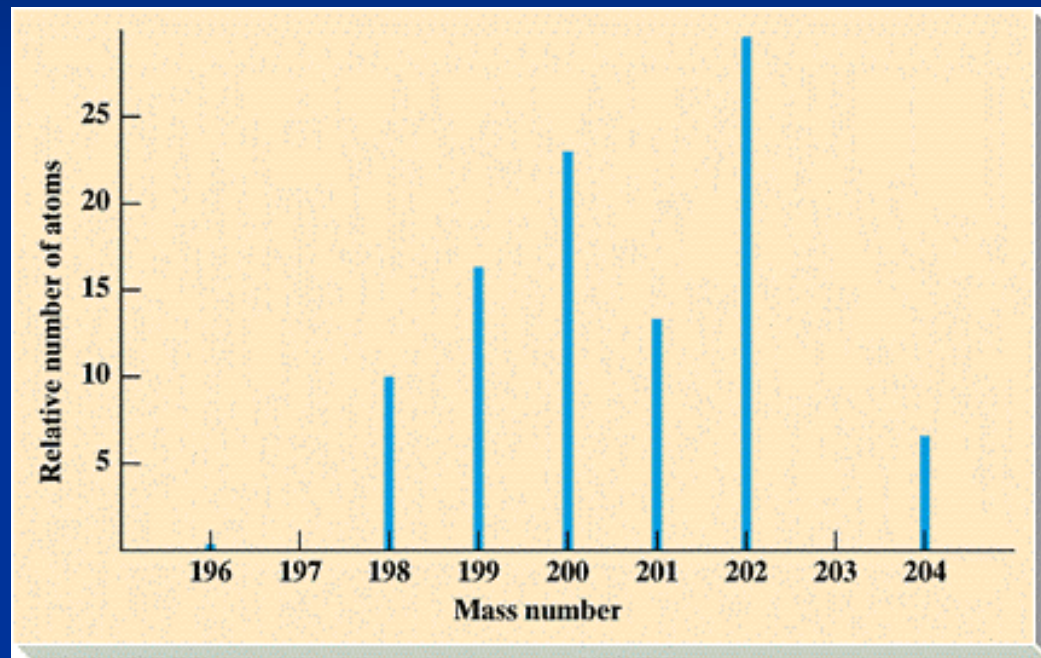
Sn has the highest number of stable isotopes – 10

112, 114, 115, 116, 117, 118, 119, 120, 122, ${}^{124}\text{Sn}$

Natural Abundance, %

Isotopic Compositions of the Elements

^AHg	I	NA%
196	0	0.146
198	0	10.02
199	1/2	16.84
200	0	23.13
201	3/2	13.22
202	0	29.80
204	0	6.850



Mass number, A

Natural Abundance, %

Isotopic Compositions of the Elements

^1H	99.985	^{16}O	99.759
^2H	0.015	^{17}O	0.037
		^{18}O	0.204
^{12}C	98.89		
^{13}C	1.11	^{32}S	95.00
		^{33}S	0.76
^{14}N	99.63	^{34}S	4.22
^{15}N	0.37	^{36}S	0.014

Variability in Isotopic Compositions

Natural Abundance, %

Isotope	Range	Average
^{10}B	18.927 - 20.337	19.9 (7)
^{11}B	81.073 - 79.663	80.1 (7)
^{16}O	99.7384 - 99.7756	99.757 (16)
^{17}O	0.0399 - 0.0367	0.038 (1)
^{18}O	0.2217 - 0.1877	0.205 (14)

Nuclear Spin

electron spin $s = 1/2$

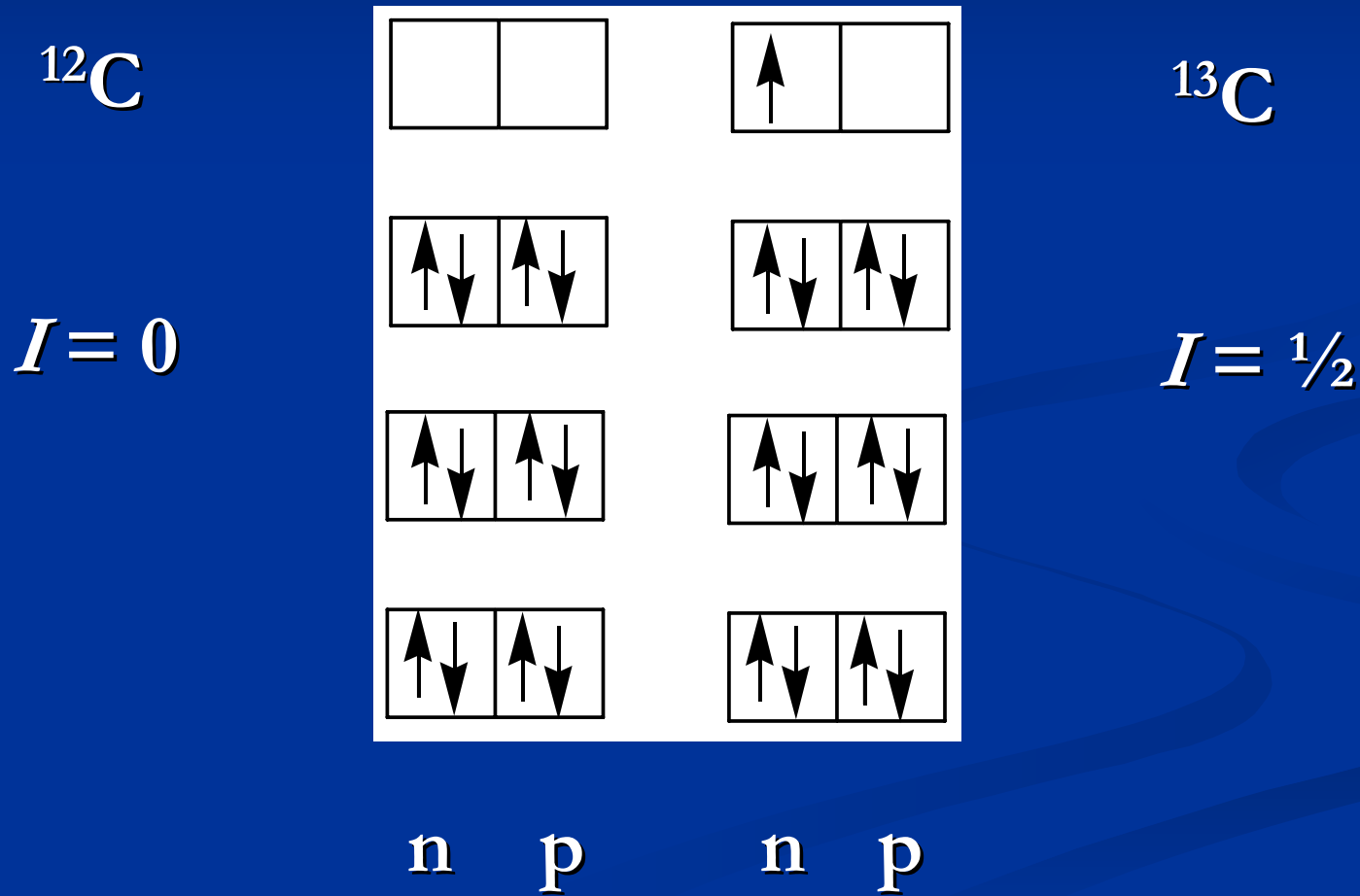
proton and neutron $I = 1/2$

nuclear spin $I = z \cdot 1/2$ $z = \text{integer } 0, 1, 2, 3, \dots$

Number of protons, Z	Number of neutrons, N	I
even	even	0
odd	odd	integer
even	odd	multiples of $1/2$
odd	even	

Nuclear Spin

protons and neutrons are Fermions, obey Pauli exclusion principle



Nuclear Spin

■ even – even: $I = 0$ ${}^4\text{He}$, ${}^{12}\text{C}$, ${}^{16}\text{O}$, ${}^{20}\text{Ne}$,
 ${}^{24}\text{Mg}$, ${}^{28}\text{Si}$, ${}^{32}\text{S}$, ${}^{36}\text{Ar}$, ${}^{40}\text{Ca}$

■ odd – odd: $I = \text{integer}$

ONLY ${}^2\text{H}$, ${}^6\text{Li}$, ${}^{10}\text{B}$, ${}^{14}\text{N}$, ${}^{40}\text{K}$, ${}^{50}\text{V}$, ${}^{138}\text{La}$, ${}^{176}\text{Lu}$

■ even – odd and odd – even:

$I = \text{multiples of } \frac{1}{2}$

${}^{13}\text{C } \frac{1}{2}$, ${}^{17}\text{O } \frac{5}{2}$, ${}^{33}\text{S } \frac{3}{2}$

Nuclear Spin

Number of protons Z	Number of neutrons N	Number of nuclides
even	even	168
odd	odd	8
odd	even	50
even	odd	57

Allowed Nuclear Multipole Moments as a function of Spin I

	$l = 0$	$l = 1$	$l = 2$	$l = 3$	$l = 4$
Spin	monopole	dipole	quadrupole	octapole	hexadecapole
$I = 0$	electric	0	0	0	0
$I = \frac{1}{2}$	electric	magnetic	0	0	0
$I = 1$	electric	magnetic	electric	0	0
$I = \frac{3}{2}$	electric	magnetic	electric	magnetic	0
$I = 2$	electric	magnetic	electric	magnetic	electric

Nuclear Magnetic Dipole Moment couples to magnetic field

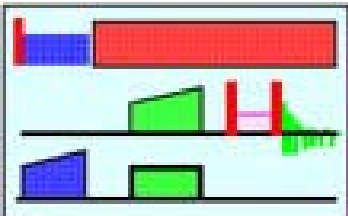


Nuclear Electric Quadrupole Moment couples to electric field gradient



IA																	VIIIA				
H																	He				
Li	Be															B	C	N	O	F	Ne
Na	Mg	IIIB	IVB	VB	VIB	VIIIB	VIII B				IB	IIB	Al	Si	P	S	Cl	Ar			
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr				
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe				
Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn				
Fr	Rd	Ac																			
		Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu						
		Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr						

Spin = $\frac{1}{2}$
Spin > $\frac{1}{2}$



Elements Accessible by NMR

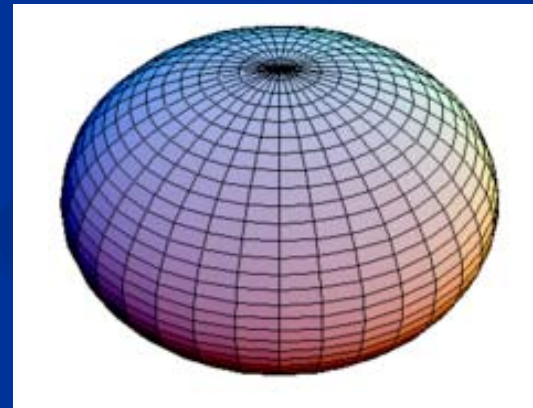
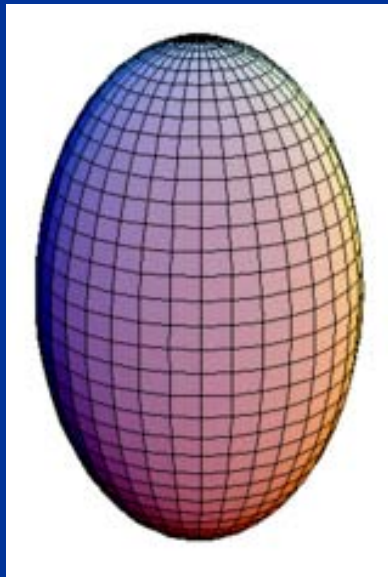
<div style="display: flex; justify-content: space-between; align-items: center;"> <div style="text-align: left;"> <p>element: Hydrogen 1</p> <p>symbol: H</p> <p>isotope: ^1H</p> <p>atomic weight: 1.007825</p> <p>spin number: 1/2</p> </div> <div style="text-align: left;"> <p>atomic number: 1</p> <p>frequency (MHz): 400</p> </div> <div style="text-align: left;"> <p> I=1/2 nuclei</p> <p> I>1/2 nuclei</p> <p> I=1/2 and I>1/2 nuclei</p> </div> </div>																					
1 IA																	18 VIIIA				
Hydrogen 1 H 1.007825 1.007825 1.007825																	Helium 2 He 4.002602				
2 IIA														13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA			
Lithium 3 Li 6.941 7.016003	Beryllium 4 Be 9.012182													Boron 5 B 10.811 11.009305	Carbon 6 C 12.011 12.010738	Nitrogen 7 N 14.007 14.003074	Oxygen 8 O 15.999 15.994815	Fluorine 9 F 18.998 18.998403	Neon 10 Ne 20.180 20.1797		
3 IIB		4 IVB		5 VB		6 VIIB		7 VIIB		8 VIIB		9 VIIB		10 VIIB		11 IB		12 IIB			
Sodium 11 Na 22.990 22.989769	Magnesium 12 Mg 24.305 24.304094													Aluminum 13 Al 26.982 26.981538	Silicon 14 Si 28.086 28.085584	Phosphorus 15 P 30.974 30.973762	Sulfur 16 S 32.065 32.0615	Chlorine 17 Cl 35.453 35.4527			
Potassium 19 K 39.098 39.096188	Calcium 20 Ca 40.078 40.0784	Scandium 21 Sc 44.956 44.955912	Titanium 22 Ti 47.883 47.883109	Vanadium 23 V 50.942 50.941501	Chromium 24 Cr 52.004 52.003876	Manganese 25 Mn 54.938 54.938045	Iron 26 Fe 55.845 55.84506	Cobalt 27 Co 58.933 58.933195	Nickel 28 Ni 58.693 58.6934	Copper 29 Cu 63.546 63.546885	Zinc 30 Zn 65.38 65.38	Gallium 31 Ga 69.723 69.7231	Germanium 32 Ge 72.631 72.630519	Arsenic 33 As 74.922 74.921603	Selenium 34 Se 78.96 78.961848	Bromine 35 Br 79.904 79.904181	Krypton 36 Kr 83.80 83.80188				
Rubidium 37 Rb 85.468 85.4678	Strontium 38 Sr 87.62 87.62	Yttrium 39 Y 88.906 88.905842	Zirconium 40 Zr 91.224 91.2242	Niobium 41 Nb 92.906 92.90638	Molybdenum 42 Mo 95.94 95.94	Technetium 43 Tc 98.906 98.906251	Ruthenium 44 Ru 101.07 101.072888	Rhodium 45 Rh 102.91 102.910354	Palladium 46 Pd 106.37 106.367554	Silver 47 Ag 107.87 107.8682	Cadmium 48 Cd 112.41 112.4144	Indium 49 In 114.82 114.8187	Tin 50 Sn 118.71 118.710	Antimony 51 Sb 121.76 121.7571	Tellurium 52 Te 127.60 127.603	Iodine 53 I 126.91 126.90509	Xenon 54 Xe 131.29 131.294				
Cesium 55 Cs 132.91 132.905451	Barium 56 Ba 137.33 137.327	Lanthanum 57 La 138.91 138.90487	Hafnium 72 Hf 178.49 178.49	Tantalum 73 Ta 180.95 180.94788	Tungsten 74 W 183.85 183.84	Rhenium 75 Re 186.21 186.207	Osmium 76 Os 190.23 190.2336	Iridium 77 Ir 192.22 192.222	Platinum 78 Pt 195.08 195.084	Gold 79 Au 196.97 196.966569	Mercury 80 Hg 200.59 200.59	Thallium 81 Tl 204.38 204.377	Lead 82 Pb 207.2 207.2	Bismuth 83 Bi 208.98 208.9804							
Lanthanum 57		Cerium 58 Ce 140.12 140.127	Praseodymium 59 Pr 140.91 140.90766	Neodymium 60 Nd 144.24 144.242	Promethium 61 Pm 144.91 144.91288	Europium 62 Eu 151.96 151.964	Gadolinium 63 Gd 157.25 157.254	Terbium 64 Tb 158.93 158.92535	Dysprosium 65 Dy 162.50 162.50014	Holmium 66 Ho 164.93 164.93033	Erbium 67 Er 167.26 167.2593	Thulium 68 Tm 168.93 168.93487	Ytterbium 70 Yb 173.05 173.0547								

Nuclear Spin

- NO stable nucleus has spin 2
- the highest value of spin for a stable nucleus is 7
 ^{176}Lu
- unstable nuclei
 - highest integral spin 16 - isomer ^{178}Hf
 - highest half-integer $37/2$ - isomer ^{177}Hf

Nuclear Spin

- Nuclei with spin $\frac{1}{2}$ - a spherical charge distribution
- Nuclei with $I > \frac{1}{2}$ - nonspherical charge distributions (prolate or oblate)

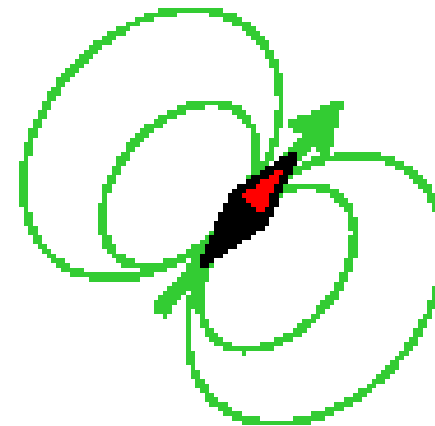
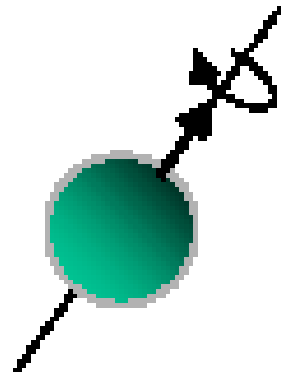


- All nuclei with non-zero spins – magnetic moments (μ)
- Nonspherical nuclei - an electric quadrupole moment (eQ)

Nuclear Spin

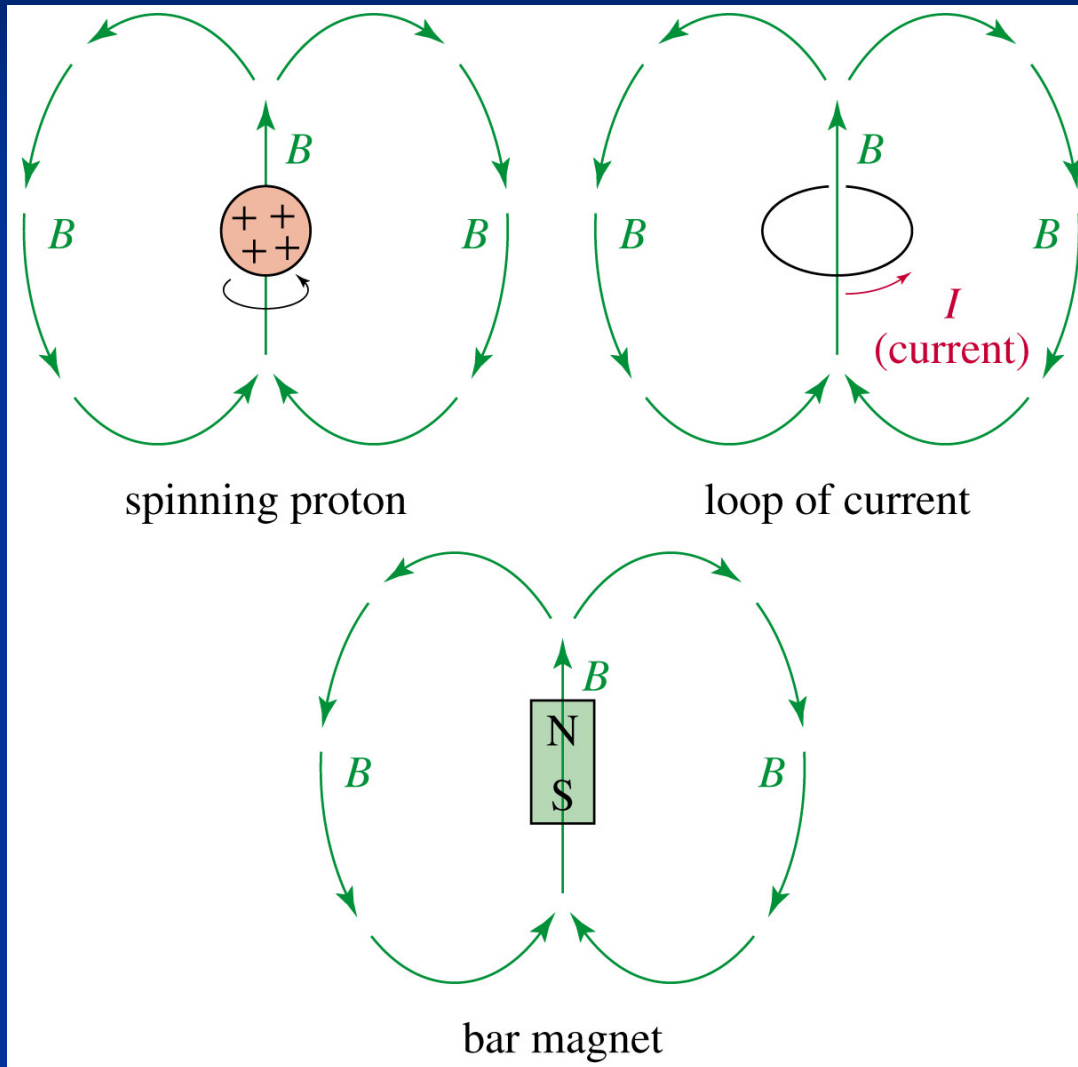
Properties of Nuclei:

Mass, Charge, Spin and Magnetism.



“The concept of spin is difficult. It was forced upon scientists by the experimental evidence”. Malcolm Levitt.

Nuclear Spin



Nuclear Spin

Nuclear spin = Spin angular momentum, P (vector)

Spin quantum number I

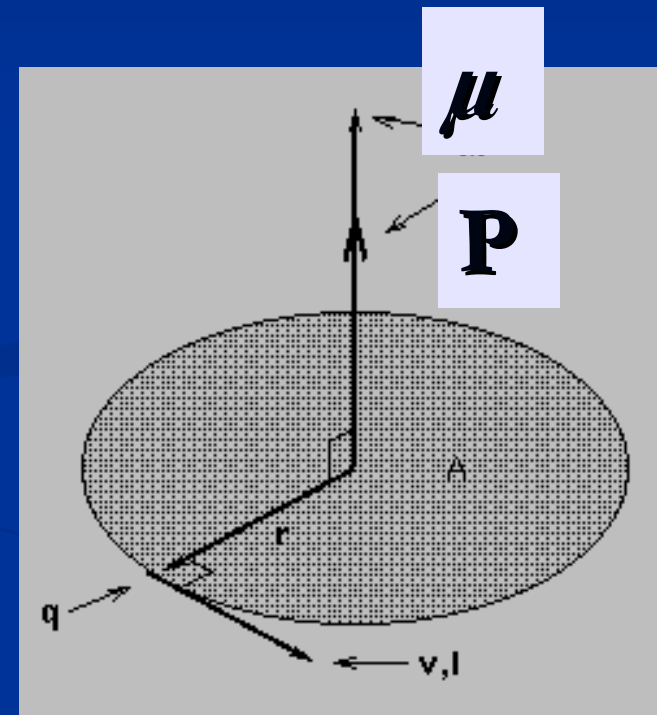
Magnetic quantum number m_I

Magnitude of P is quantized:

$$P = \frac{h}{2\pi} \sqrt{I(I+1)}$$

Direction with respect to the magnetic field B_0 is quantized:

$$P_z = \frac{h}{2\pi} m_I$$



Spin Angular Momentum, P

$$P = \frac{h}{2\pi} \sqrt{I(I+1)}$$

$$P_z = \frac{h}{2\pi} m_I$$

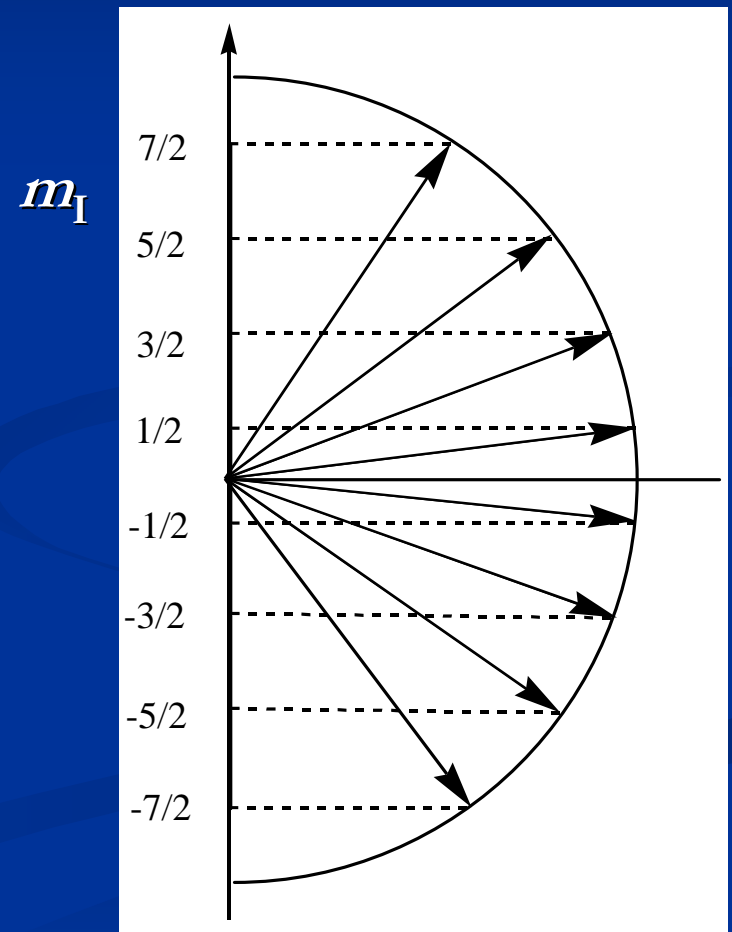
I = Nuclear spin quantum number
 $I = 0, 1/2, 1, 3/2, 5/2, 3, 7/2, \dots$

m_I = Nuclear spin magnetic quantum number

Multiplicity, M $2I + 1$ values
 $m_I = I, I-1, I-2, \dots, -I+2, -I+1, -I$

$$\cos \theta = \frac{P_z}{P} = \frac{m_I}{\sqrt{I(I+1)}}$$

\uparrow B_0 $^{59}\text{Co}, I = 7/2$



Spin Angular Momentum, P

$$P = \frac{h}{2\pi} \sqrt{I(I+1)}$$

I	$[I(I+1)]^{1/2}$
$1/2$	0.866
1	1.414
$3/2$	1.936
$5/2$	2.958
3	3.464
$7/2$	3.969
4	4.472
$9/2$	4.975

Spin Magnetic Moment, μ

The electrons, nucleons (protons, neutrons) and some nuclei possess intrinsic magnetism, which is not due to a circulating current.

Permanent magnetic moment similarly as spin angular momentum.

Magnetic moment, μ , is directly proportional to the spin angular momentum, P :

$$\mu = \gamma P$$

γ is the gyromagnetic (magnetogyric) ratio

Magnetogyric Ratio

γ - the magnetogyric ratio is the ratio of the nuclear magnetic moment μ to the nuclear angular momentum P .

$$\mu = \gamma P$$

γ - Important characteristic of nuclei

$$[\text{rad T}^{-1} \text{s}^{-1}]$$

Spin Magnetic Moment, μ

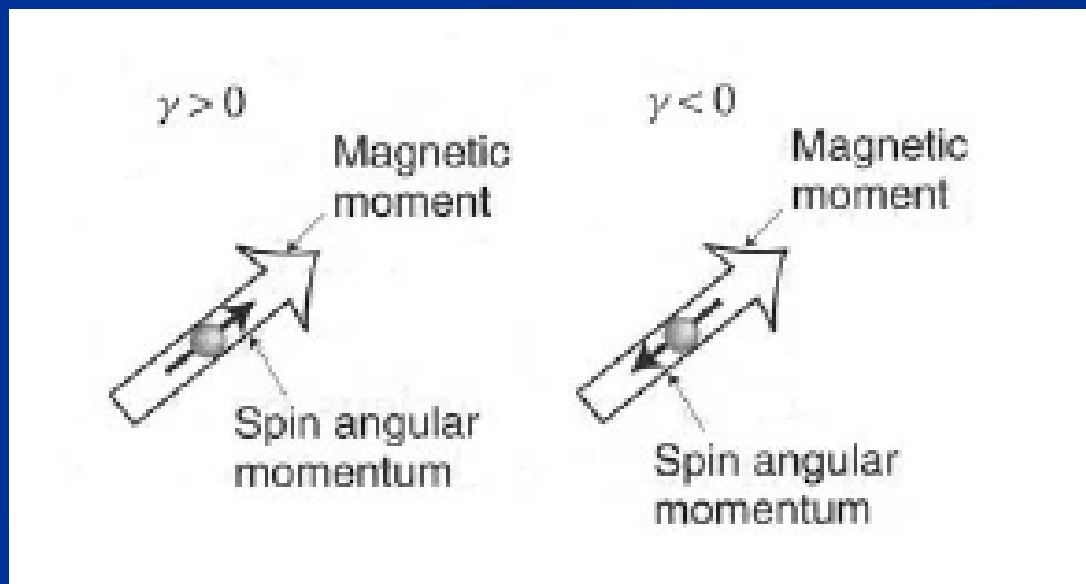
$$\mu = \gamma P = \gamma (h/2\pi) [I(I+1)]^{1/2}$$

$$\mu_z = \gamma P_z = \gamma (h/2\pi) m_I$$

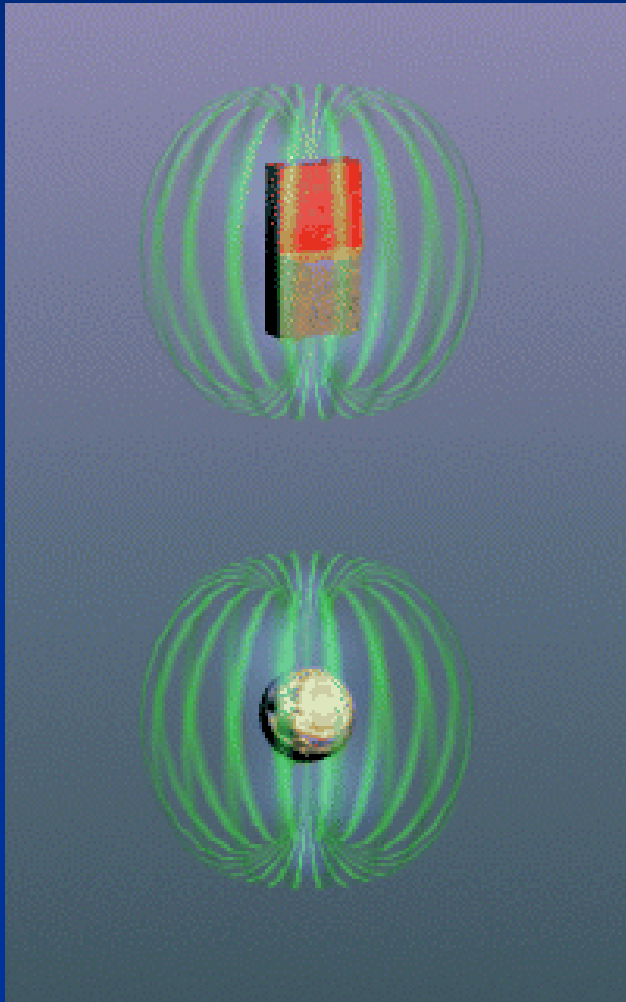
Nucleus	^1H	^2H	^{13}C	^{15}N	^{19}F	^{29}Si	^{31}P
γ [10^{-7} rad $\text{T}^{-1}\text{s}^{-1}$]	26.75	4.11	6.73	-2.71	25.18	-5.32	10.84

electron

$$\gamma_e = 17\,609\,10^7 = 658\,\gamma(\text{H})$$



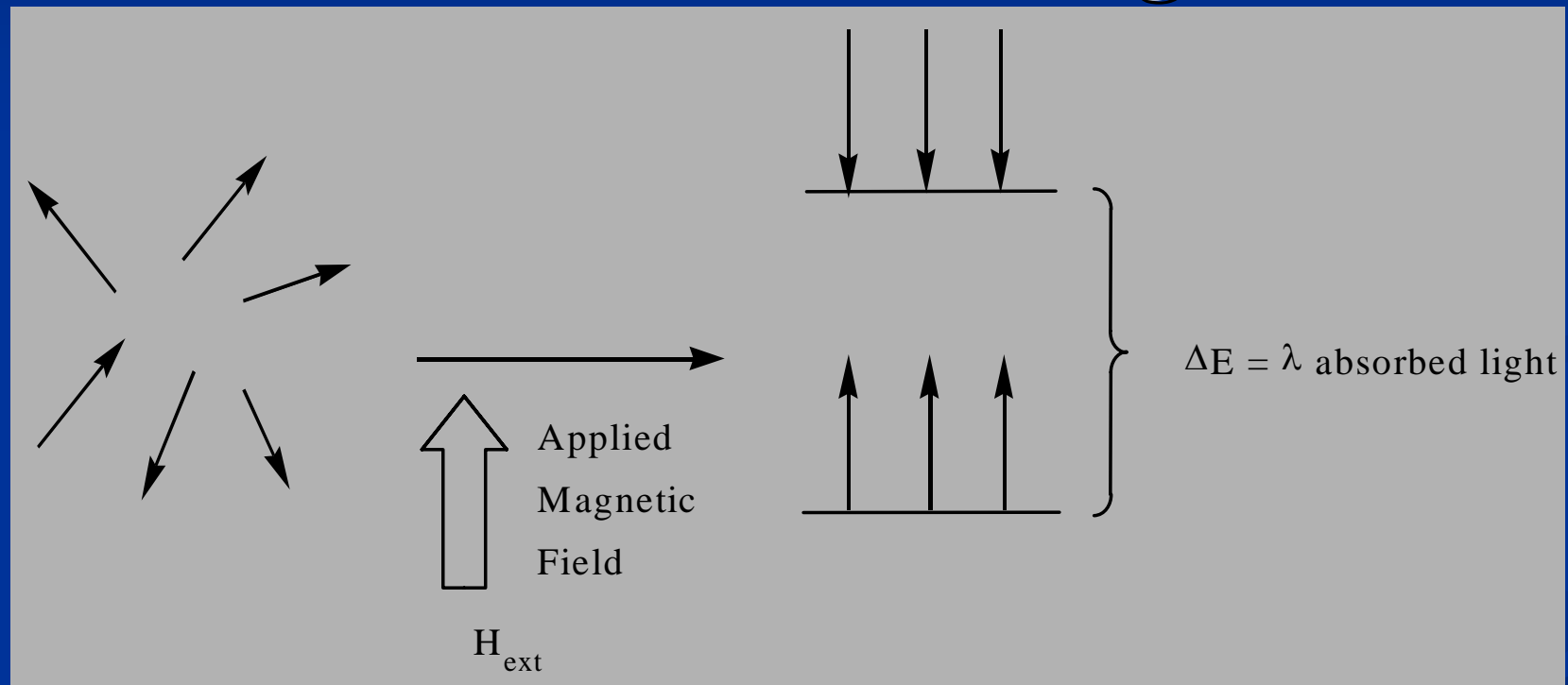
Nuclear Spin in Magnetic Field



Nuclear Spin in Magnetic Field

Random orientation

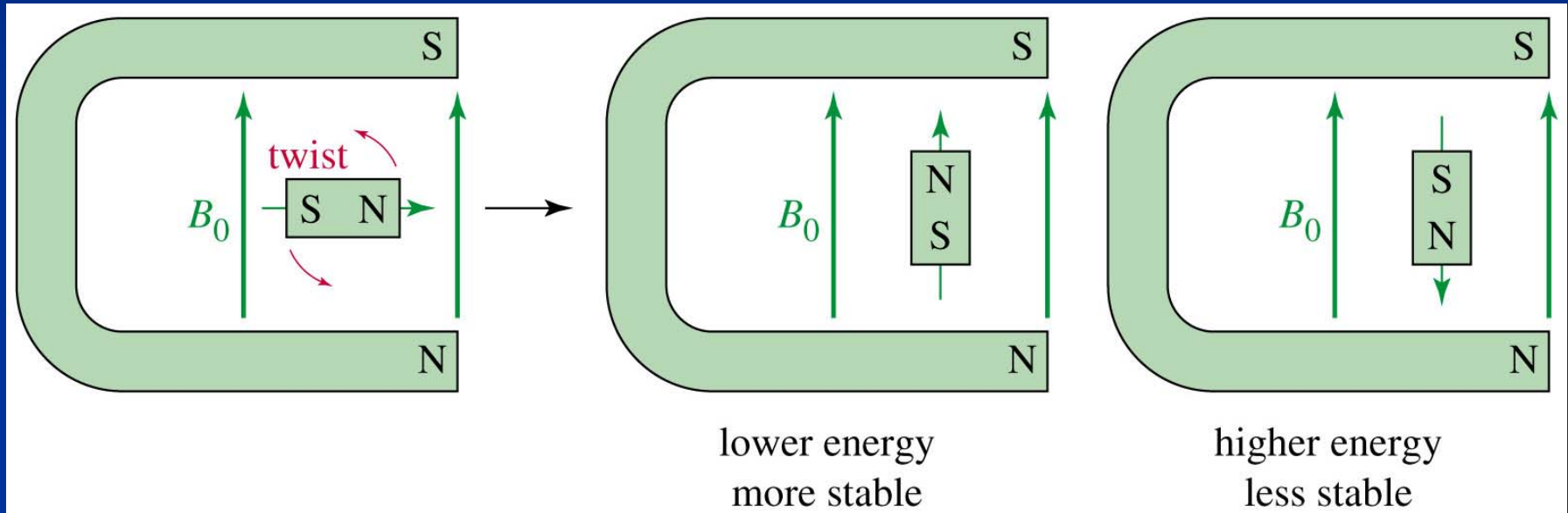
Alignment



No Field

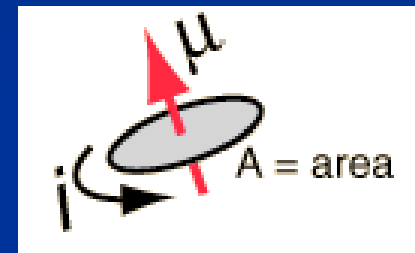
Magnetic Field

Nuclear Spin in Magnetic Field



Nuclear Spin in Magnetic Field

- An angular momentum is associated with each rotating object
- A nuclear spin possesses a magnetic moment arising from the angular momentum of the nucleus
- The magnetic moment is a vector perpendicular to the current loop
- In a magnetic field (**B**) the magnetic moment behaves as a magnetic dipole



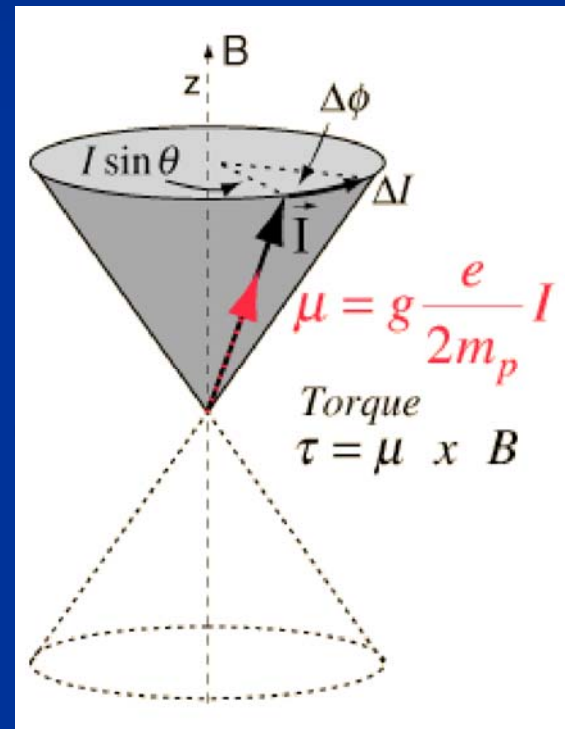
$$\mu = i A$$

Nuclear Spin in Magnetic Field

In B_0 , a magnetic moment is directed at some angle w.r.t. B_0 direction

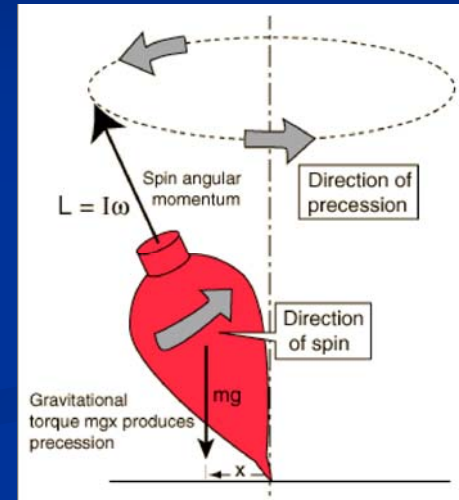
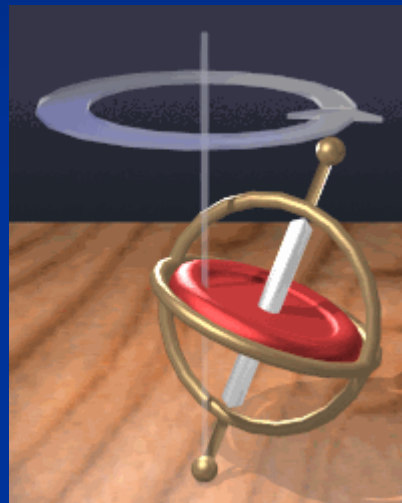
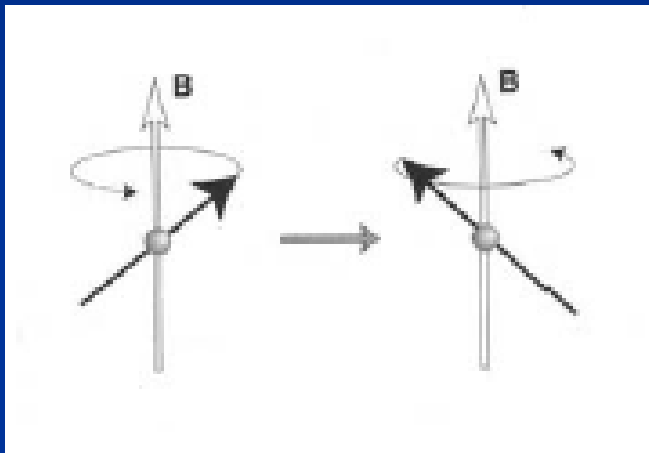
the B_0 field will exert a torque on the magnetic moment. This causes it to precess about the magnetic field direction

Torque is the rate of change of the nuclear spin angular momentum



Nuclear Spin in Magnetic Field

Spin precession in the external magnetic field.



Quantum description of precession shows that the frequency of the motion is:

$$\omega_0 = -\gamma B_0 \text{ [rad s}^{-1}\text{]} \text{ or } \nu_0 = -\gamma B_0 / 2\pi \text{ [Hz]}$$

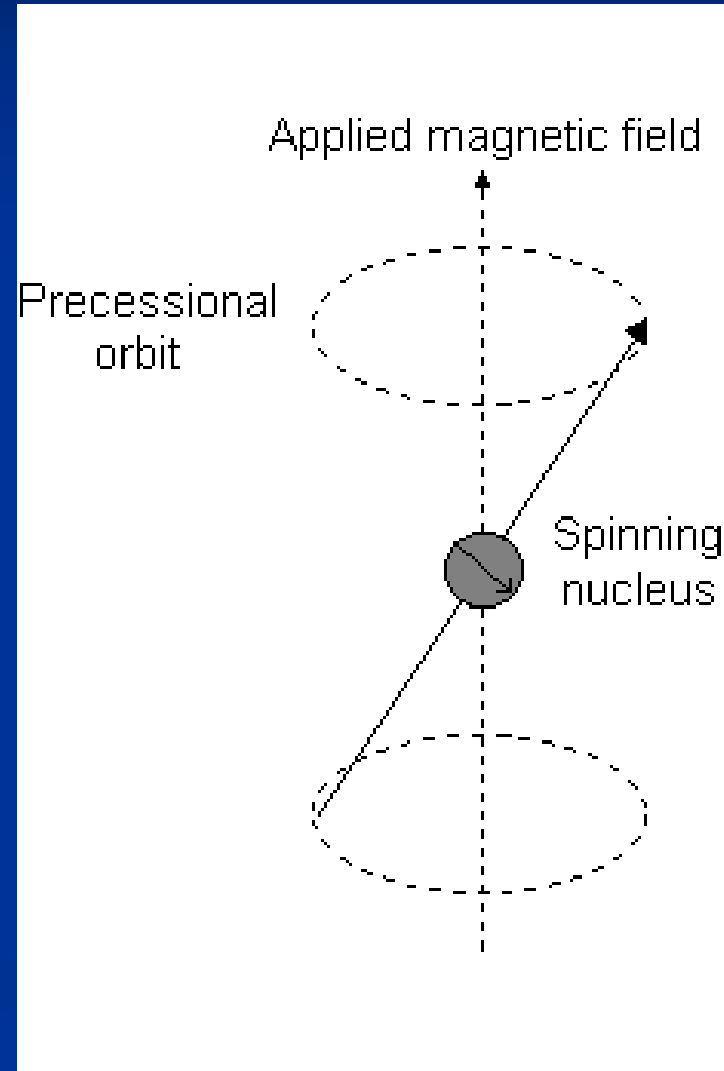
It is called the Larmor frequency (if $\gamma > 0$ then $\nu_0 < 0$)

Larmor Frequency

$$\omega_0 = -\gamma B_0 \text{ [rad s}^{-1}\text{]}$$

$$\nu_0 = -\gamma B_0 / 2\pi \text{ [Hz]}$$

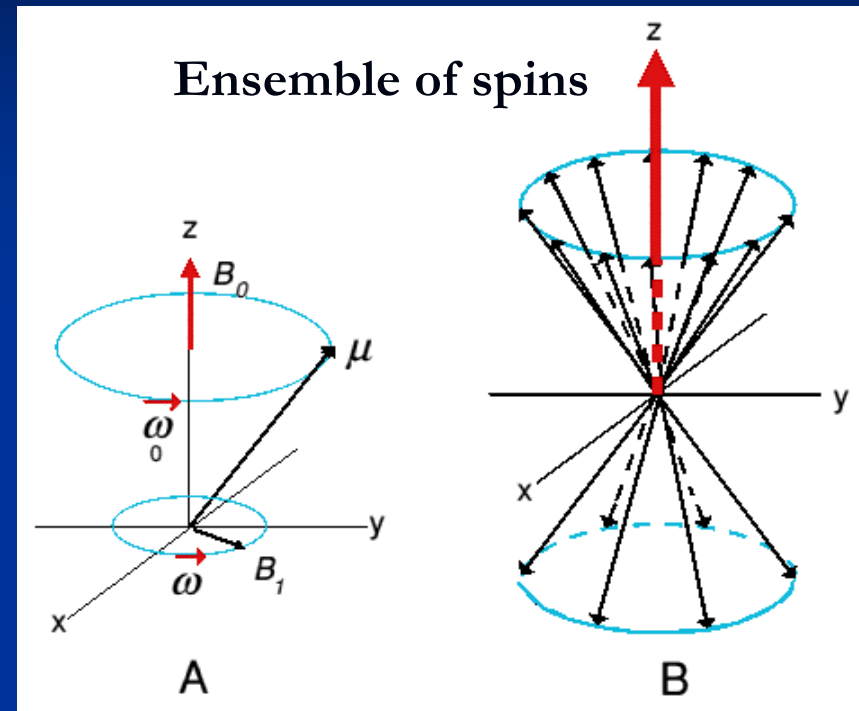
$$\nu_0 = -\frac{\gamma B_0}{2\pi}$$



Larmor Frequency



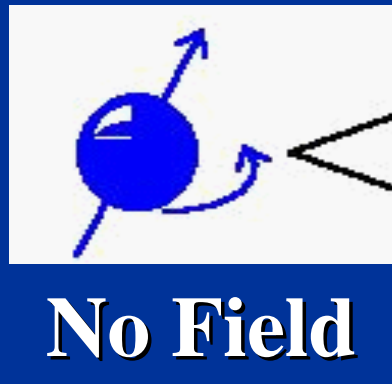
Sir Joseph Larmor
(1857-1942)



$$\omega_0 = -\gamma B_0 \text{ [rad s}^{-1}\text{]} \text{ or } \nu_0 = -\gamma B_0 / 2\pi \text{ [Hz]}$$

Nuclear Zeeman Effect - Splitting

Nuclei are charged and if they have spin, they are magnetic



Energy of transition = energy of radiowaves

Higher energy state: magnetic field opposes applied field

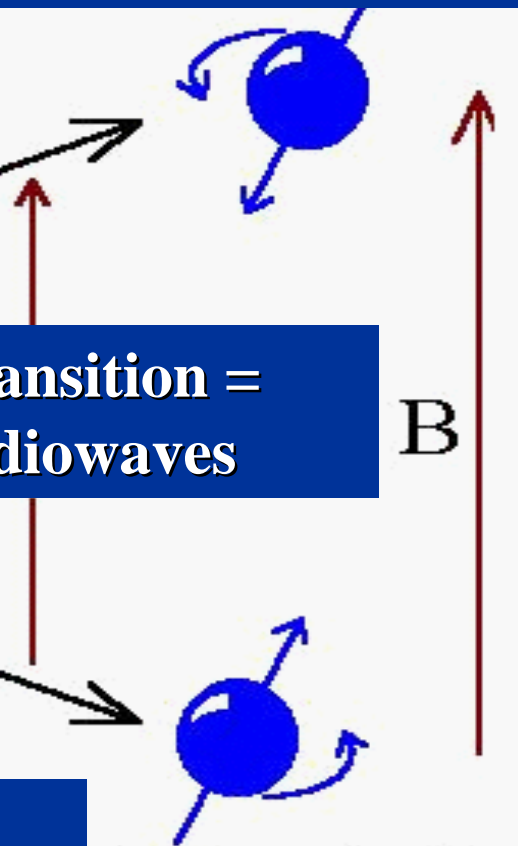
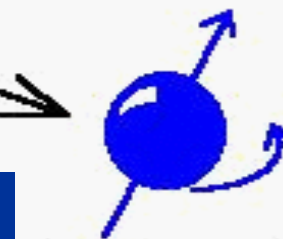
$$m_I - 1/2$$

Applied Magnetic Field = B_0

B

Lower energy state: magnetic field aligned with applied field

$$m_I + 1/2$$



Nuclear Spin in Magnetic Field

The magnetic energy depends on the interaction between the magnetic moment and B_0 field:

$$E_{\text{mag}} = -\boldsymbol{\mu} \cdot \mathbf{B}_0 \quad (\text{scalar product})$$

$$E_{\text{mag}} = -\mu_z B = -\gamma P_z B$$

$$E_{\text{mag}} = -m_I \hbar \gamma B$$

NMR selection rule $\Delta m_I = \pm 1$

CALVIN AND HOBBS By Bill Watterson

WHAT A RIP-OFF! THEY SAY IF YOU CONNECT THESE DOTS YOU GET A PICTURE, BUT LOOK! I DID IT AND IT'S JUST A BIG MESS!



I THINK YOU'RE SUPPOSED TO CONNECT THEM IN THE ORDER THAT THEY'RE NUMBERED.



OH.



EVERYTHING'S GOTTA HAVE RULES, RULES, RULES!

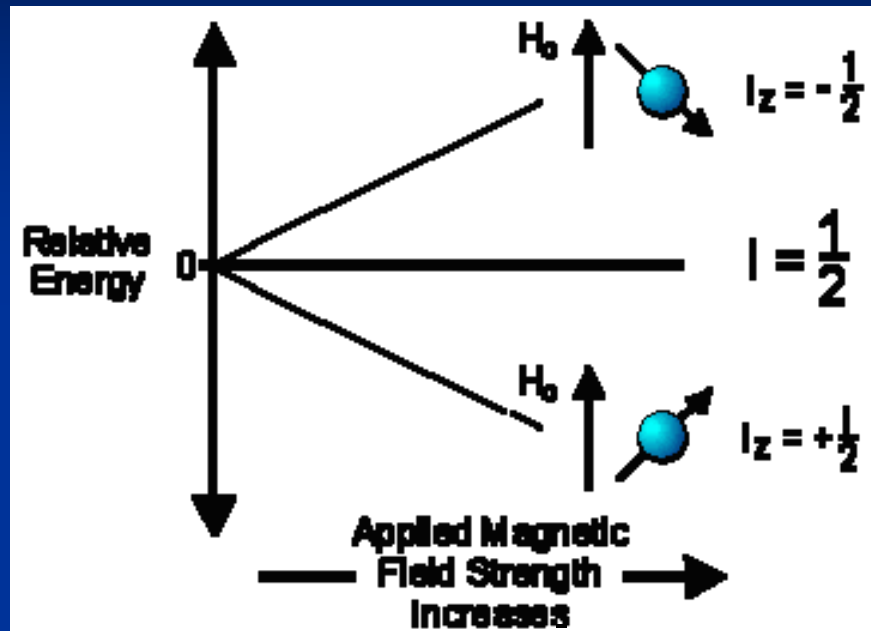


Spin in Magnetic Field

$$I = 1/2$$

$$E_{m=-1/2}$$

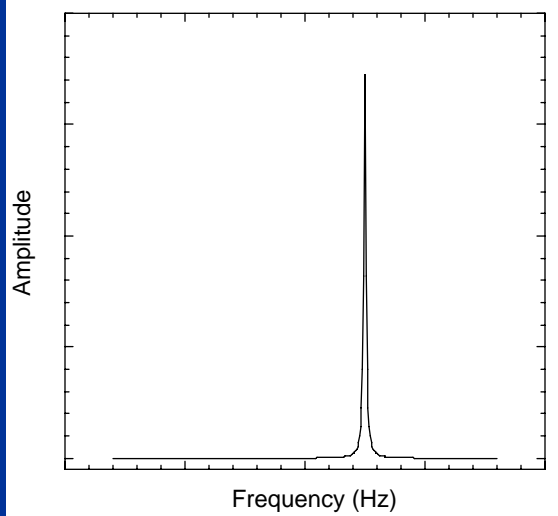
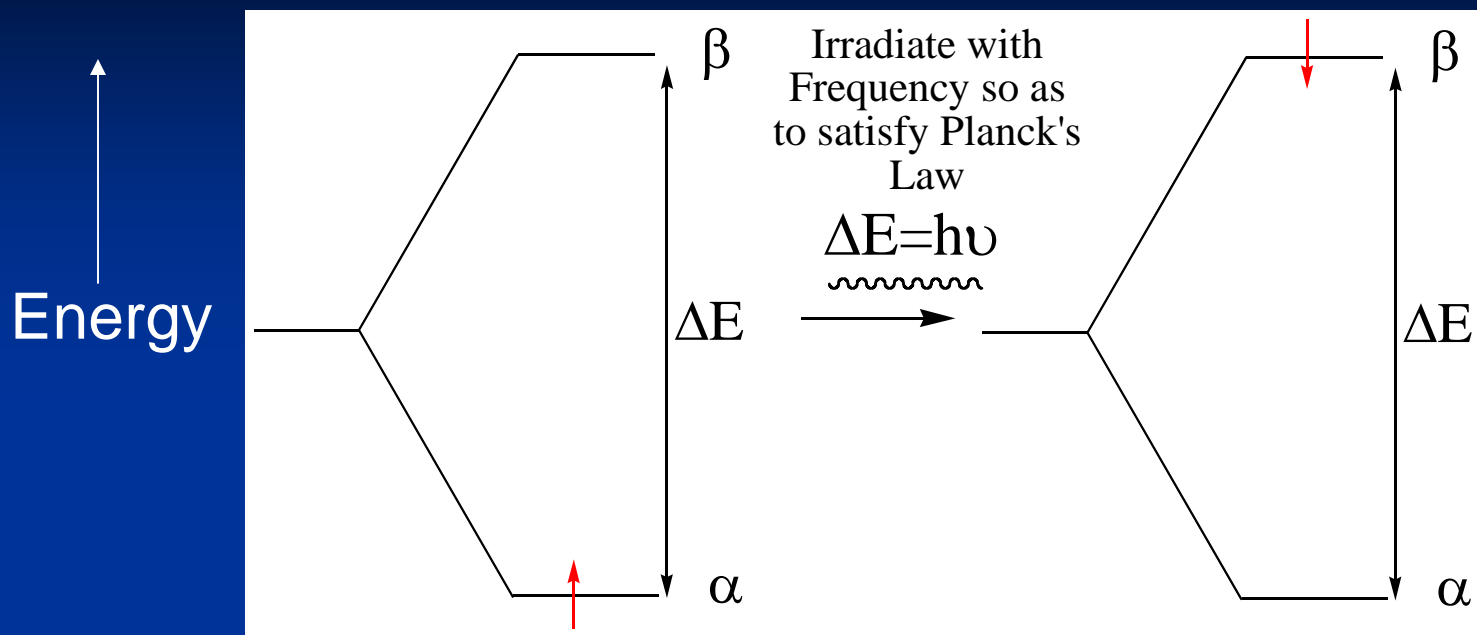
$$E_{m=1/2}$$

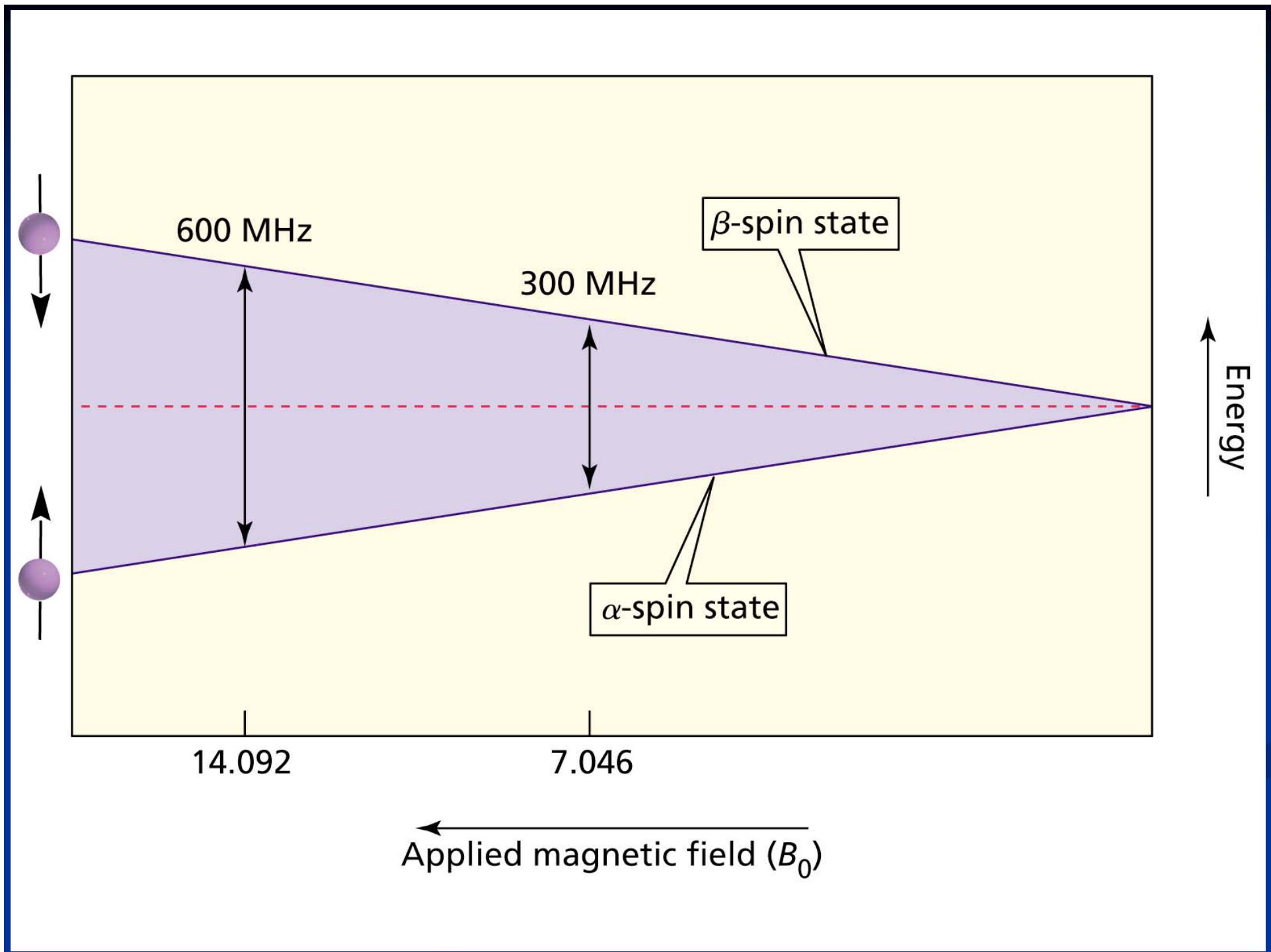


$$\Delta E_{\text{mag}} = E_{m=-1/2} - E_{m=1/2} = \Delta m_I \hbar \gamma B = h \nu \Rightarrow \nu = \gamma B / 2\pi$$

The frequency of the electromagnetic radiation that corresponds to the energy difference between the two energy levels is equal to the precessional frequency of the nuclei.

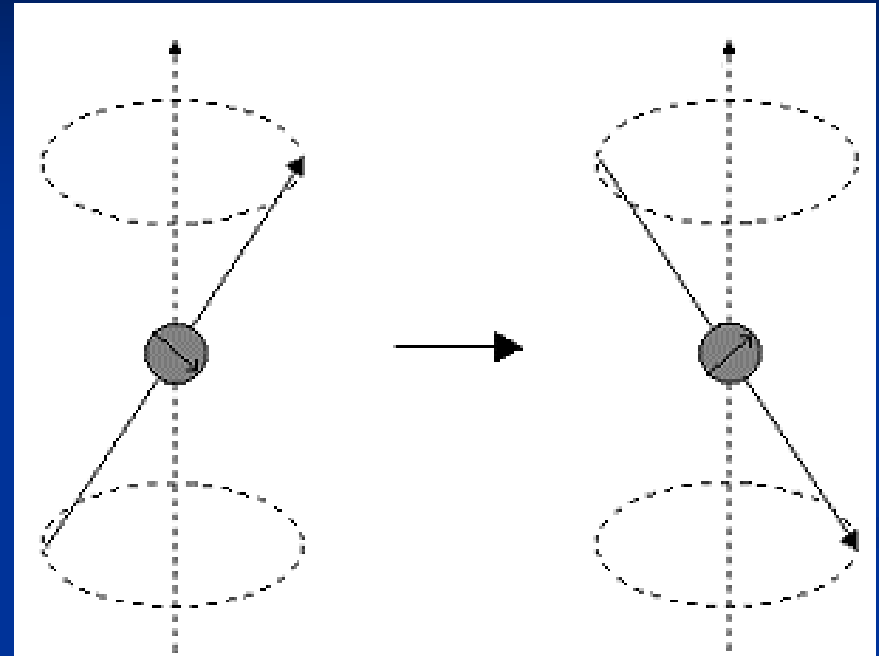
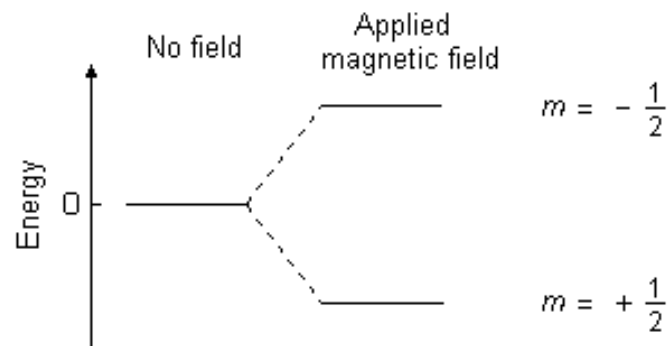
Excitation of NMR Spin





Energy Levels for $I = 1/2$

Energy levels for a nucleus with spin quantum number $1/2$

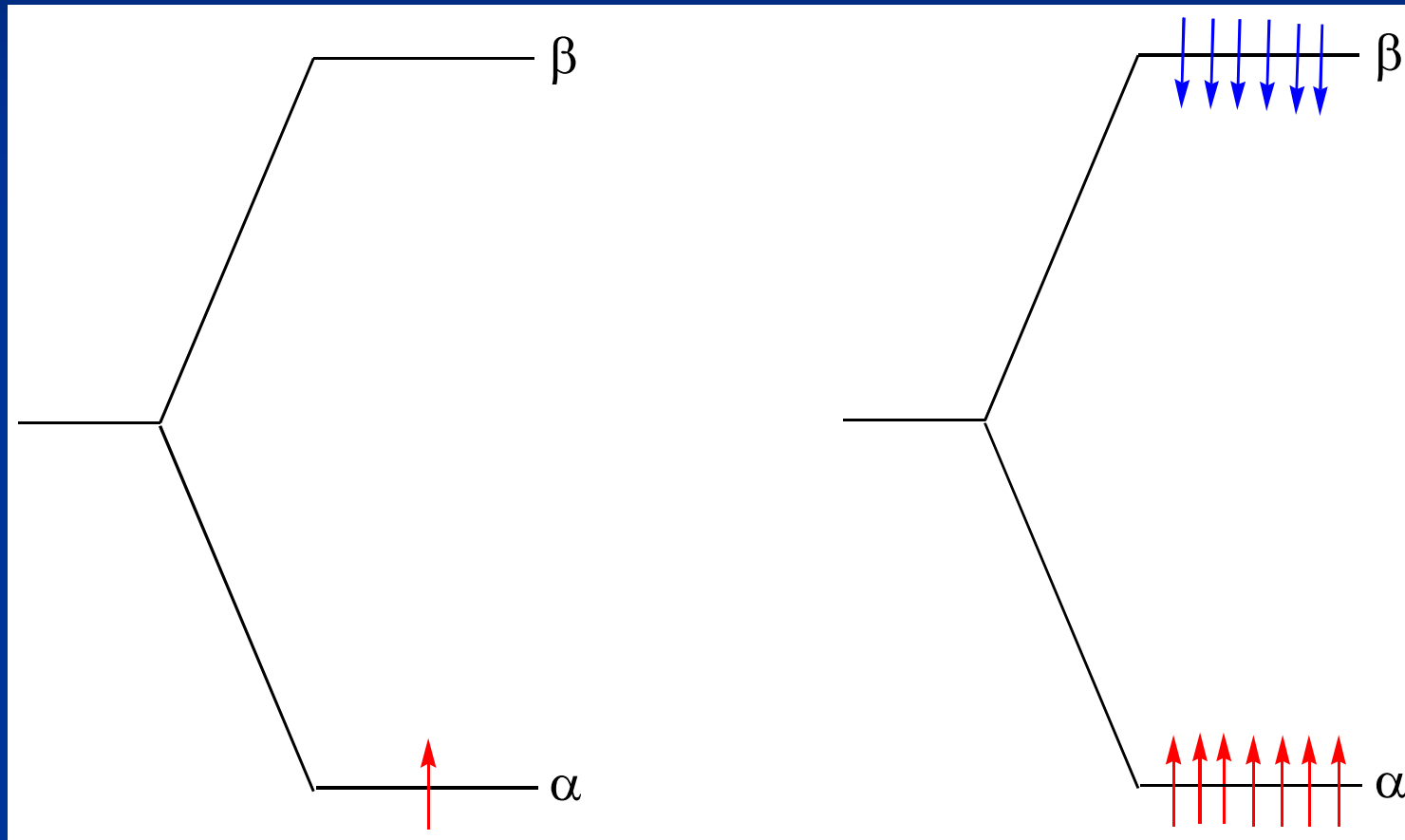


$$\Delta E_{\text{mag}} = E_{m=-1/2} - E_{m=1/2} = \Delta m_I \hbar \gamma B = \hbar \gamma B / 2\pi$$

$$\Delta E = (6.626 \times 10^{-34} \text{ J s } 26.75 \times 10^7 \text{ rad T}^{-1}\text{s}^{-1} 11.743 \text{ T}) / 2\pi = 3.313 \times 10^{-25} \text{ J}$$

very small energy difference

Energy Levels for $I = 1/2$



Boltzmann Distribution

The excess of nuclei on the lower energy level is given by Boltzmann distribution:

$$N_{\uparrow\downarrow} / N_{\uparrow\uparrow} = \exp(-\Delta E / k_B T) = \exp(-\hbar \gamma B / k_B T)$$
$$= \exp(-3.313 \cdot 10^{-25} / 4.101 \cdot 10^{-21}) = \exp(-8.078 \cdot 10^{-5}) = 0.99991922$$

If $N_{\uparrow\uparrow} = 1\,000\,000$ then $N_{\uparrow\downarrow} = 999919$

Only 81 out of million ^1H nuclei contribute to NMR signal at 500 MHz!

$$\hbar = 1.055 \cdot 10^{-34} \text{ J s}$$

$$\gamma_{\text{H}} = 26.75 \cdot 10^7 \text{ rad T}^{-1}\text{s}^{-1}$$

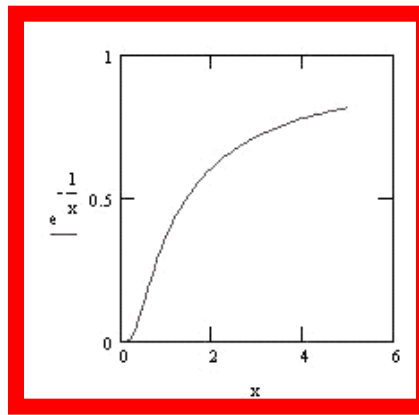
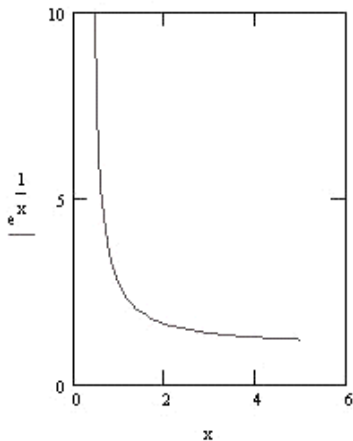
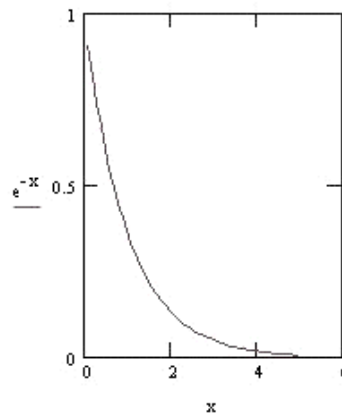
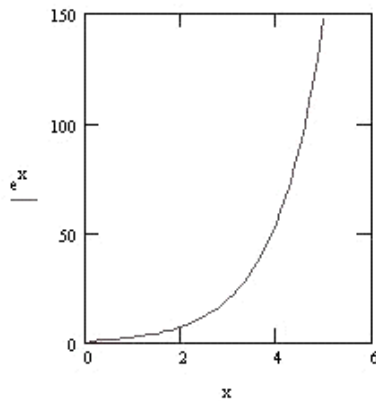
$$B = 11.7433 \text{ T (500 MHz)}$$

$$k_B = 1.3807 \cdot 10^{-23} \text{ J K}^{-1}$$

$$T = 297 \text{ K}$$

Boltzmann Distribution

$$N_{\uparrow\downarrow}/N_{\uparrow\uparrow} = \exp(-\Delta E/k_B T) = \exp(-\hbar \gamma B / k_B T)$$

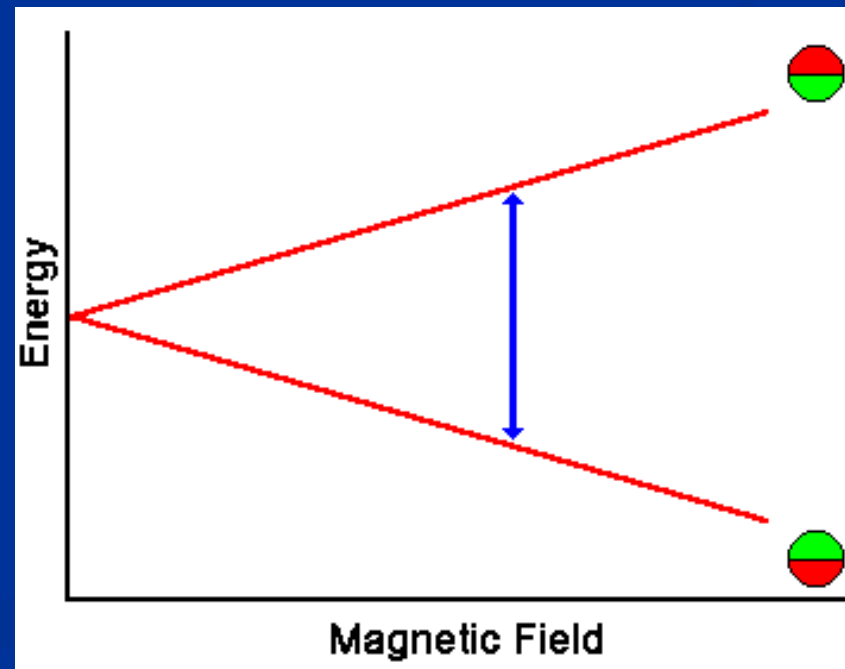


the stronger the field and the higher the magnetogyric ratio, the larger the population difference

the higher the temperature, the smaller the population difference

Boltzmann Distribution

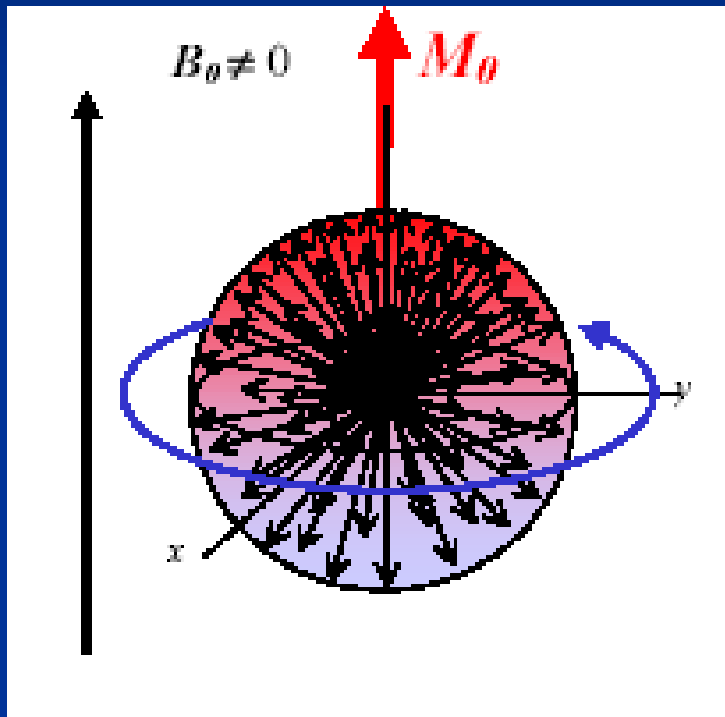
The higher the field,
the larger the energy difference,
the larger the population difference,
the larger the net magnetization,
and the bigger the NMR signal



Nuclear Magnetic Resonance (NMR)

- Nuclear – spin $\frac{1}{2}$ nuclei (e.g. protons) behave as tiny bar magnets.
- Magnetic – a strong magnetic field causes a small energy difference between $+\frac{1}{2}$ and $-\frac{1}{2}$ spin states.
- Resonance – photons of radio waves can match the exact energy difference between the $+\frac{1}{2}$ and $-\frac{1}{2}$ spin states resulting in absorption of photons as the protons change spin states.

Magnetization

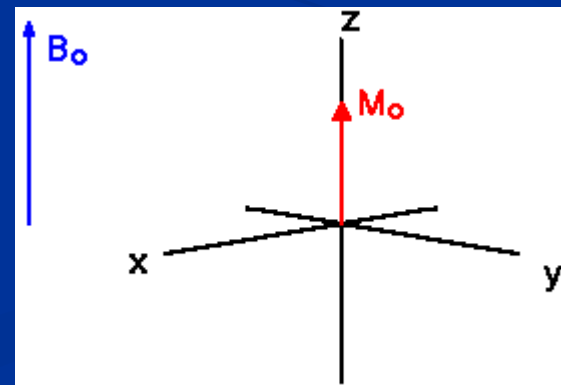


In-Field

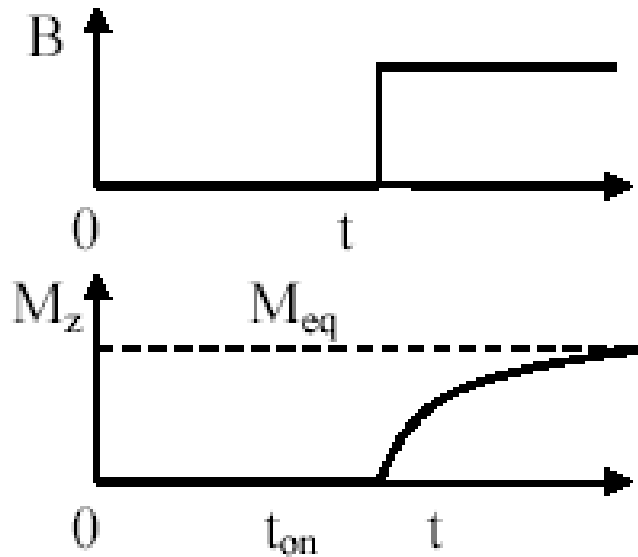
More nuclei point in parallel to the static magnetic field.

The macroscopic magnetic moment, M_0

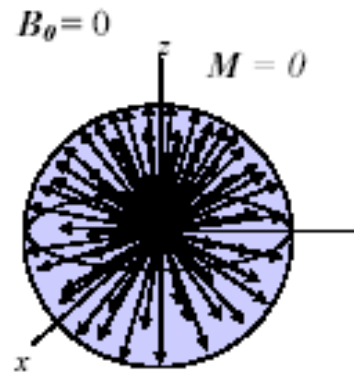
$$M_0 = \sum \mu_i$$



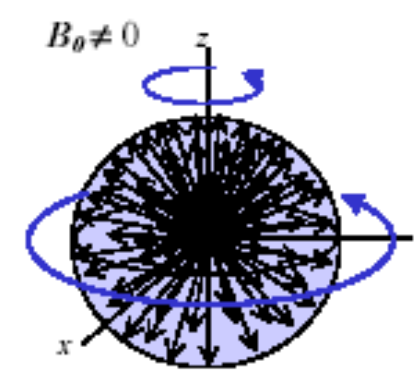
Longitudinal Magnetization



No magnetic field

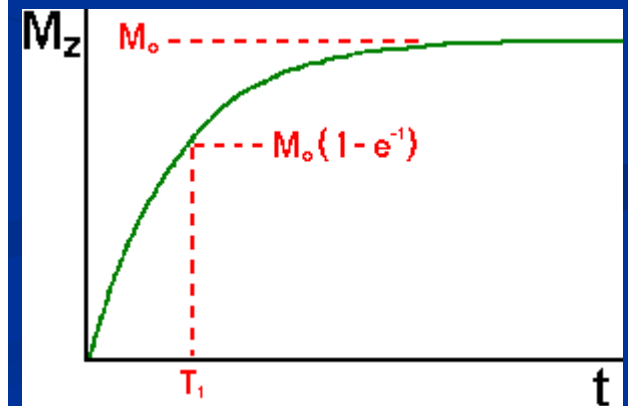


Magnetic field



$$\frac{d}{dt} M_z = R_1 (M_{eq} - M_z) = \frac{1}{T_1} (M_{eq} - M_z)$$

$$M_z(t) = M_{eq} \left(1 - e^{-\frac{-(t-t_{on})}{T_1}} \right)$$



Spin-Lattice Relaxation Time

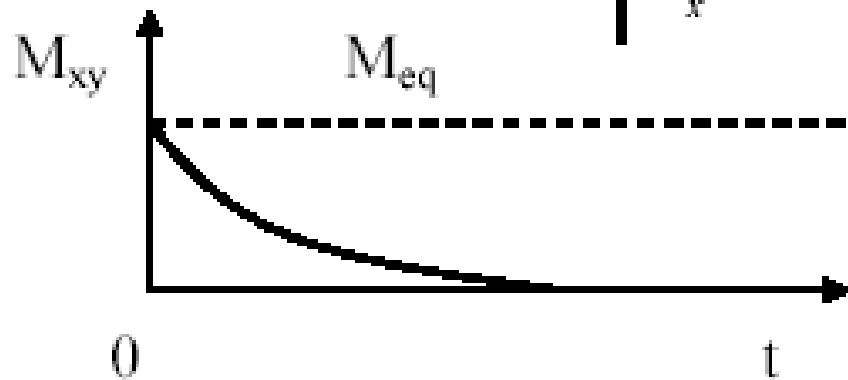
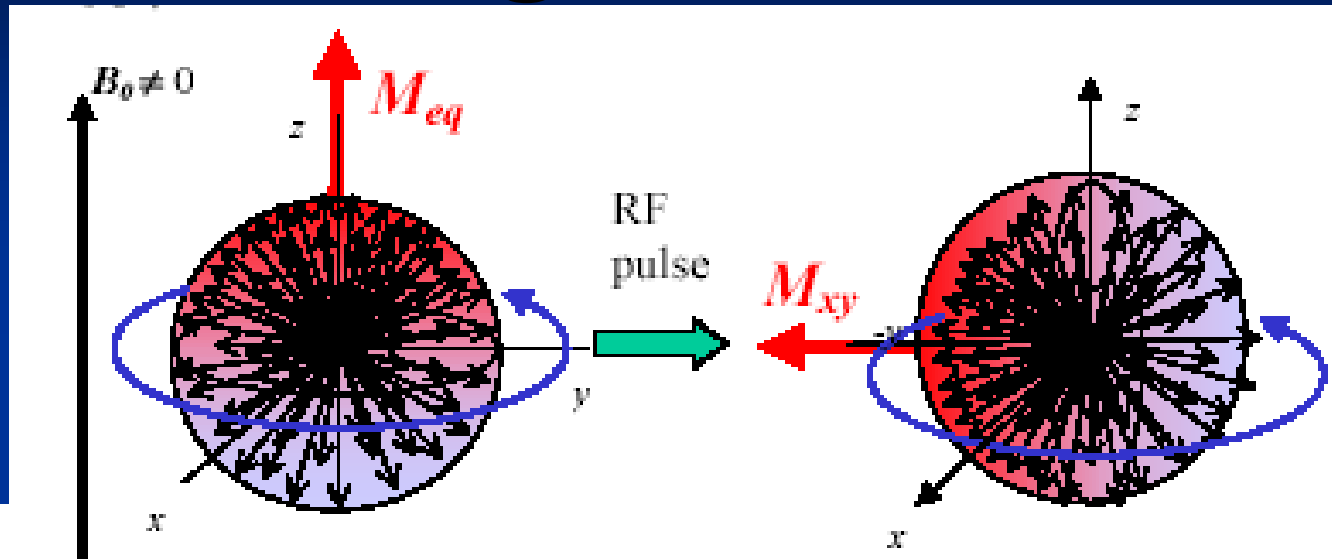
$R_1 = 1/T_1$ [Hz] longitudinal relaxation rate constant

T_1 [s] longitudinal relaxation time

spin-lattice relaxation time

enthalpy

Transverse Magnetisation



$$\frac{d}{dt} M_{xy} = -R_2 M_{xy} = \frac{-1}{T_2} M_{xy} \quad M_{xy}(t) = M_{eq} e^{\frac{-t}{T_2}}$$

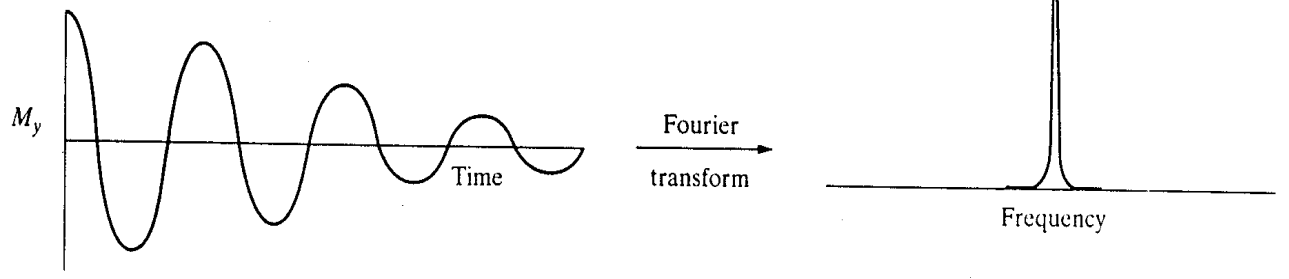
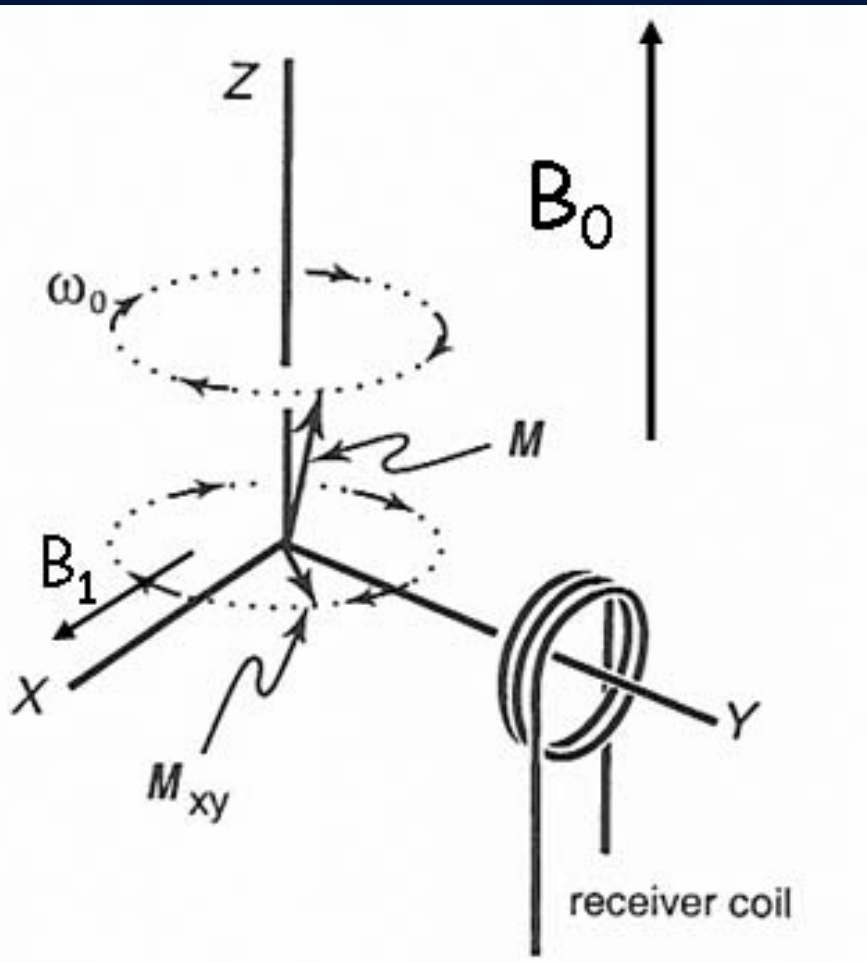
Spin-Spin Relaxation Time

$R_2 = 1/T_2$ [Hz] transverse relaxation rate constant

T_2 [s] transverse relaxation time constant

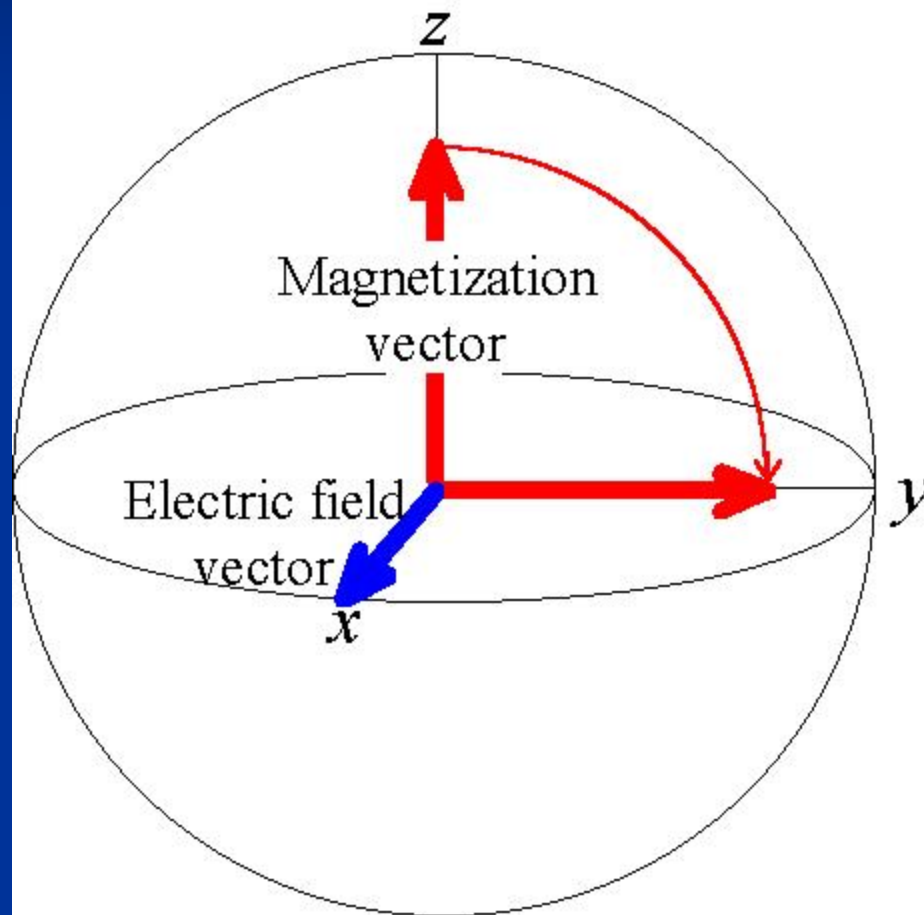
spin-spin relaxation time

entropy

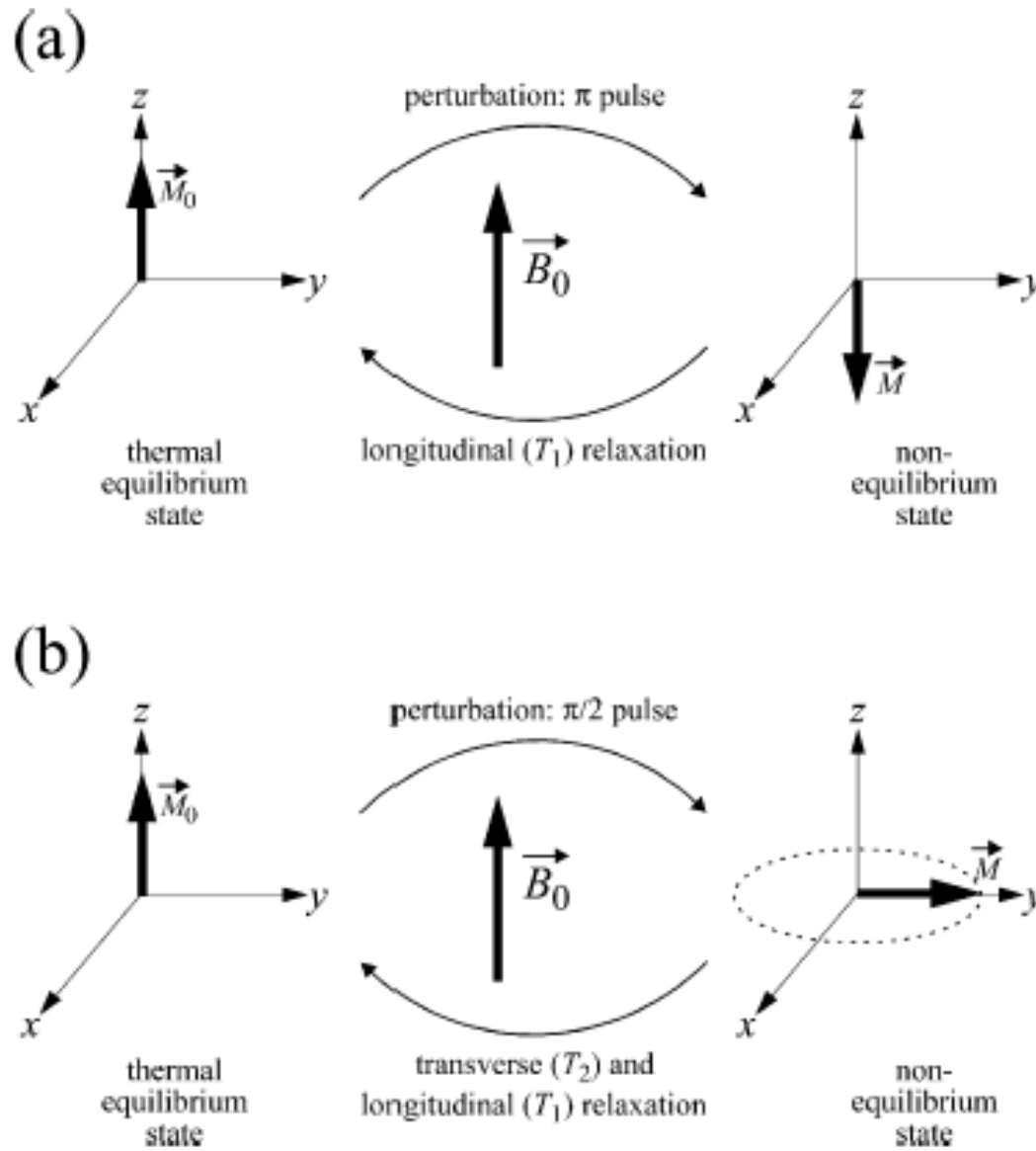


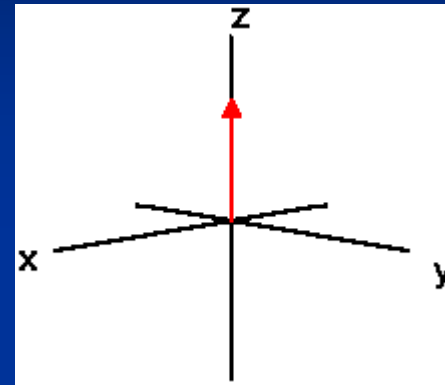
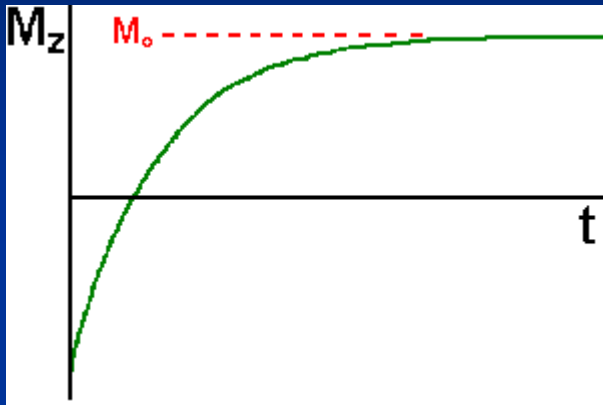
Effect of a $90^\circ x$ pulse

The magnetization vector is rotated to the y axis

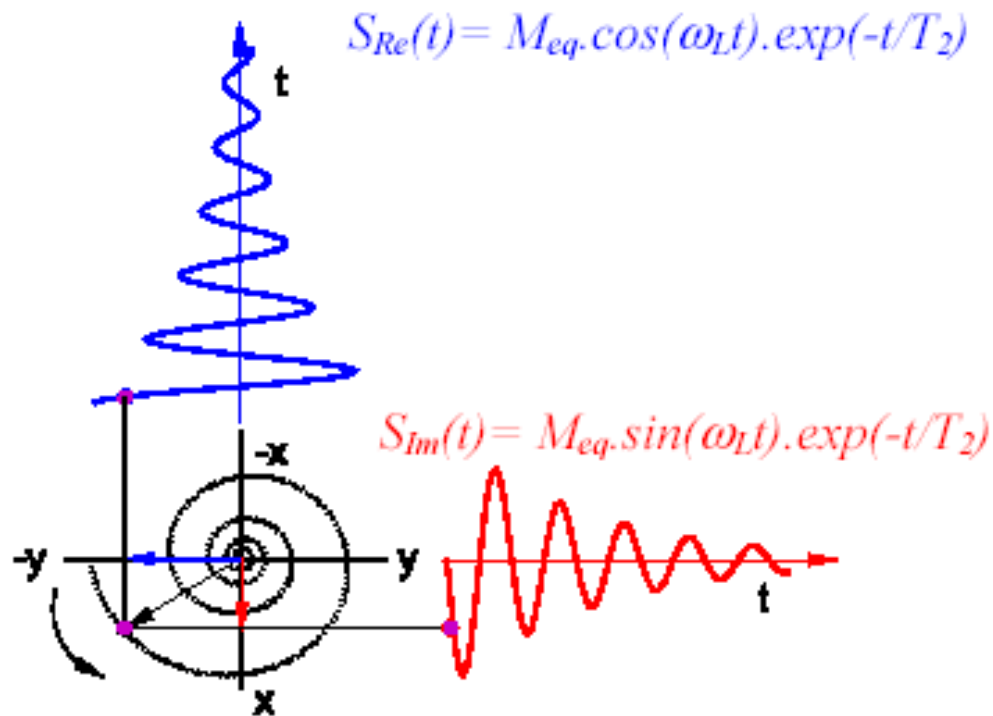


Relaxation





Free Induction Decay FID



$$S(t) = S_{Re}(t) + iS_{Im}(t)$$

$$= [\cos(\omega_L t) + i\sin(\omega_L t)] \exp(-t/T_2)$$

$$= \exp(i\omega_L t) \exp(-t/T_2) = \exp[-(1/T_2 - i\omega_L)t]$$

