

Pr 23/3c)

$$xy' + y = y^2, \quad y(-1) = \frac{1}{2}$$

$$y' = \frac{y^2 - y}{x}$$

$$\int \frac{1}{y^2 - y} dy = \int \frac{1}{x} dx$$

Rozklad na parciální zlomky:

$$\frac{1}{y^2 - y} = \frac{A}{y} + \frac{B}{y-1}$$

$$1 = A(y-1) + By$$

$$1 = Ay - A + By$$

$$y^1: 0 = A + B$$

$$y^0: 1 = -A$$

$$\Rightarrow A = -1$$

$$B = 1$$

$$\int \left(\frac{-1}{y} + \frac{1}{y-1} \right) dy = \int \frac{1}{x} dx$$

$$-\ln|y| + \ln|y-1| = \ln|x| + c$$

$$\ln \left| \frac{y-1}{y} \right| = \ln|x| + c$$

$$\left| \frac{y-1}{y} \right| = k \cdot |x|$$

$$\frac{y-1}{y} = K \cdot x$$

$$\underline{y = \frac{1}{1 - K \cdot x}}$$

Počáteční podmínka: $y(-1) = \frac{1}{2}$

$$\frac{1}{2} = \frac{1}{1 - K \cdot (-1)}$$

$$2 = 1 + K$$

$$\underline{K = 1}$$

$$\underline{\underline{\text{Řešení: } y = \frac{1}{1-x}}}$$