HOMEWORK 7

Exercise 1. Let $\times : H^*(X) \otimes H^*(Y) \to H^*(X \times Y)$ be the cross product defined by the formula

$$\alpha \times \beta = p_1^*(\alpha) \cup p_2^*(\beta)$$

where $p_1 : X \times Y \to X$ and $p_2 : X \times Y \to Y$ are the projections on the first and the second component, respectively. Let $\Delta : X \to X \times X$ be the diagonal $\Delta(x) = (x, x)$. Prove that for $\alpha, \beta \in H^*(X)$

$$\alpha \cup \beta = \Delta^*(\alpha \times \beta).$$

Exercise 2. Let $f : X \to Y$ be a constant map. Prove that $f_* : H_n(X) \to H_n(Y)$ and $f^* : H^n(Y) \to H^(X)$ are zero maps for $n \geq 1$. (Hint: One can do it from the definition, but much easier is to factor f as a composition of suitable two maps and use the fact that H_* and H^* are a functor and a cofunctor, respectively.)

Exercise 3. Let the cohomology rings of the spaces X and Y are the following

$$H^*(X) \cong \mathbb{Z}[x]/\langle x^n \rangle, \quad H^*(Y) \cong \mathbb{Z}[y]/\langle y^m \rangle$$

where $x \in H^1(X)$ and $y \in H^1(Y)$. Prove that

$$H^*(X \vee Y) \cong \mathbb{Z}[u, v]/\langle u^n, v^m, uv \rangle.$$