

Homework number 6

This time only one exercise.

Exercise 15. Show that the spaces $U = S^n \times S^k$ and $V = S^n \vee S^k \vee S^{n+k}$ have the same homology and cohomology groups but they are not homotopy equivalent. (Hint: Compute their structures as graded cohomology rings. To compute the cup product on $X \vee Y$ use its naturality with respect to inclusions of X and Y into $X \vee Y$ and retractions $X \vee Y$ to X and Y , respectively.)