Lecture 3: Multiplication and inverse matrix

http://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/video-lectures/lecture-3-multiplication-and-inverse-matrices/

Pre-listening

1) What is a matrix, where can you use it in maths?
2) What is the plural form of a matrix (two ....)?
3) What are the basic operations you can apply to a matrix?

Listening – Answer these questions.

1) In how many ways can you multiply matrices?...
2) What does the point in matrix C indicate?...
3) Why should we write C i j and not C j i ?..
4) Why does the professor use C 3 4 instead of C i j?....................................
5) What does a dot indicate?..
6) What are indices?...
7) What does B 2 4 indicate?..
8) What happens when the shape of a matrix is rectangular?.........................
9) What is n?...
10) What is p?..

Try to explain these expressions the professor uses or supply the synonyms.

1) That´s a big deal. ....
2) That gives us a chance to do it. ....
3) I very seldom get down to the details. ....
4) Then comes the next guy. ....

Try to read this notation.

$$[\mathbf{AB}]_{i,j} = A_{i,1}B_{1,j} + A_{i,2}B_{2,j} + \cdots + A_{i,n}B_{n,j} = \sum_{r=1}^{n} A_{i,r}B_{r,j},$$

....

Matrix (mathematics)

From Wikipedia, the free encyclopedia

1) Read the text and explain the following terms.

a) an element ....

b) commutative ....

c) tensor ....

d) eigenvalues ....

e) dodecaphonism ....

In mathematics, a **matrix** (plural **matrices**, or less commonly **matrixes**) is a rectangular array of numbers, such as

$$\begin{bmatrix} 1 & 9 & 13 \\ 20 & 55 & 6 \end{bmatrix}.$$

An item in a matrix is called an entry or an element. The example has entries 1, 9, 13, 20, 55, and 6. Entries are often denoted by a variable with two subscripts, as shown on the right. Thus in the matrix above, $a_{2,1} = 20$. Matrices of the same size can be added and subtracted entrywise and matrices of compatible sizes can be multiplied. These operations have many of the properties of ordinary arithmetic, except that matrix multiplication is not commutative, that is, **AB** and **BA** are not equal in general. Matrices consisting of only one column or row define the components of vectors, while higher-dimensional (e.g., three-dimensional) arrays of numbers define the components of a generalization of a vector called a tensor. Matrices with entries in other fields or rings are also studied.

Matrices are a key tool in linear algebra. One use of matrices is to represent linear transformations, which are higher-dimensional analogs of linear functions of the form $f(x) = cx$, where c is a constant; matrix multiplication corresponds to composition of linear transformations. Matrices can also keep track of the coefficients in a system of linear equations. For a square matrix, the determinant and inverse matrix (when it exists) govern the behavior of solutions to the corresponding system of linear equations, and eigenvalues and eigenvectors provide insight into the geometry of the associated linear transformation.

Matrices find many applications. Physics makes use of matrices in various domains, for example in geometrical optics and matrix mechanics; the latter led to studying in more detail matrices with an infinite number of rows and columns. Graph theory uses matrices to keep track of distances between pairs of vertices in a graph. Computer graphics uses matrices to project 3-dimensional space onto a 2-dimensional screen. Matrix calculus generalizes classical analytical notions such as derivatives of functions or exponentials to matrices. The latter is a recurring need in solving ordinary differential equations. Serialism and dodecaphonism are musical movements of the 20th century that use a square mathematical matrix to determine the pattern of music intervals.

2) There is an example of the matrix multiplication. Read it and try to fill in the missing items.

multiply

dot product

column

last

comma

product

height

row

width

coordinate

Ordinary matrix product

The ordinary matrix product is the most often used and the most important way to a)........ matrices. It is defined between two matrices only if the width of the first matrix equals the b)....... of the second matrix. Multiplying an $m \times n$ matrix with an $n \times p$ matrix results in an $m \times p$ matrix. If many matrices are multiplied together, and their dimensions are written in a list in order, e.g. $m \times n$, $n \times p$, $p \times q$, $q \times r$, the size of the result is given by the first and the c)....... numbers ($m \times r$), and the values surrounding each d)........must match for the result to be defined.

$$\underset{3\times 4 \text{ matrix}}{\begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 1 & 2 & 3 & 4 \end{bmatrix}} \underset{4\times 5 \text{ matrix}}{\begin{bmatrix} \cdot & \cdot & \cdot & a & \cdot \\ \cdot & \cdot & \cdot & b & \cdot \\ \cdot & \cdot & \cdot & c & \cdot \\ \cdot & \cdot & \cdot & d & \cdot \end{bmatrix}} = \underset{3\times 5 \text{ matrix}}{\begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & x_{3,4} & \cdot \end{bmatrix}}$$

The element $x_{3,4}$ of the above matrix e)......... is computed as follows

$$x_{3,4} = (1,2,3,4) \cdot (a,b,c,d) = 1 \times a + 2 \times b + 3 \times c + 4 \times d.$$

The first f).......... in matrix notation denotes the g)........ and the second the column; this order is used both in indexing and in giving the dimensions. The element x_{ij}at the intersection of row *i* and h)...........*j* of the product matrix is the dot product (inner product) of row *i* of the first matrix and column *j* of the second matrix. This explains why the i)......... and the height of the matrices being multiplied must match: otherwise the j)....... is not defined.

3) WORD STUDY there are some words from the text which have been mixed up, Try to sort them out.

oatonin	ryara................	lunomc ....
misodenisn.............	copurdt............	finefsocitec ....

Work out each of the following problems.

1. Mary found that her new car averaged 18.2 per miles per gallon the first week, 19.0 the second week, 17.6 the third week, and 18.5 the fourth week. Write this information as a row matrix and then as a column matrix.

2. Joan' s scores on the first three tests in her math class were 82, 77, and 85. Paula scored 91, 80, and 82 on the same three tests. Janet' s scores on the three tests were 90, 82, and 79. Write this information as a 3x3 square matrix in two different ways.

3. Richard Marcias bought 7 shares of Sears stock, 9 shares of IBM stock, and 8 shares of Chrysler stock. The following month, he bought 2 shares of Sears stock, no IBM, and 6 shares of Chrysler. Write this information first as a 3x2 matrix and then as a 2x3 matrix.

4. Margie Bezzone works in a computer store. The first week she sold 5 computers, 3 printers, 4 disc drives, and 6 monitors. The next week she sold 4 computers, 2 printers, 6 disc drives, and 5 monitors, Write this information first as a 2x4 matrix and then as a 4x2 matrix.