



CEITEC

Central European Institute of Technology
BRNO | CZECH REPUBLIC

Image analysis III & 3D Reconstruction

C9940 3-Dimensional Transmission Electron Microscopy
S1007 Doing structural biology with the electron microscope

March 23, 2015



EUROPEAN UNION
EUROPEAN REGIONAL DEVELOPMENT FUND
INVESTING IN YOUR FUTURE



OP Research and
Development for Innovation



Outline

Image analysis III

- ◆ Still more Fourier transforms
 - Convolution
 - Step function
 - Power spectrum
 - Friedel's Law
- ◆ More orientation alignment
- ◆ More interpolation
- ◆ Classification

3D Reconstruction

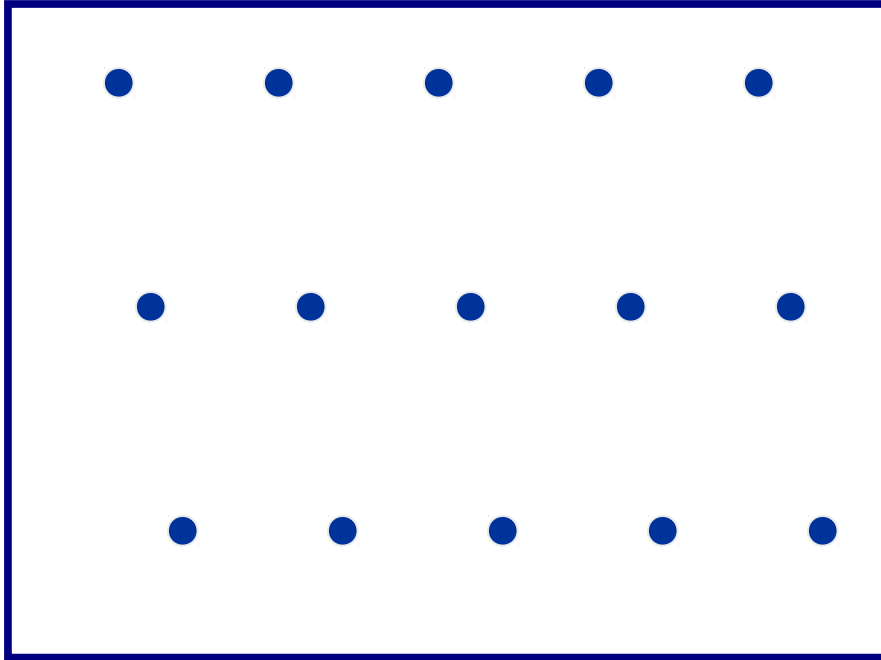
- ◆ Principles
- ◆ Reference-based alignment
- ◆ RCT

*Current events:
Convolutions*

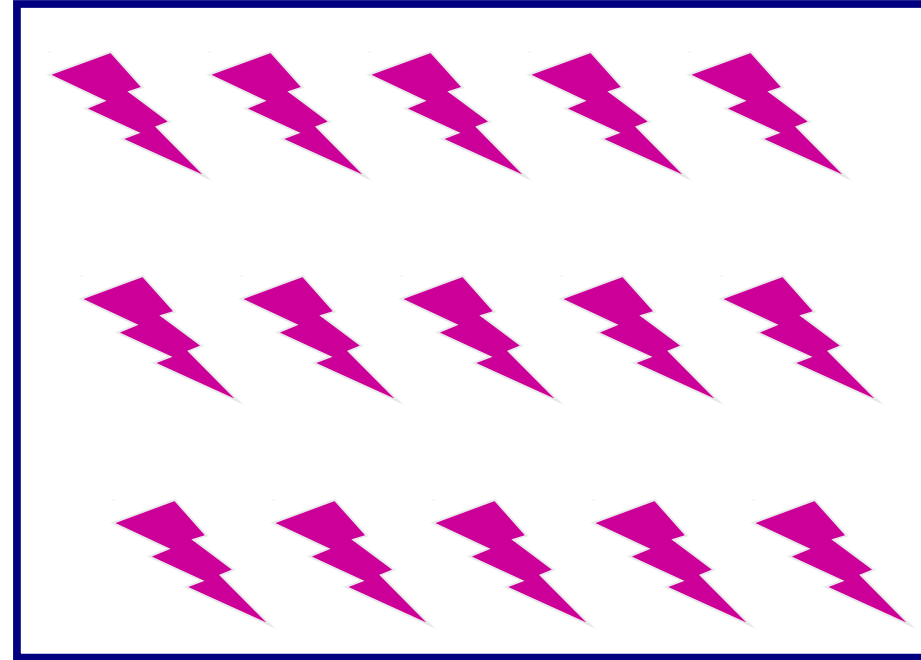
Convolution of a molecule with a lattice generates a crystal.

Notation: $f(x)*l(x)$

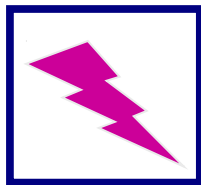
From David DeRosier



lattice = $l(x)$



Set a molecule down at every lattice point.



Molecule = $f(x)$

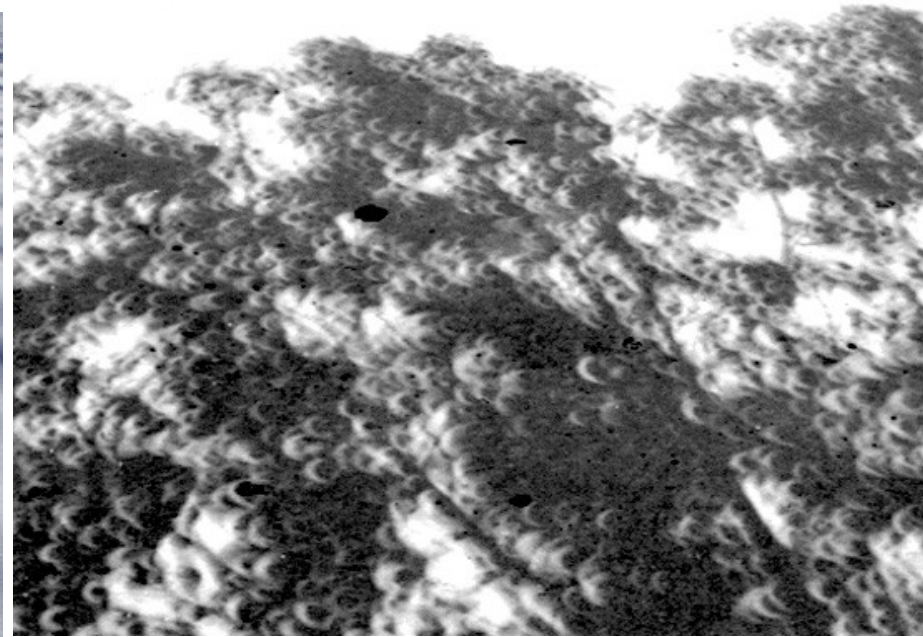
Convolution of a molecule with a lattice generates a crystal.

Notation: $f(x)*l(x)$

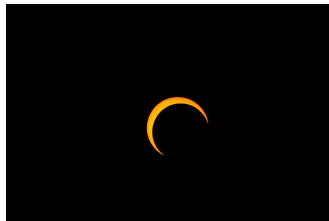


lattice = $l(x)$

(<http://www.photos-public-domain.com>)



<http://www.symbolicmessengers.com>



Molecule = $f(x)$

<http://en.wikipedia.org>

Set a molecule down at every
lattice point.

Cross-correlation vs. convolution

Complex conjugate:

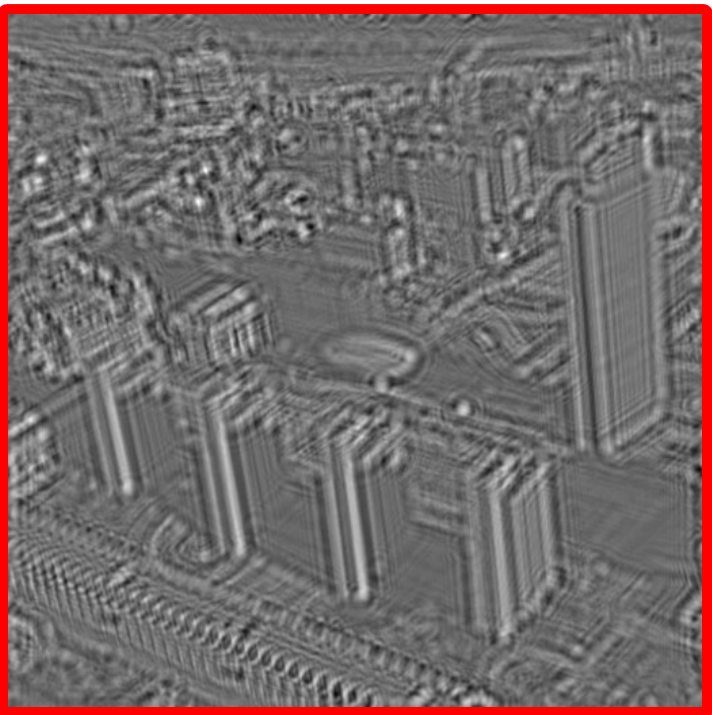
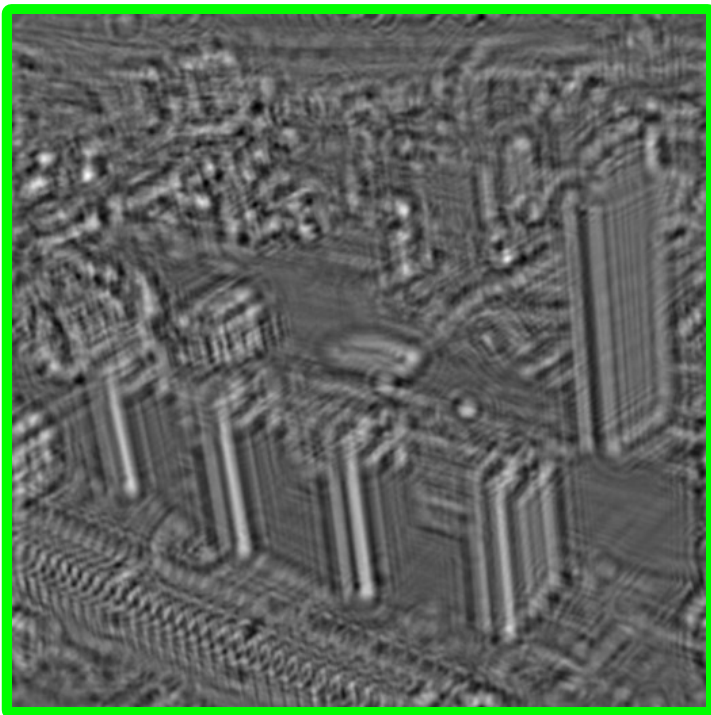
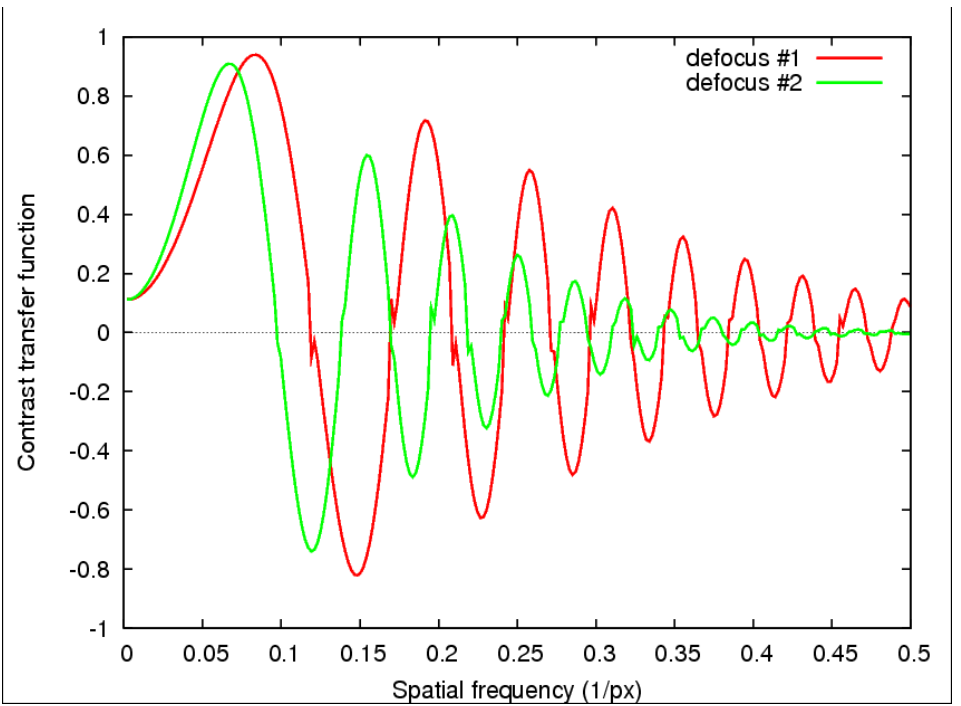
If a Fourier coefficient $F(X)$ has the form: $a + bi$

The complex conjugate $F^*(X)$ has the form: $a - bi$

Cross-correlation: $F^*(X) G(X)$

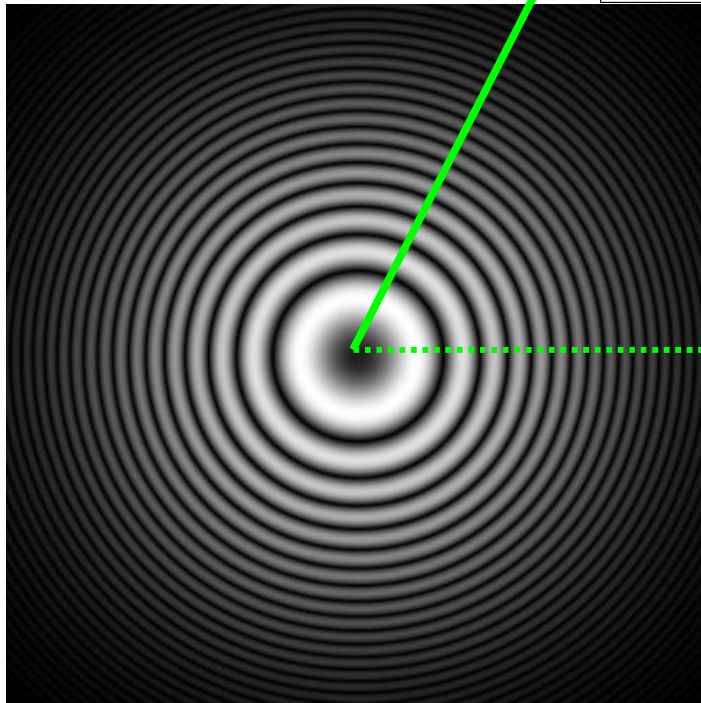
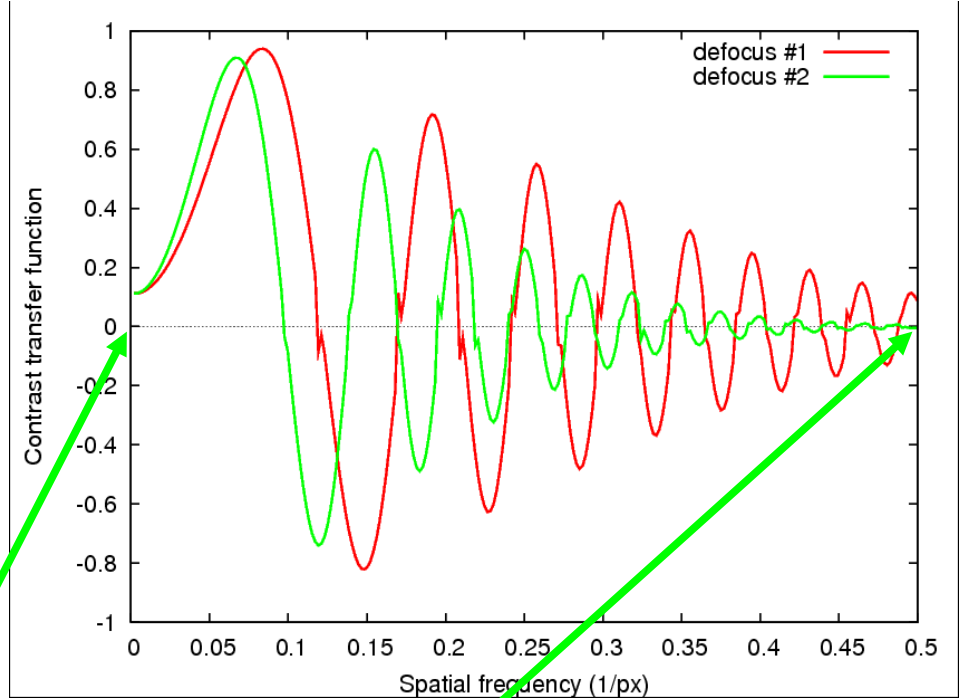
Convolution: $F(X) G(X)$

original



CTF

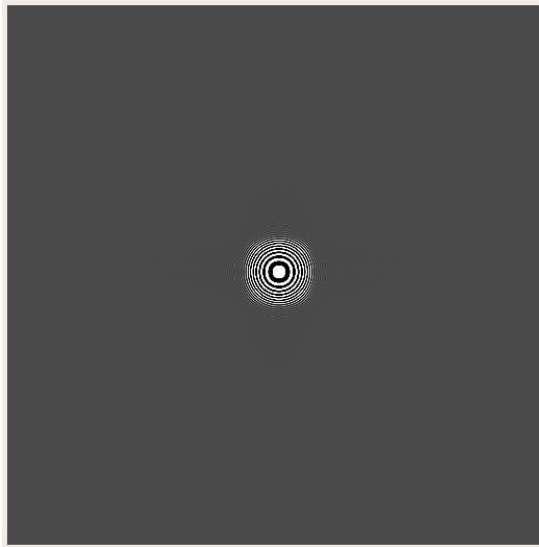
1D profile



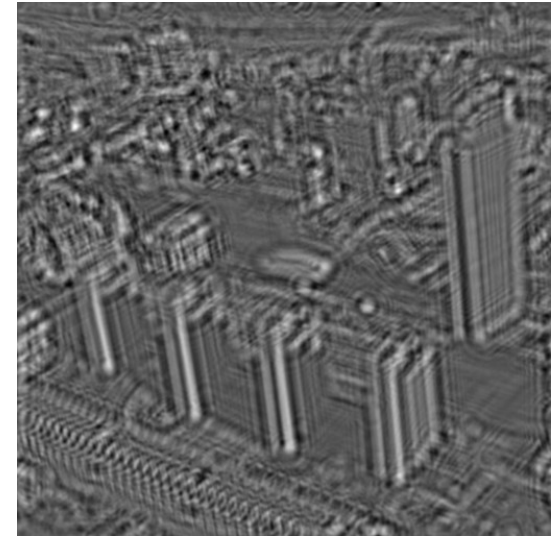
2D power spectrum
 $G(X)$



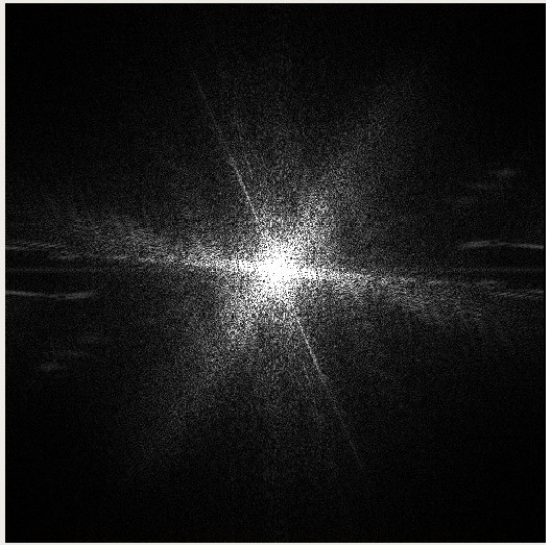
$f(x)$



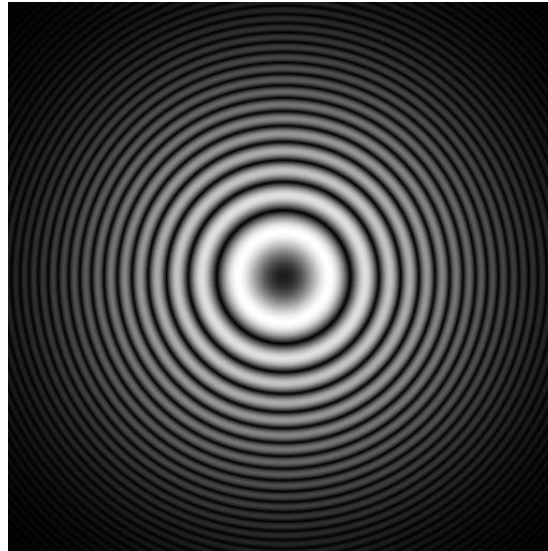
$g(x)$



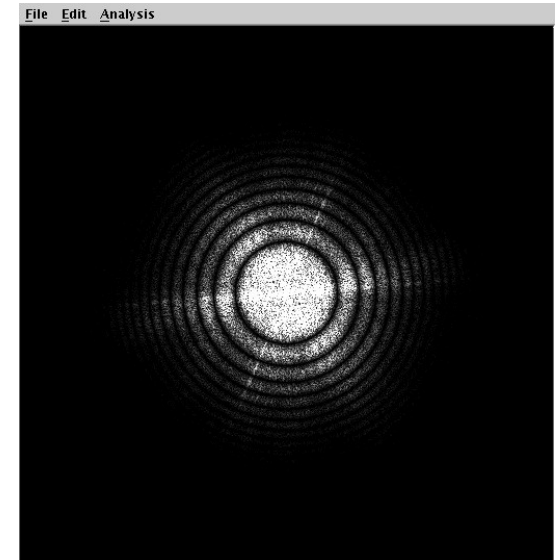
$f(x) g(x)$



$F(X)$

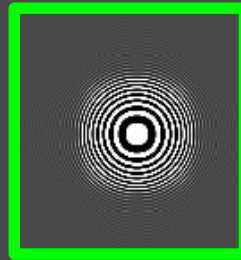


$G(X)$

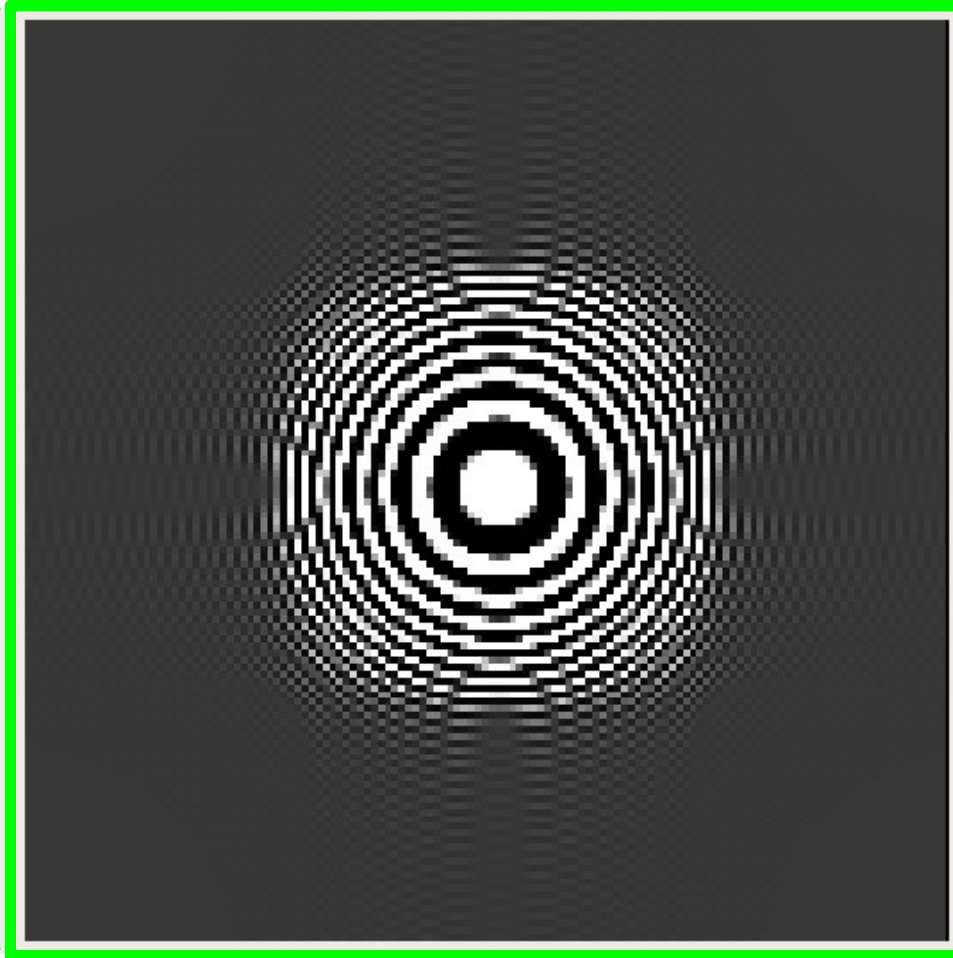


$F(X) G(X)$

Point spread function



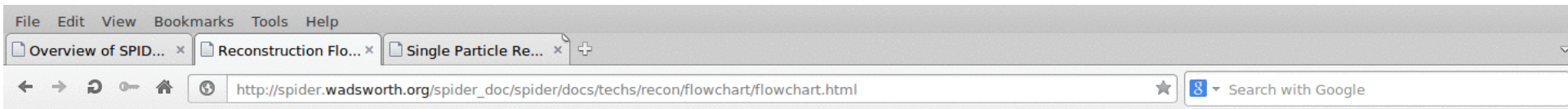
$g(x)$



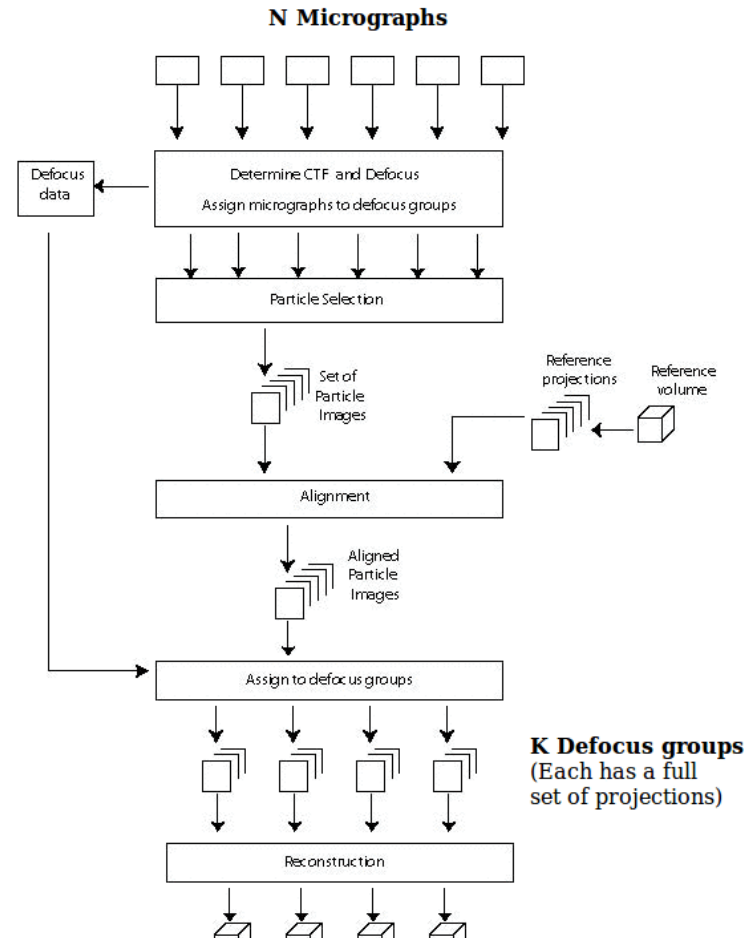
zoomed

An ideal point spread function would be an infinitely-sharp point.

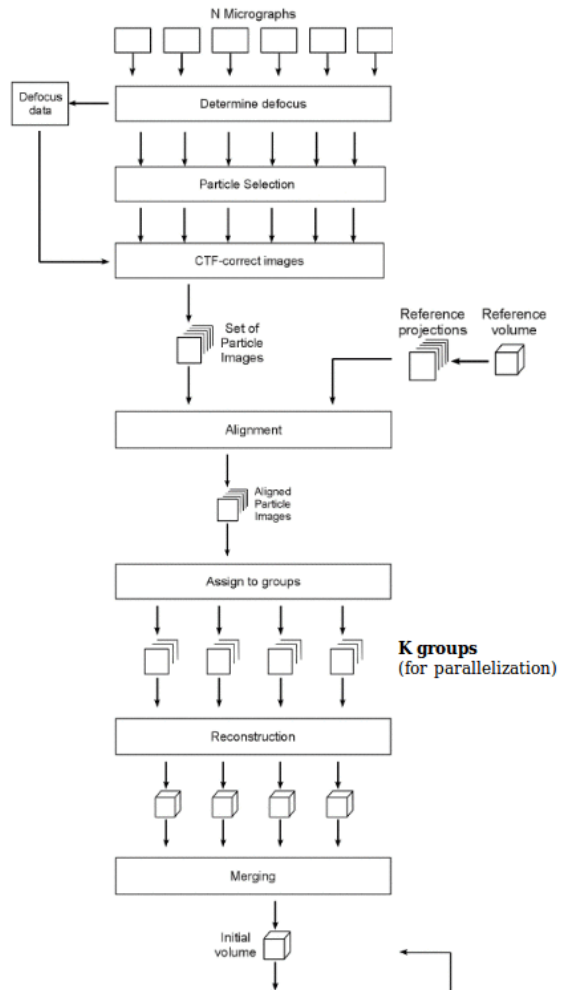
Defocus groups



Reference-based Reconstruction



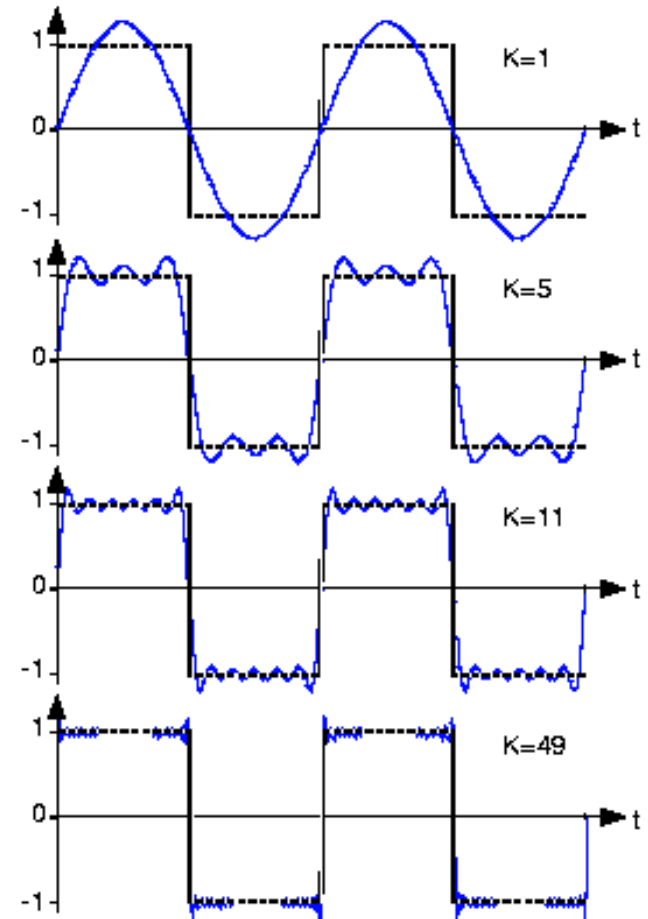
Defocus groups



Step function revisited

Fourier transforms: plot of step function

The higher the spatial frequencies (i.e., higher resolution) that are included, the more faithful the representation of the original function will be.



<http://cnx.org>

The power spectrum is the a real
(as opposed to complex)
map of the amplitudes of the Fourier transform

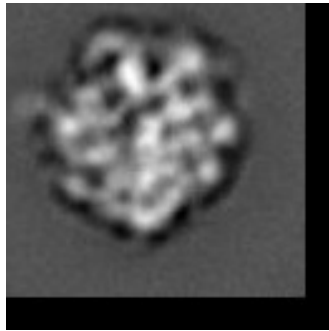


Image $f(x)$

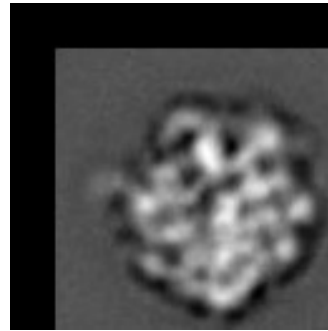
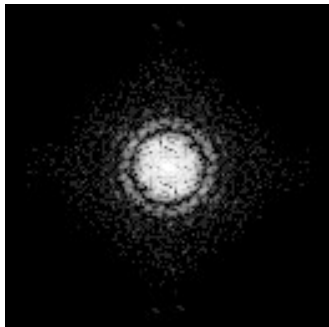
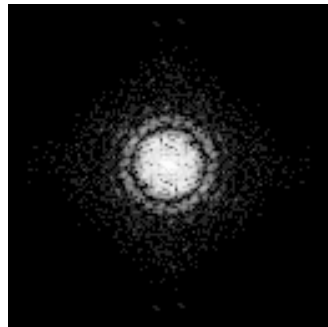


Image $g(x)$



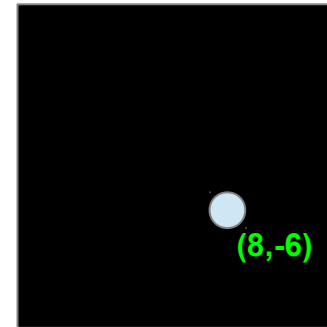
F.T. $F^*(X)$
(complex conjugate)

x



F.T. $G(X)$

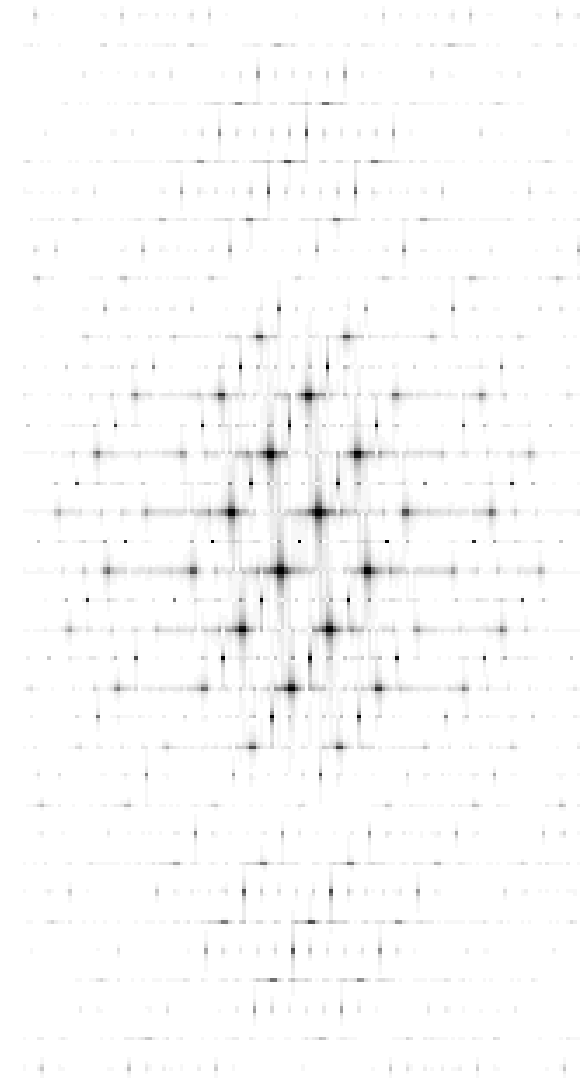
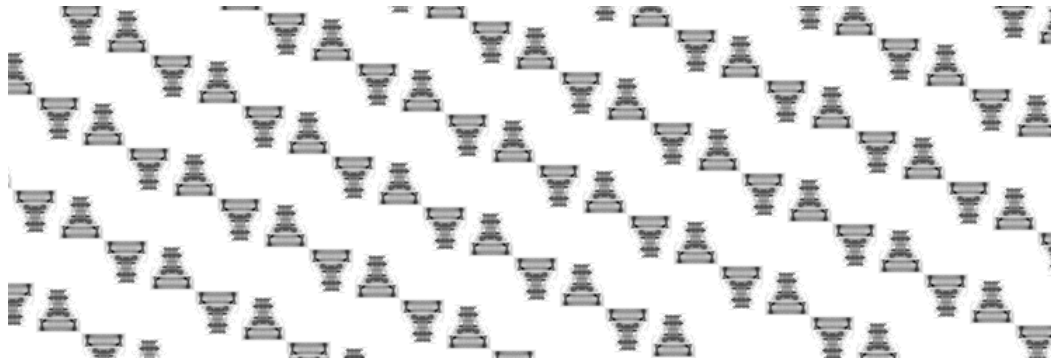
=



CCF

The position of the peak gives us the shifts that give the best match, e.g., (8,-6).
It's more difficult to plot a 2D F.T. showing both amplitude and phase.

Fourier transform of a 2D crystal



h	k	Amp	Phase
0	0	500	0
1	-1	40	45
1	0	50	5
1	1	30	5
2	-2	2	54
2	-1	4	57

QUESTION:

Why did I not list the Fourier data where h was negative?

h	k	Amp	Phase
0	0	500	0
1	-1	40	45
1	0	50	5
1	1	30	5
2	-2	2	54
2	-1	4	57

Friedel's Law

If the complex part of $f(x)$ is zero, then

$$F(-X) = F^*(X)$$

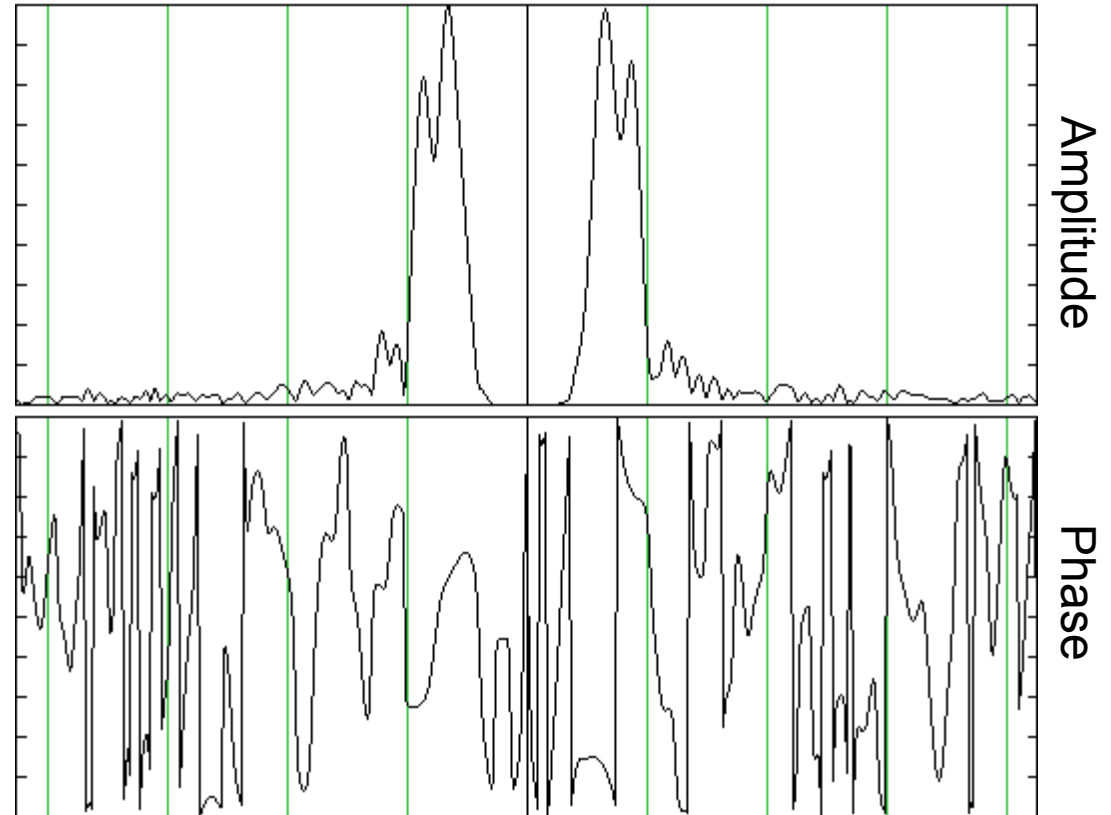
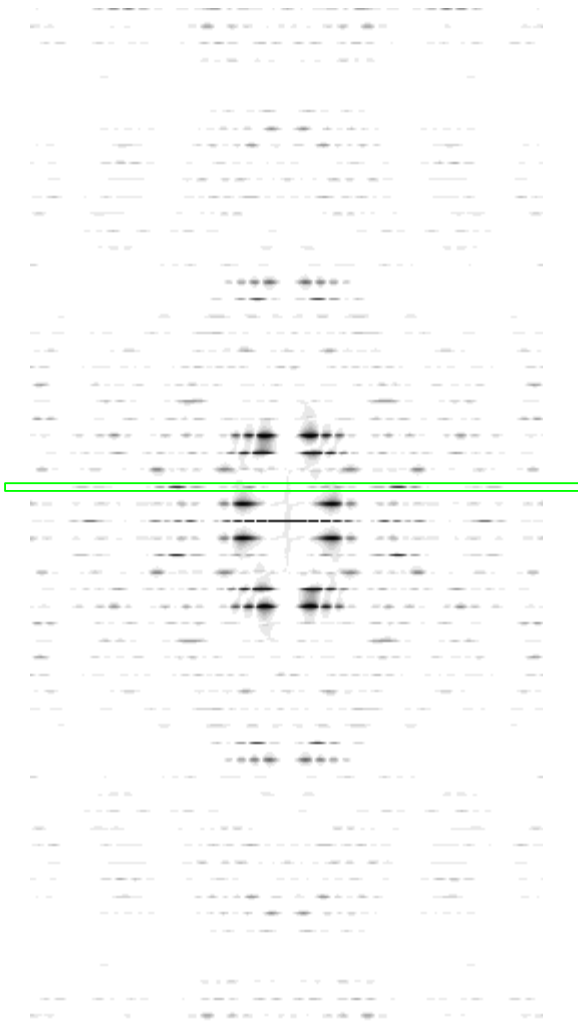
where * indicates the complex conjugate.

USE: Thus, centrosymmetrically related reflections have the same amplitude but opposite phases (Friedel's law).

When calculating a transform of an image, one only has to calculate half of it. The other half is related by Friedel's law.

From David DeRosier

2D Fourier transform of a helix



More orientational alignment

Orientation alignment

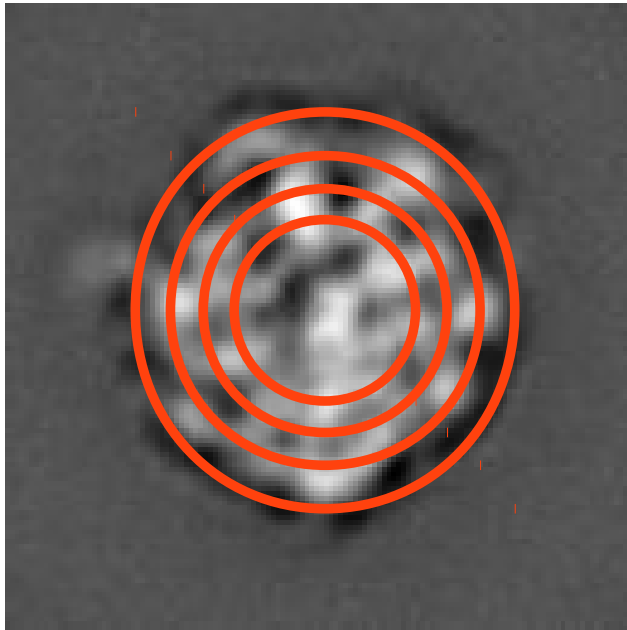


Image 1

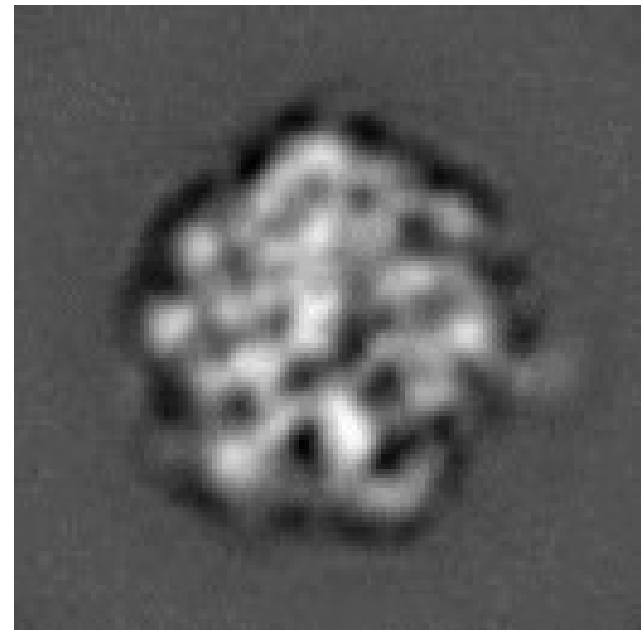


Image 2

We take a series of rings from each image, unravel them, and compute a series of 1D cross-correlation functions.

Shifts along these unraveled CCFs is equivalent to a rotation in Cartesian space.

Orientation alignment

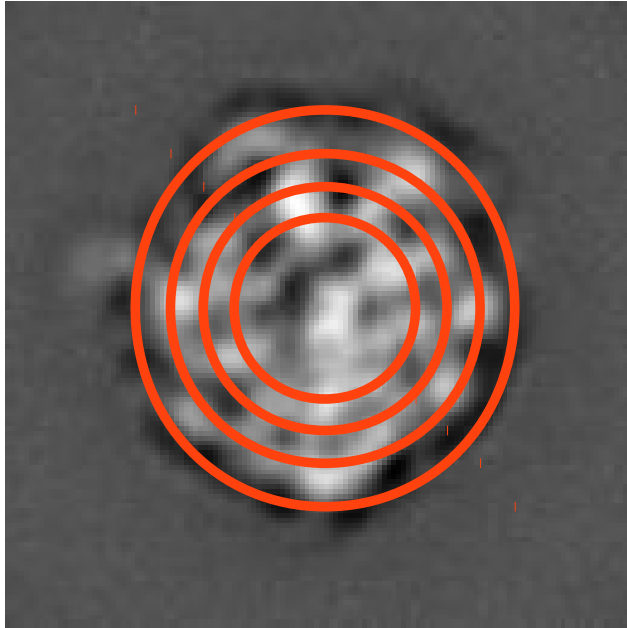


Image 1

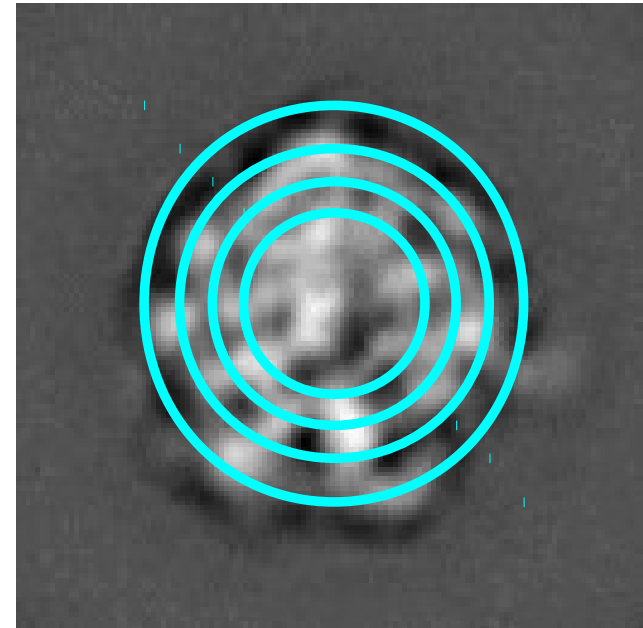
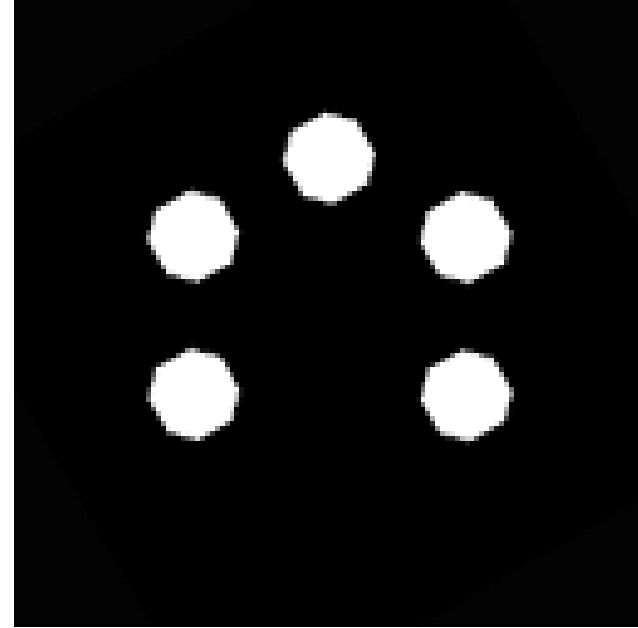
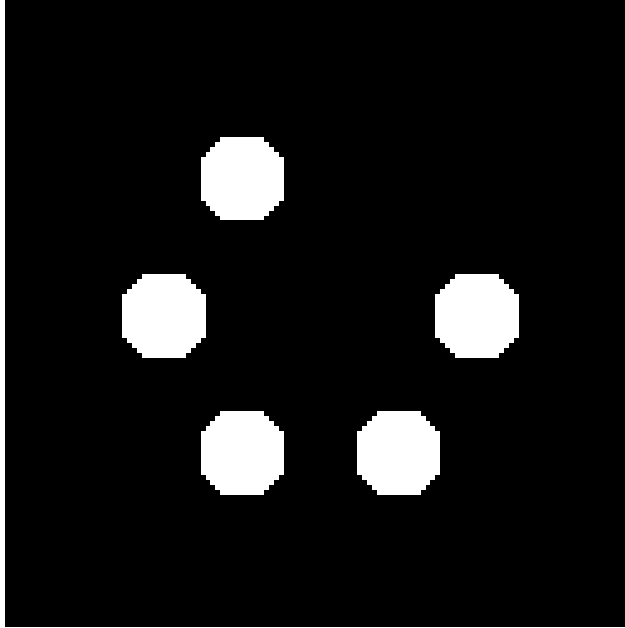


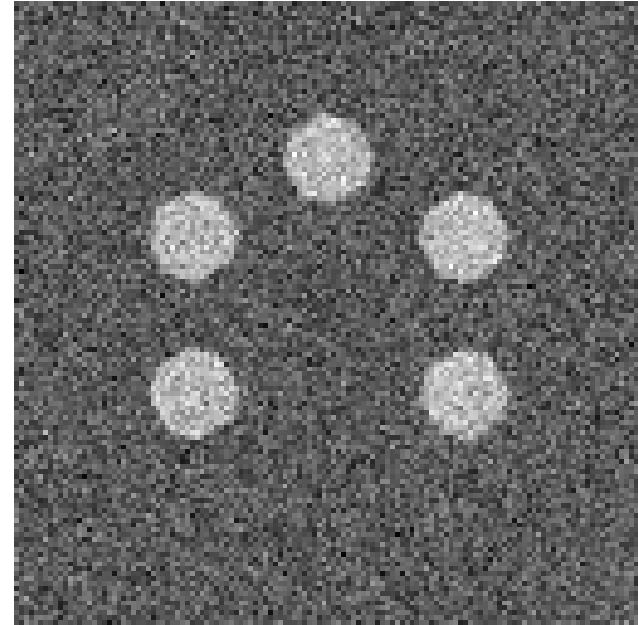
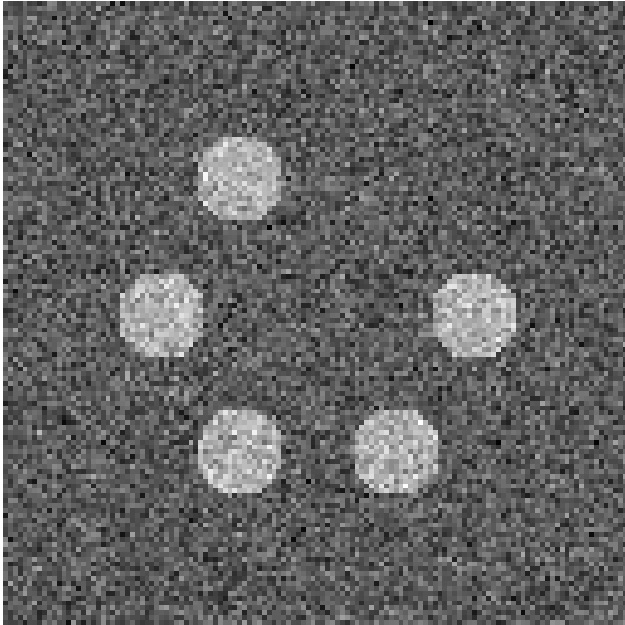
Image 2



Reference image



Noise added



Orientation alignment

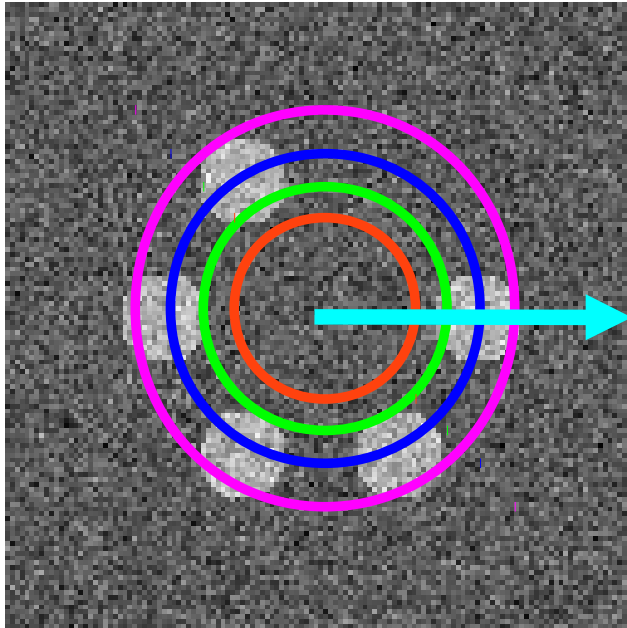


Image 1

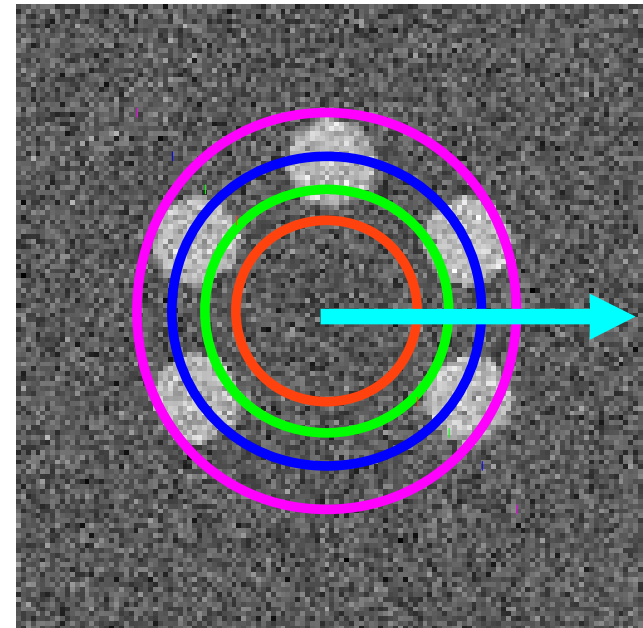
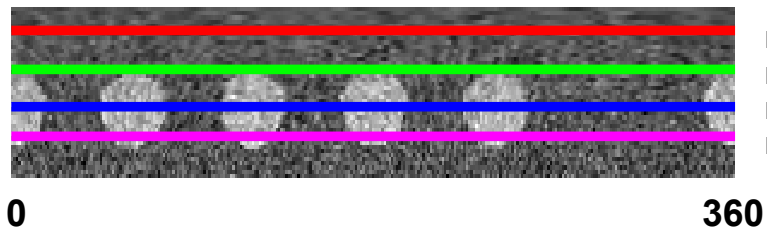


Image 2

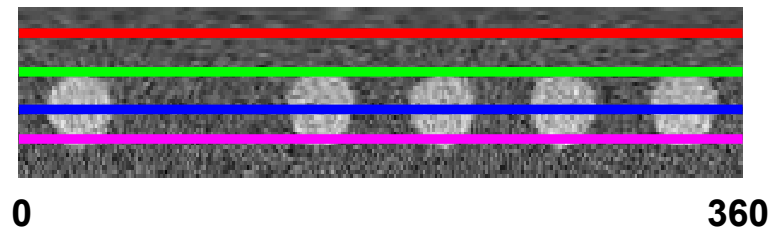


radius 1
radius 2
radius 3
radius 4

0

360

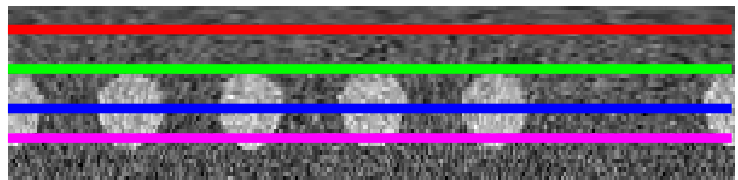
Polar representation



0

360

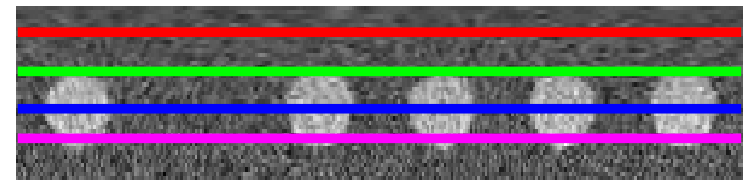
Orientation alignment



radius 1
radius 2
radius 3
radius 4

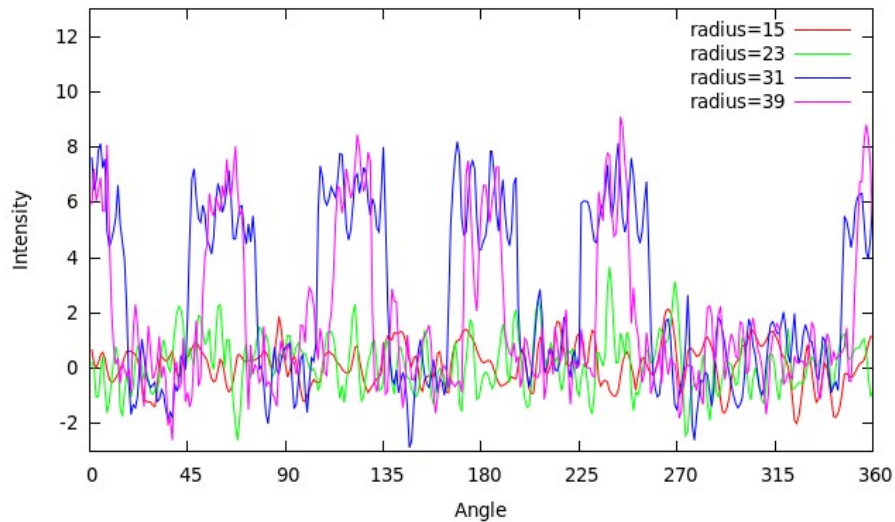
0

360

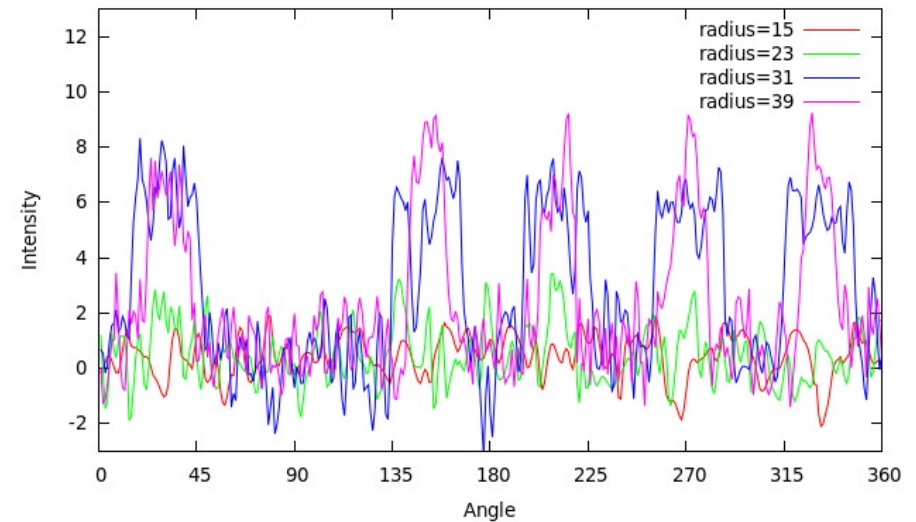


0

360

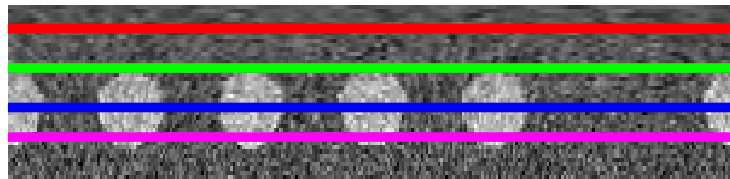


374.951, 4.53721

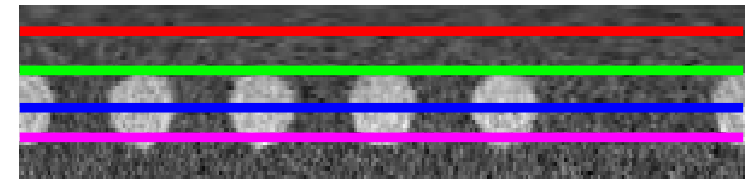


356.141, -2.50024

Orientation alignment: After rotation



radius 1
radius 2
radius 3
radius 4

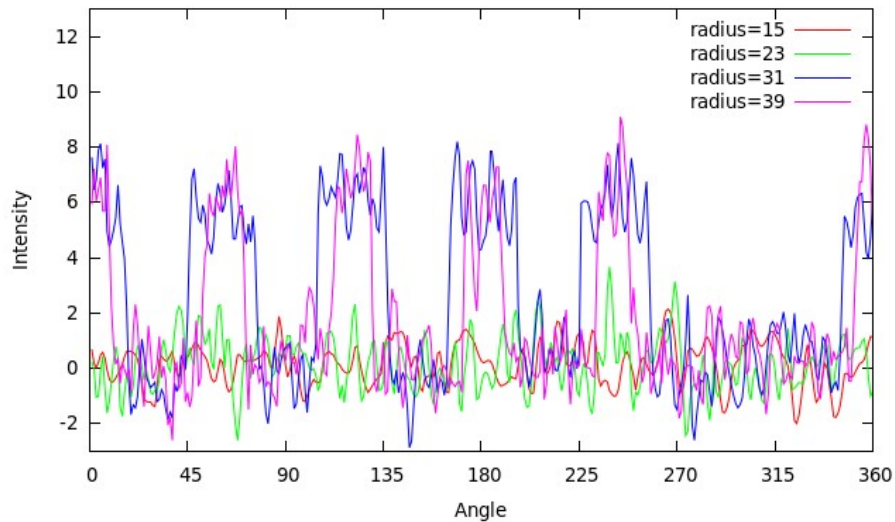


0

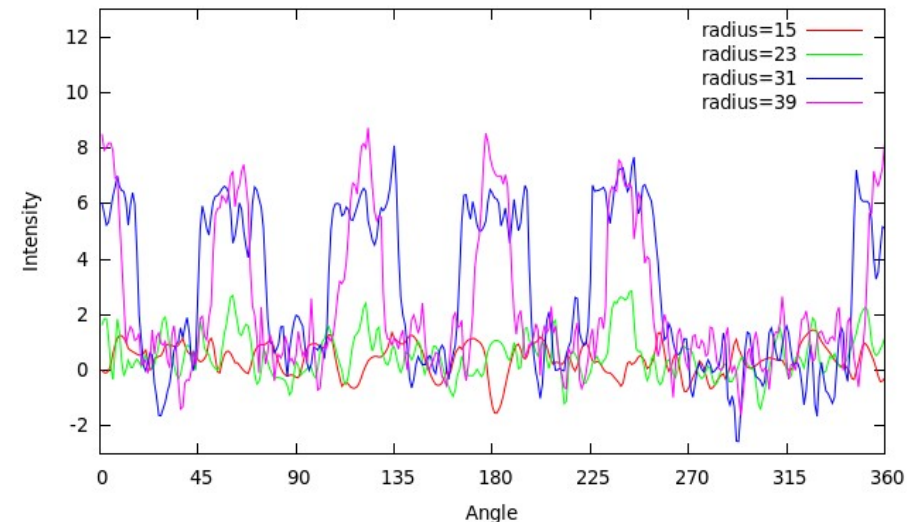
360

0

360



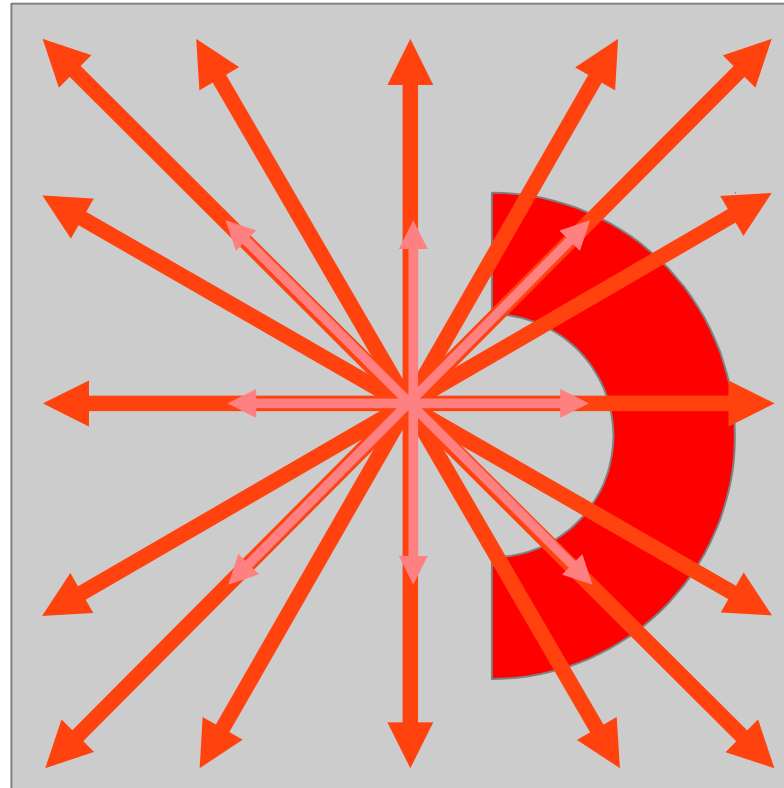
374.951, 4.53721



372.357, -3.21418

Another strategy for translation and orientation alignment

Translational and orientation alignment are interdependent



Set of all shifts of up to 1 pixel

Set of all new shifts of up to 2 pixels

Shifts of $(0, +/-1, +/-2)$ pixels results in 25 orientation searches.

*The power spectrum is translationally invariant.
If we shift the object in real space,
the power spectrum is unchanged.*

Cross-correlation function (CCF)

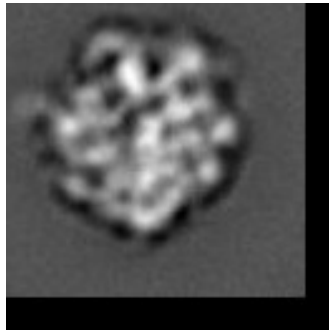


Image $f(x)$

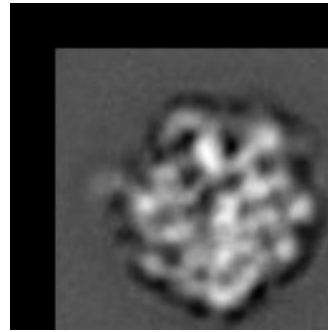
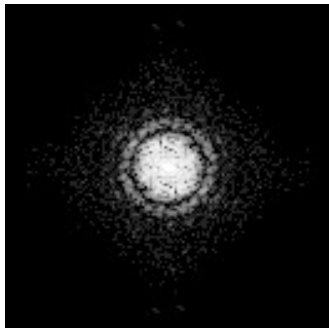
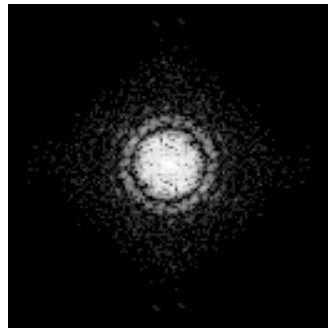


Image $g(x)$



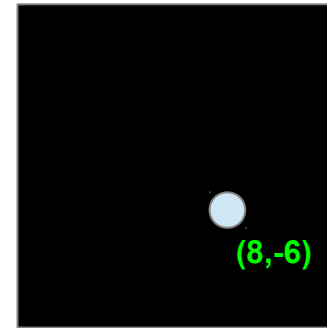
F.T. $F^*(X)$
(complex conjugate)

x



F.T. $G(X)$

=



CCF

The position of the peak gives us the shifts that give the best match, e.g., (8,-6).

Cross-correlation function (CCF)

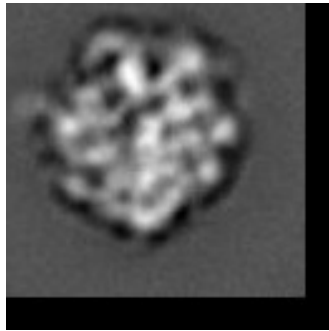


Image $f(x)$

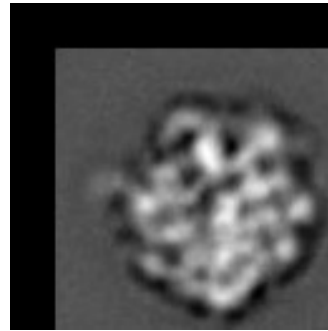
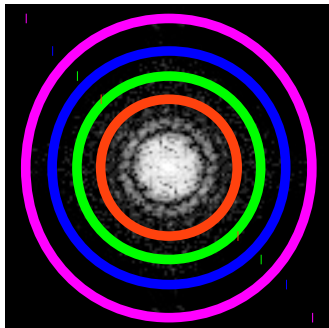
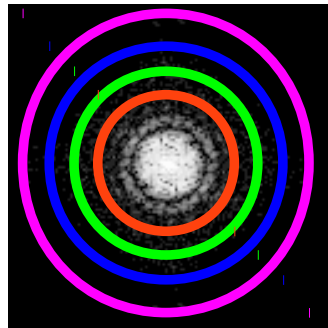


Image $g(x)$



F.T. $F^*(X)$
(complex conjugate)

x



F.T. $G(X)$

=

Series of
1D CCFs

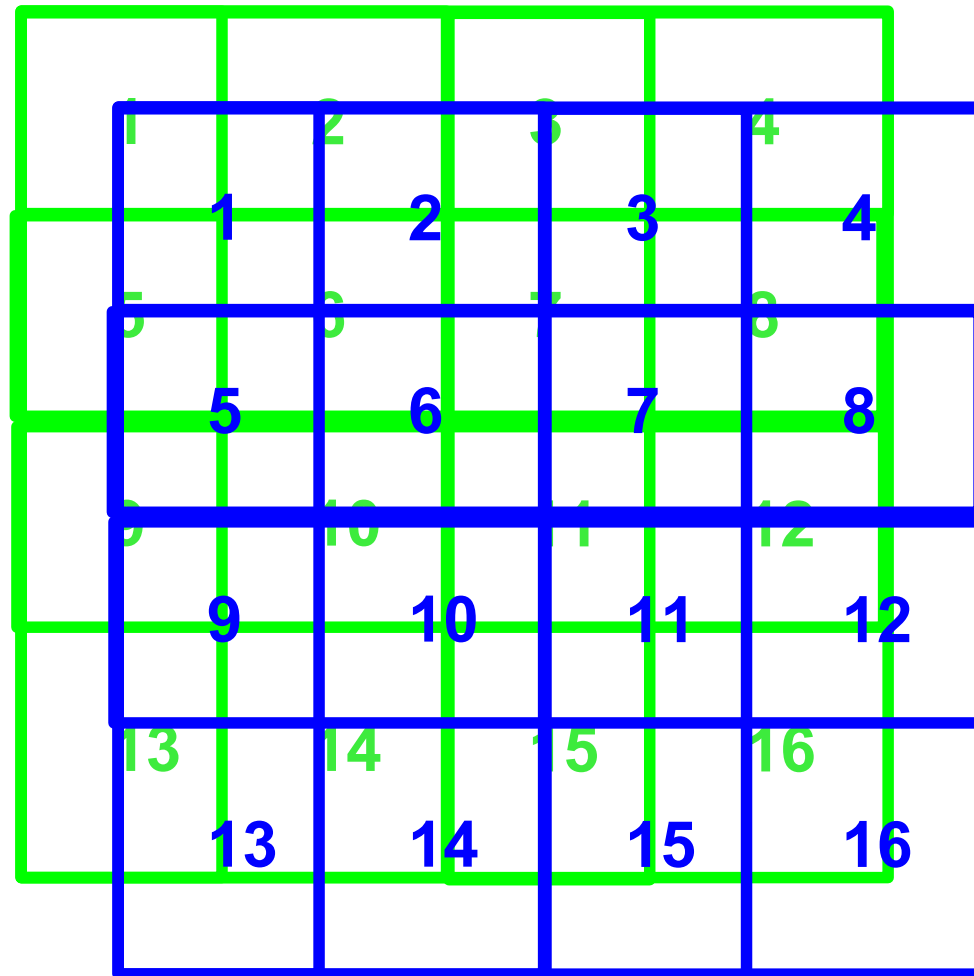
Problems:

1. The power spectrum of a roughly round object is roughly round.
2. The amplitudes fall off quickly, so you don't have many rings of useful data.

More interpolation

Bammes... Chiu (2012) J. Struct. Biol.

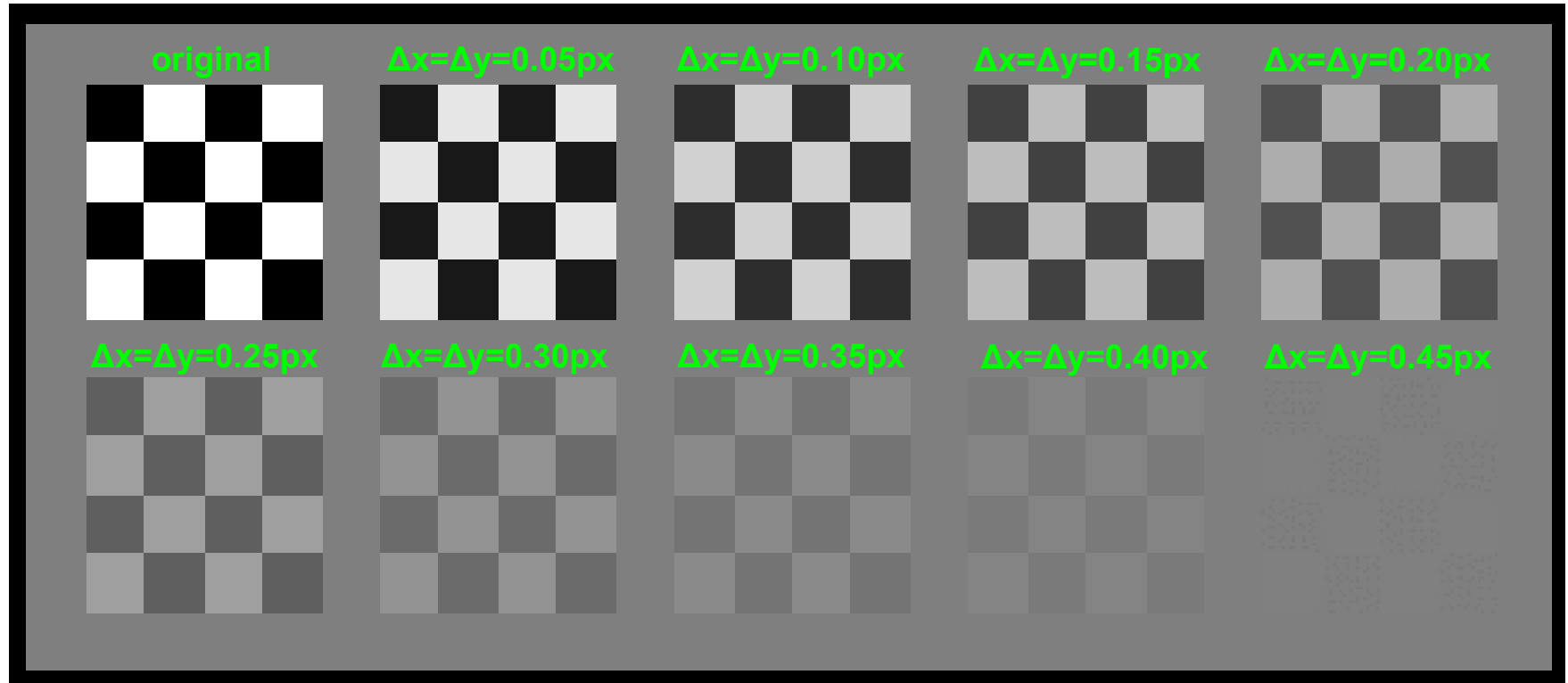
Shifts



Suppose we shift the image in x & y .

The new pixels will be weighted averages of the old pixels.

Effect of shifts



1 ₁	2	2 ₃	3	4 ₃	5	4 ₆	6
7	8	9	10	11	12		
5		6		7		8	
13	14	15	16	17	18		
19 ₉	20	10 ₂₁	22 ₁₁	23	12 ₂₄		
25	26	27	28	29	30		
13		14		15		16	
31	32	33	34	35	36		

	1	2	3	4	5	6
7	5	8	9	6	10	7
13	14	15	16	17	18	
	9		10		11	12
19	20	21	22	23	24	
25	13	26	27	14	28	15
	29		30		31	16
31	32	33	34	35	36	

Questions

1) If the pixel size is $3 \text{ \AA}/\text{px}$, what is the Nyquist frequency?

◆ **ANSWER:** $1/6 \text{ \AA}$

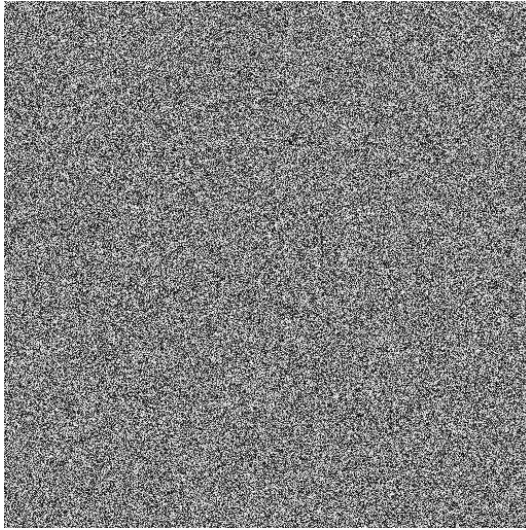
2) If we oversample/upscale the image by a factor of $1.5X$, what is the new pixel size?

◆ **ANSWER:** $2 \text{ \AA}/\text{px}$

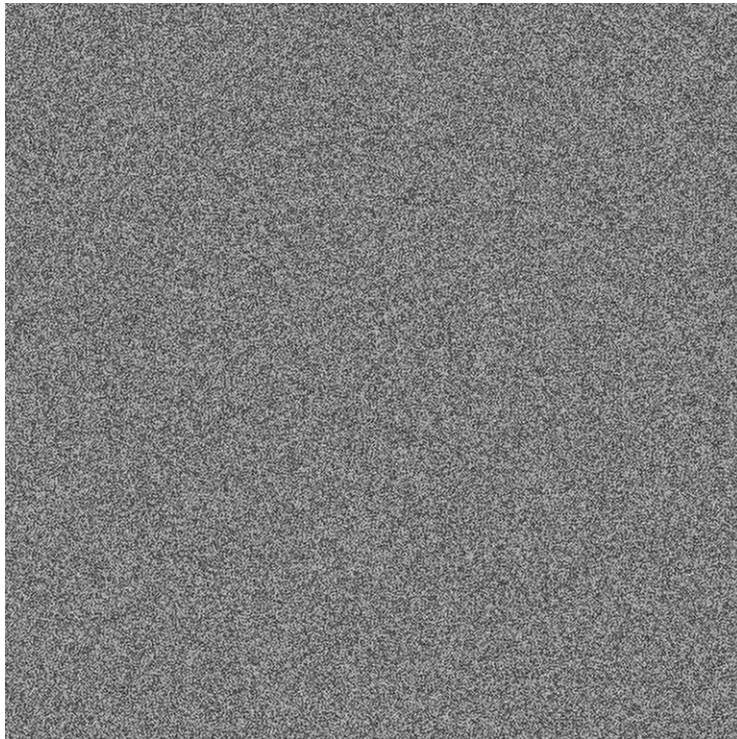
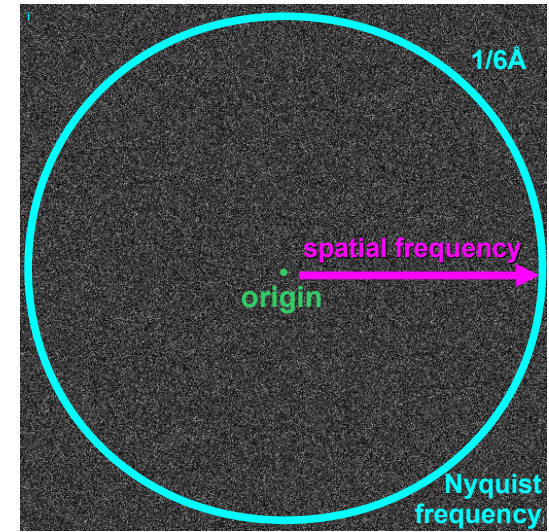
3) What will be the new Nyquist frequency in the oversampled image?

◆ **ANSWER:** $1/4 \text{ \AA}$

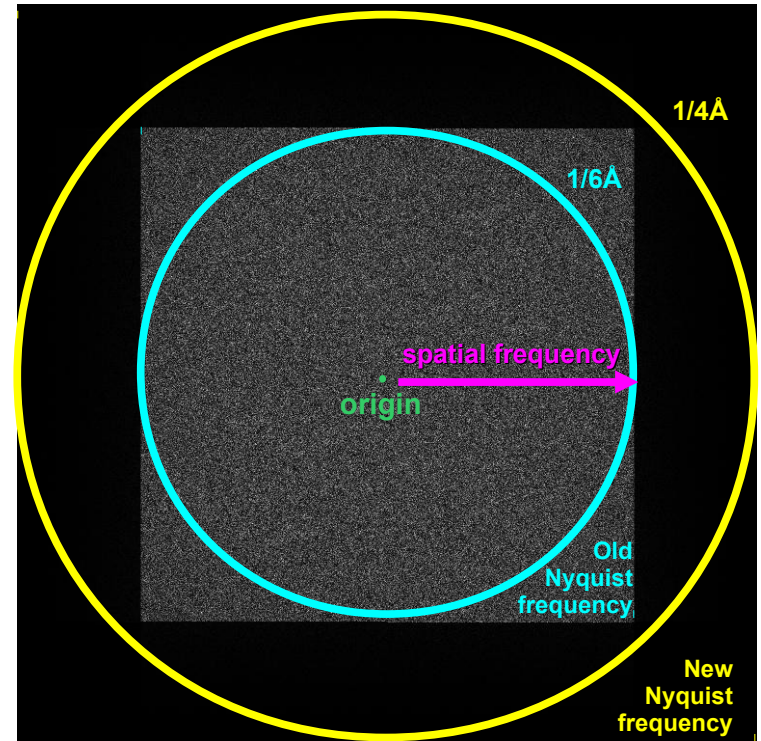
White noise



Power spectrum



Upscaled

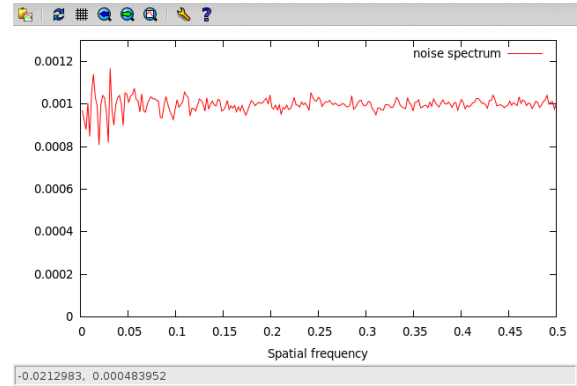
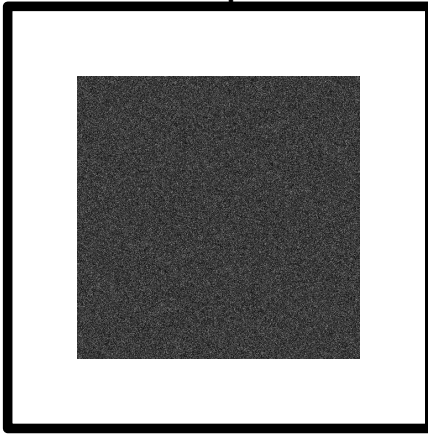
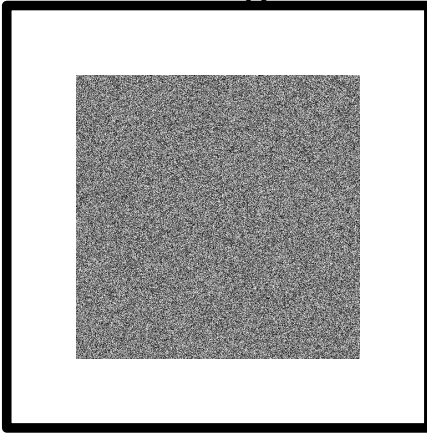


Image

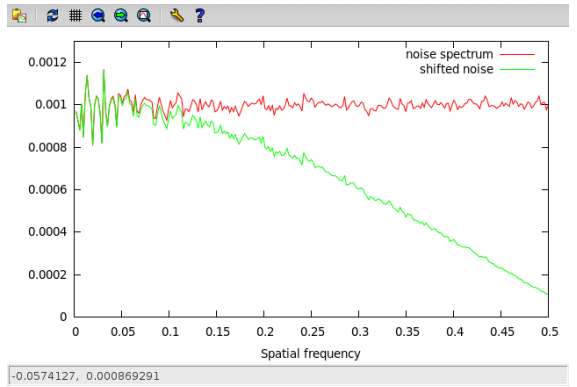
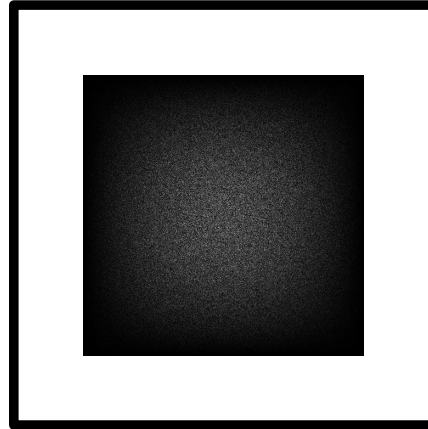
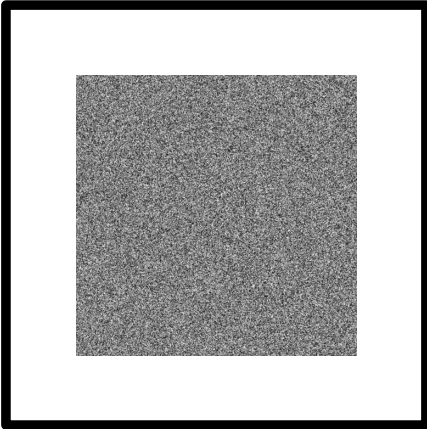
Power spectrum

Power spectrum profile

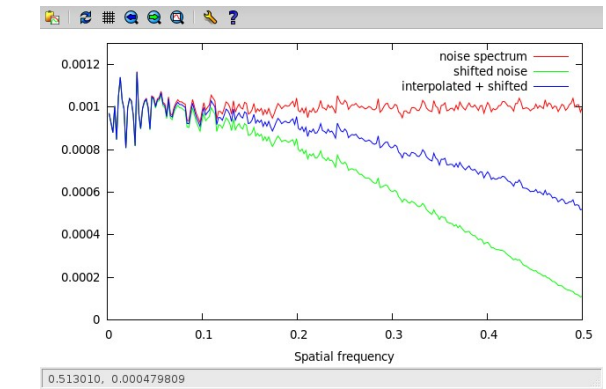
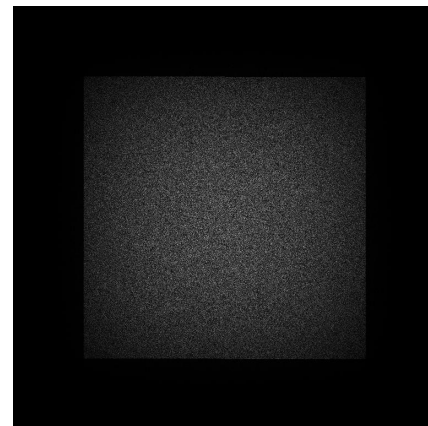
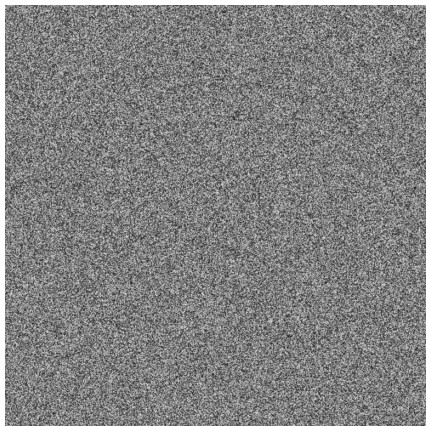
Original



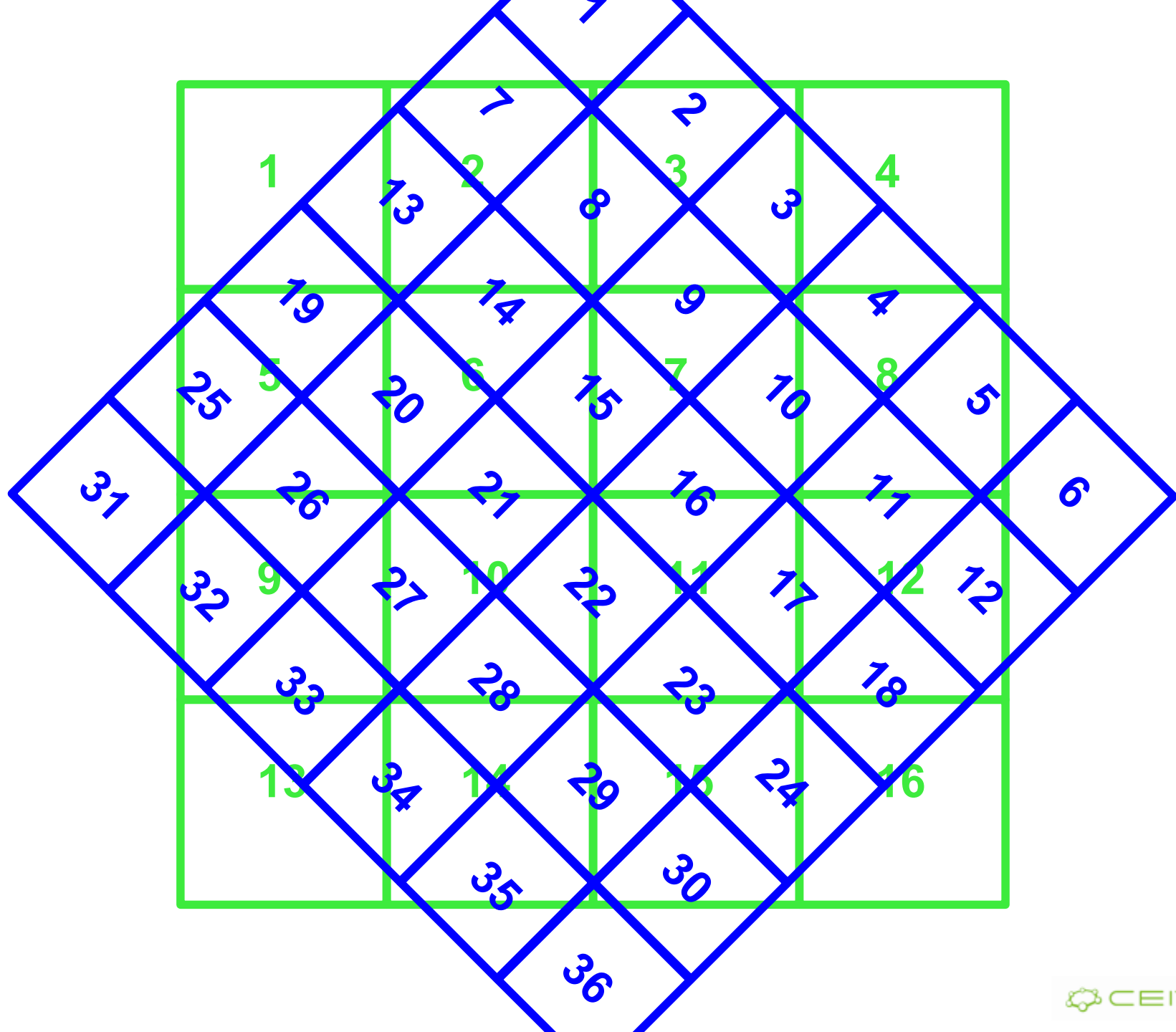
Shifted by (0.5,0.5)



Shifted + Interpolated



1 ₁	2	2 ₂	3	4 ₃	5	4 ₄	6
7	8	9	10	11	12		
5		6		7		8	
13	14	15	16	17	18		
19	20	10 ₂₁	22 ₁₁	23	12 ₂₄		
25	26	27	28	29	30		
13		14		15		16	
31	32	33	34	35	36		

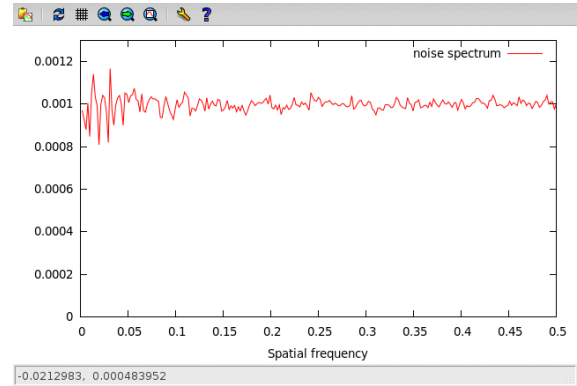
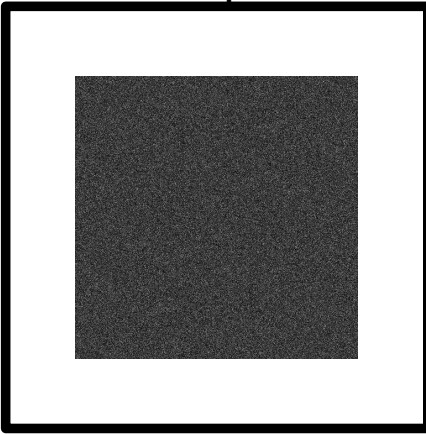
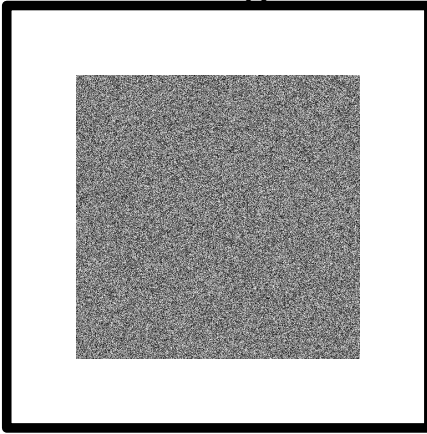


Image

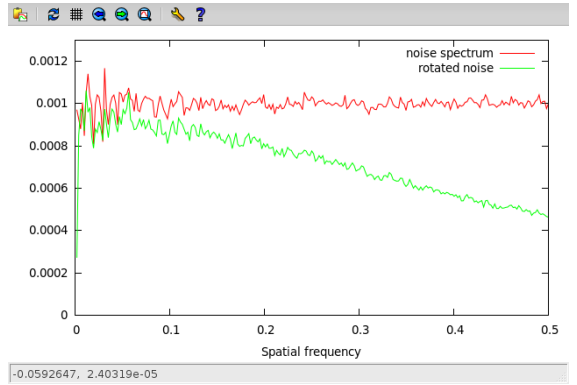
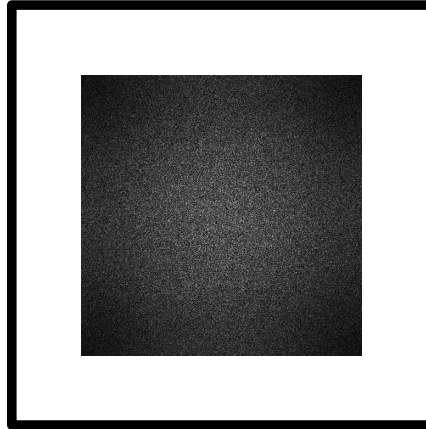
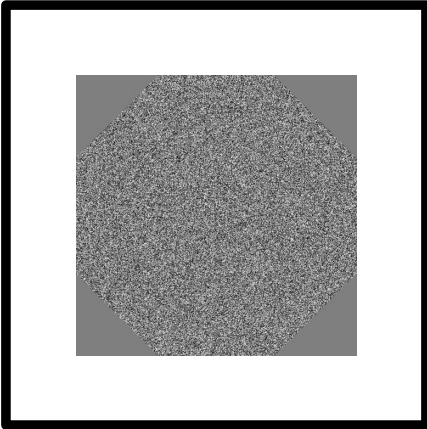
Power spectrum

Power spectrum profile

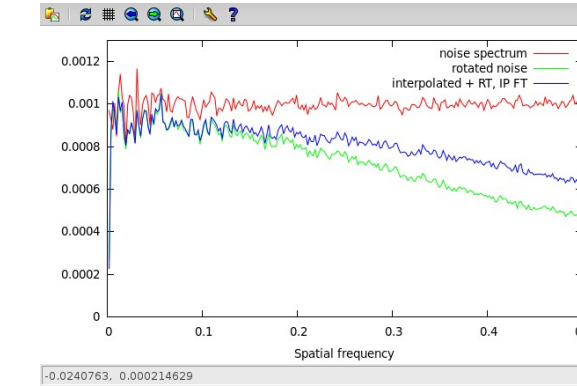
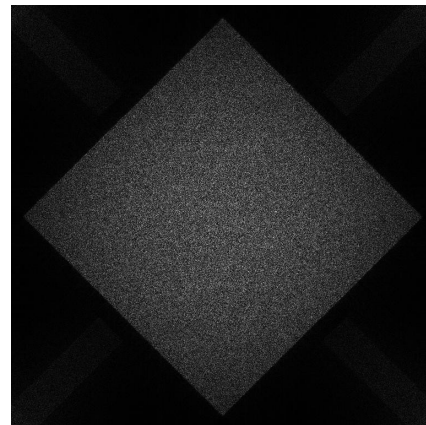
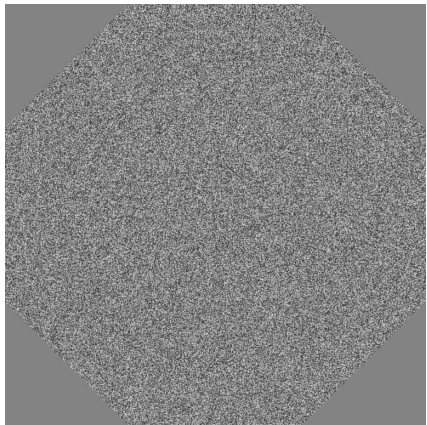
Original



Rotated by 45°



Rotated +
Interpolated



Conclusion:

You can do a little better by oversampling.

Bammes... Chiu (2012) J. Struct. Biol.

Classification

Multivariate data analysis (MDA)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

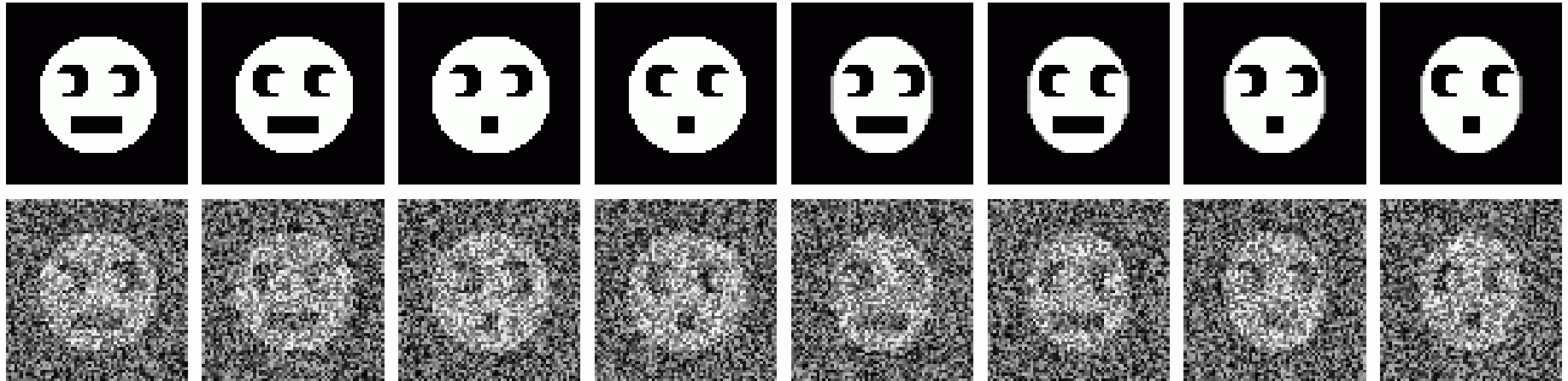
Multivariate data analysis (MDA), or Multivariate statistical analysis (MSA)



Our 16-pixel image can be reorganized into a 16-coordinate vector.

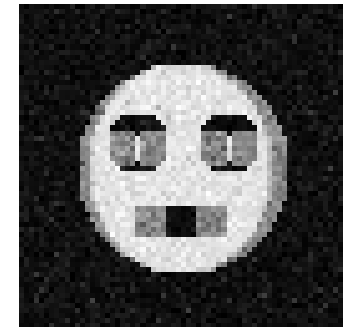
MDA: An example

8 classes of faces, 64x64 pixels



With noise added

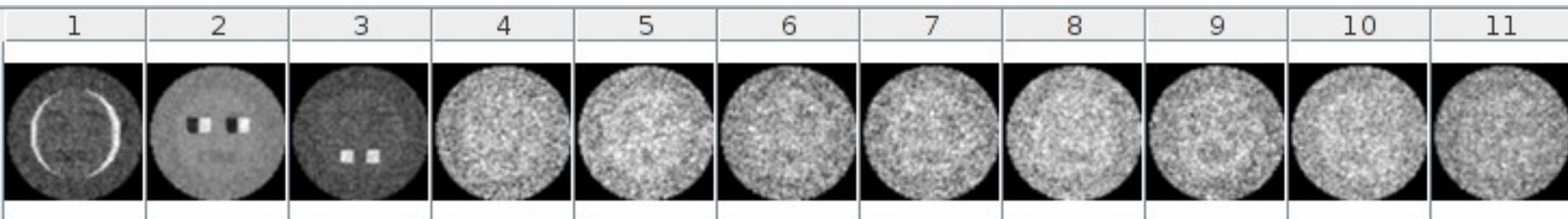
Average:



From http://spider.wadsworth.org/spider_doc/spider/docs/techs/classification/tutorial.html

Principal component analysis (PCA) or Correspondence analysis (CA)

- ◆ For a 4096-pixel image, we will have a 4096x4096 covariance matrix.
- ◆ Row-reduction of the covariance matrix gives us “eigenvectors.”
 - The eigenvectors describe correlated variations in the data.
 - The eigenvectors have 64 elements and can be converted back into images, called “eigenimages.”
 - The first eigenvectors will account for the most variation. The later eigenvectors may only describe noise.
 - Linear combinations of these images will give us approximations of the classes that make up the data.



Eigenimages

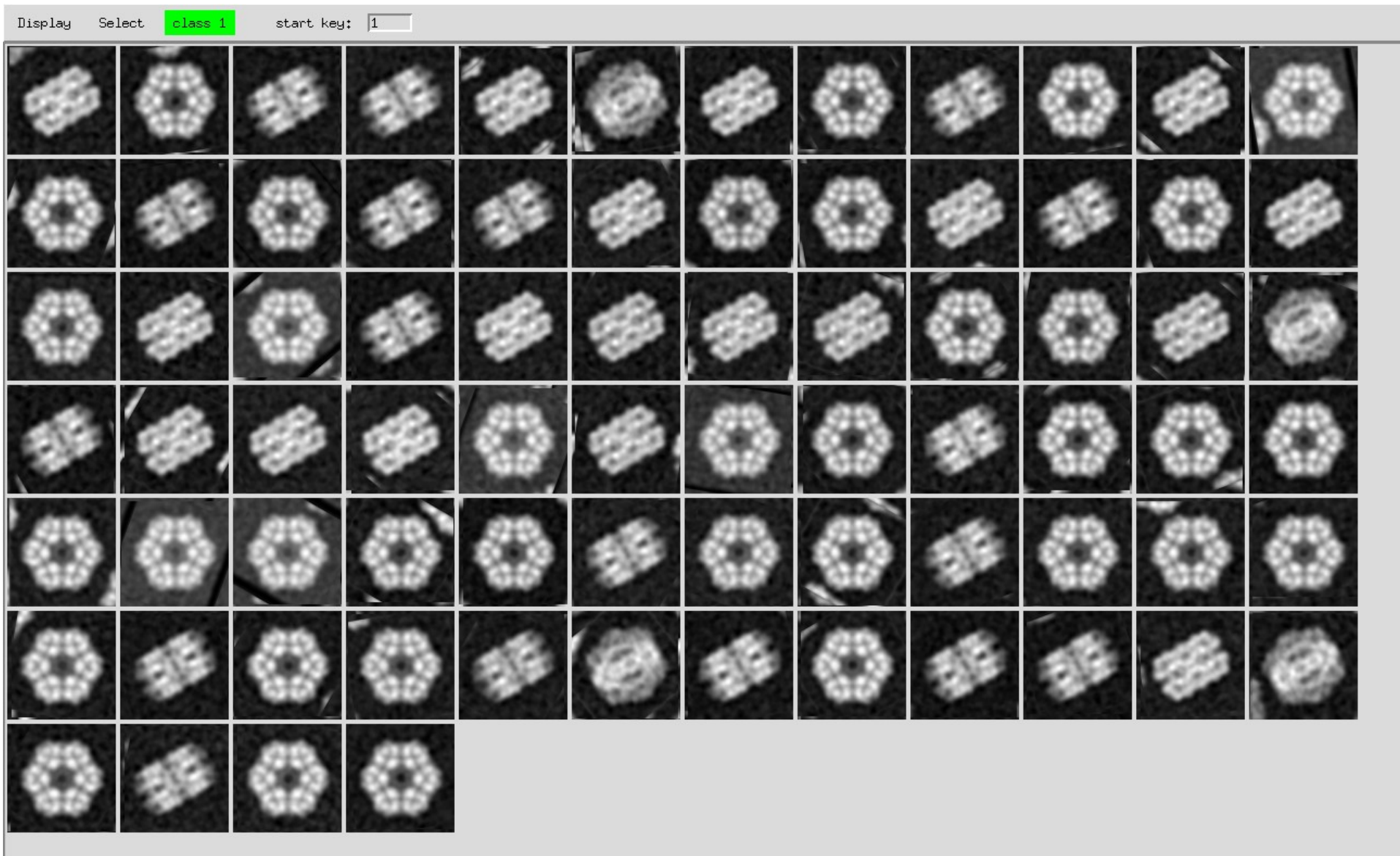
Reconstituted images

Linear combinations of these images will give us approximations of the classes that make up the data.

$$c_0 \begin{array}{c} \text{Average} \\ \text{Eigenimage \#1} \\ \text{Eigenimage \#2} \\ \text{Eigenimage \#3} \end{array} + c_1 \begin{array}{c} \text{Eigenimage \#1} \\ \text{Eigenimage \#2} \\ \text{Eigenimage \#3} \end{array} + c_2 \begin{array}{c} \text{Eigenimage \#2} \\ \text{Eigenimage \#3} \end{array} + c_3 \begin{array}{c} \text{Eigenimage \#3} \end{array} + \dots$$

Average Eigenimage #1 Eigenimage #2 Eigenimage #3

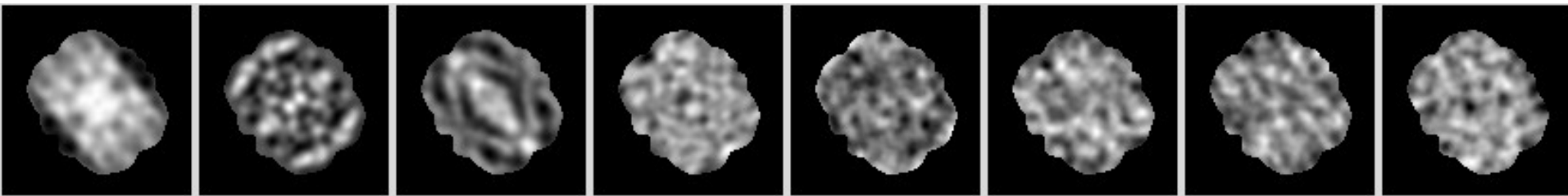
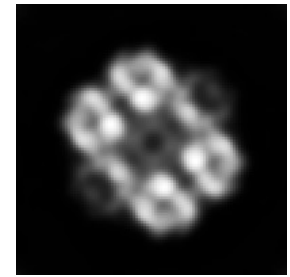
Another example: worm hemoglobin



Phantom images of worm hemoglobin

PCA of worm hemoglobin

Average:



stkeigenimg@1 stkeigenimg@2 stkeigenimg@3 stkeigenimg@4 stkeigenimg@5 stkeigenimg@6 stkeigenimg@7 stkeigenimg@8

$+c_0$

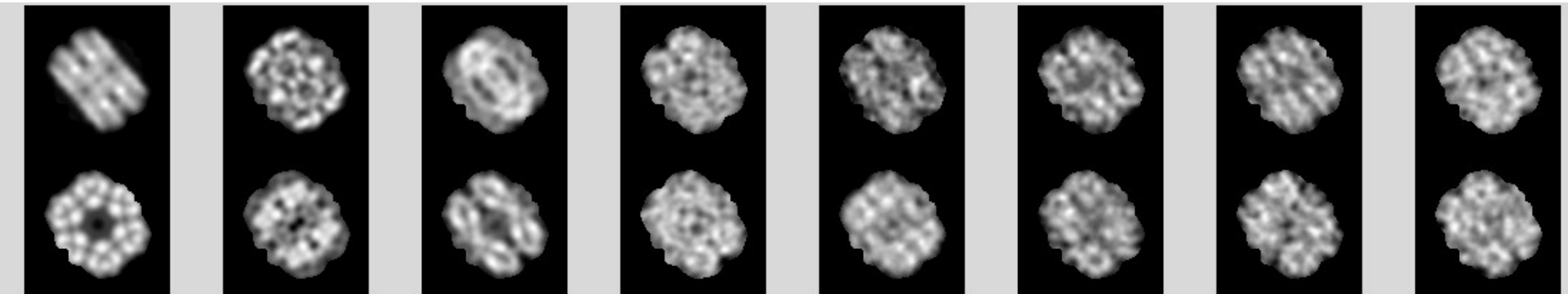
$+c_1$

$+c_2$

$+c_3$

$+c_4$

$+c_5$



stkreconstituted@1 stkreconstituted@2 stkreconstituted@3 stkreconstituted@4 stkreconstituted@5 stkreconstituted@6 stkreconstituted@7 stkreconstituted@8

$-c_0$

$-c_1$

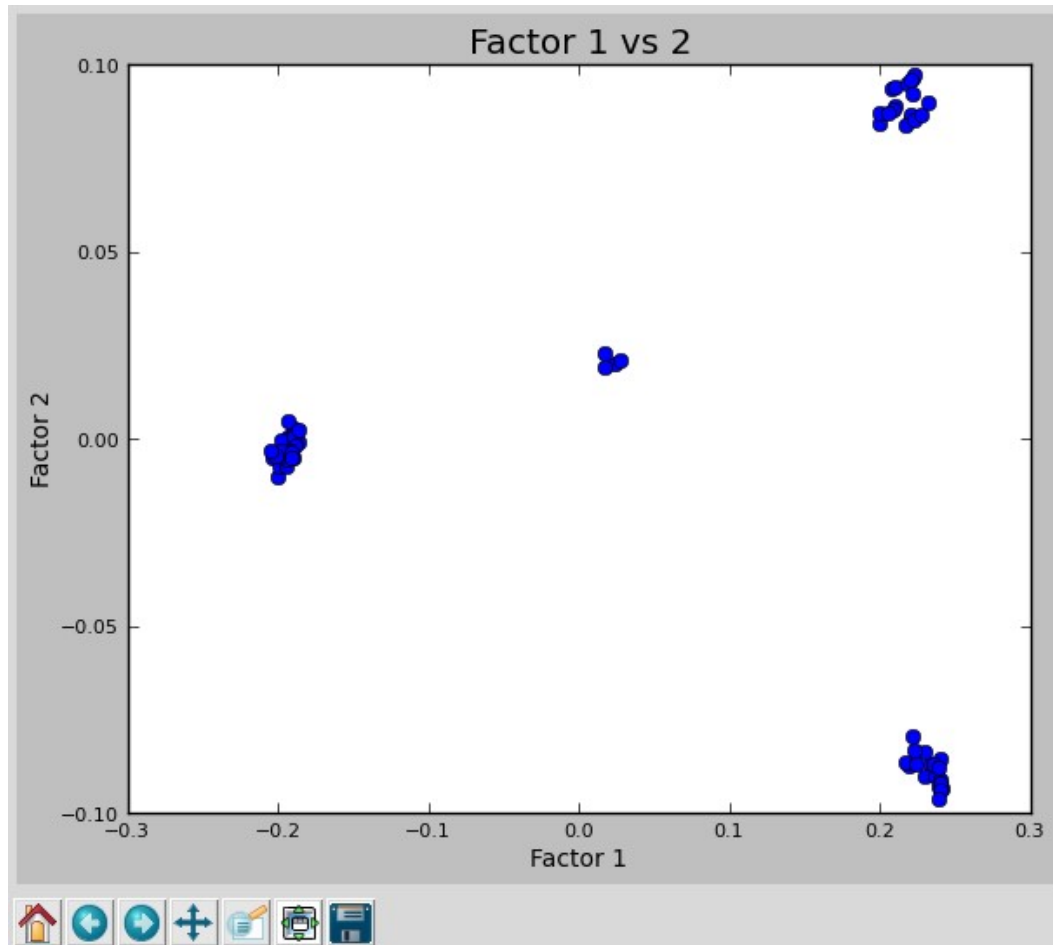
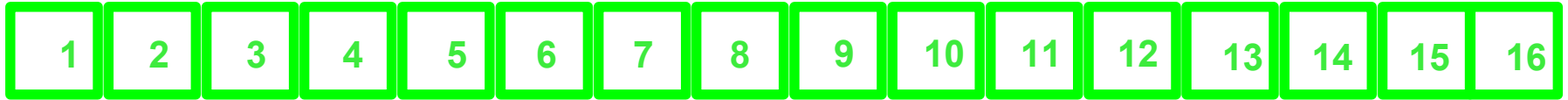
$-c_2$

$-c_3$

$-c_4$

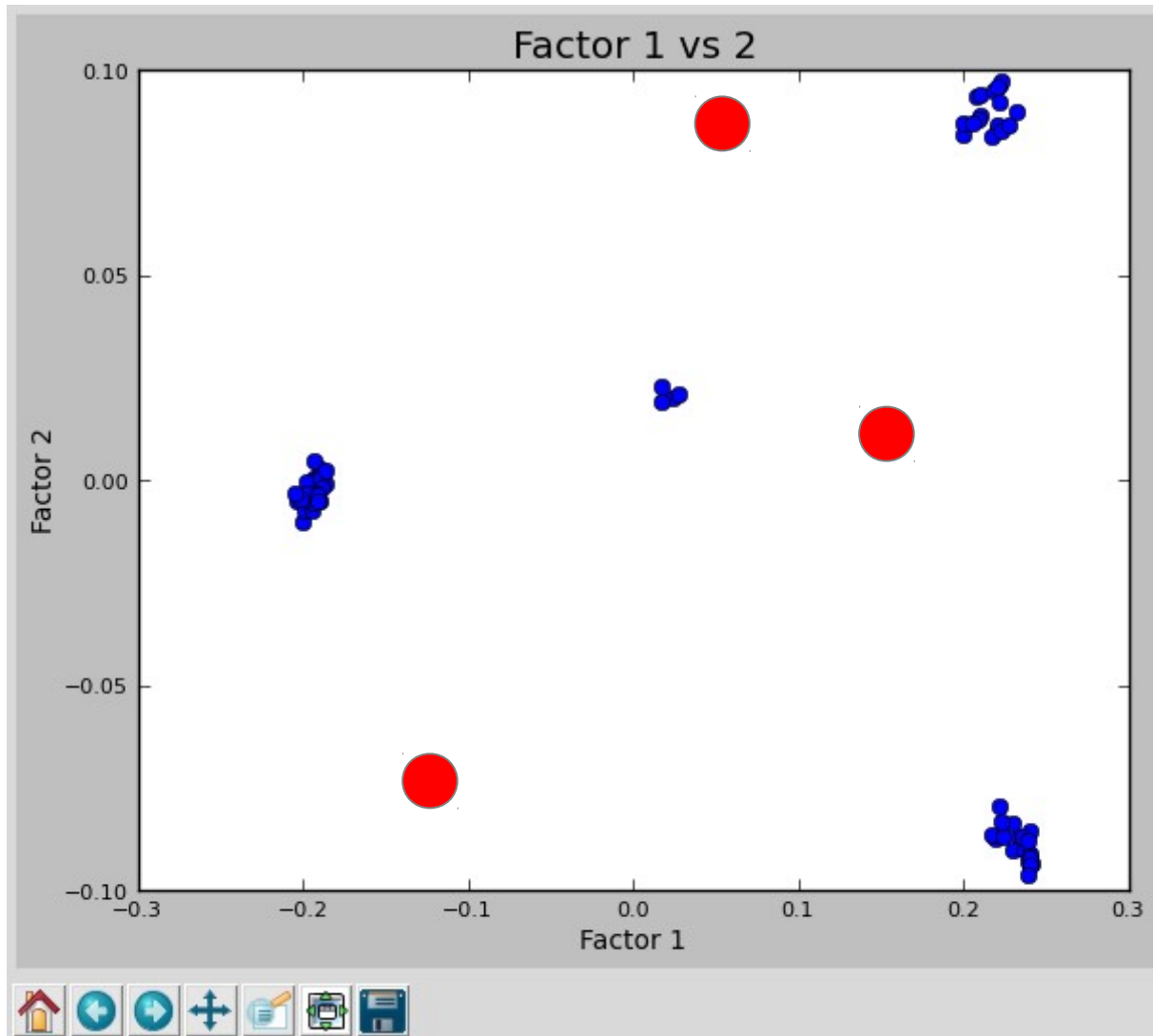
$-c_5$

Classification

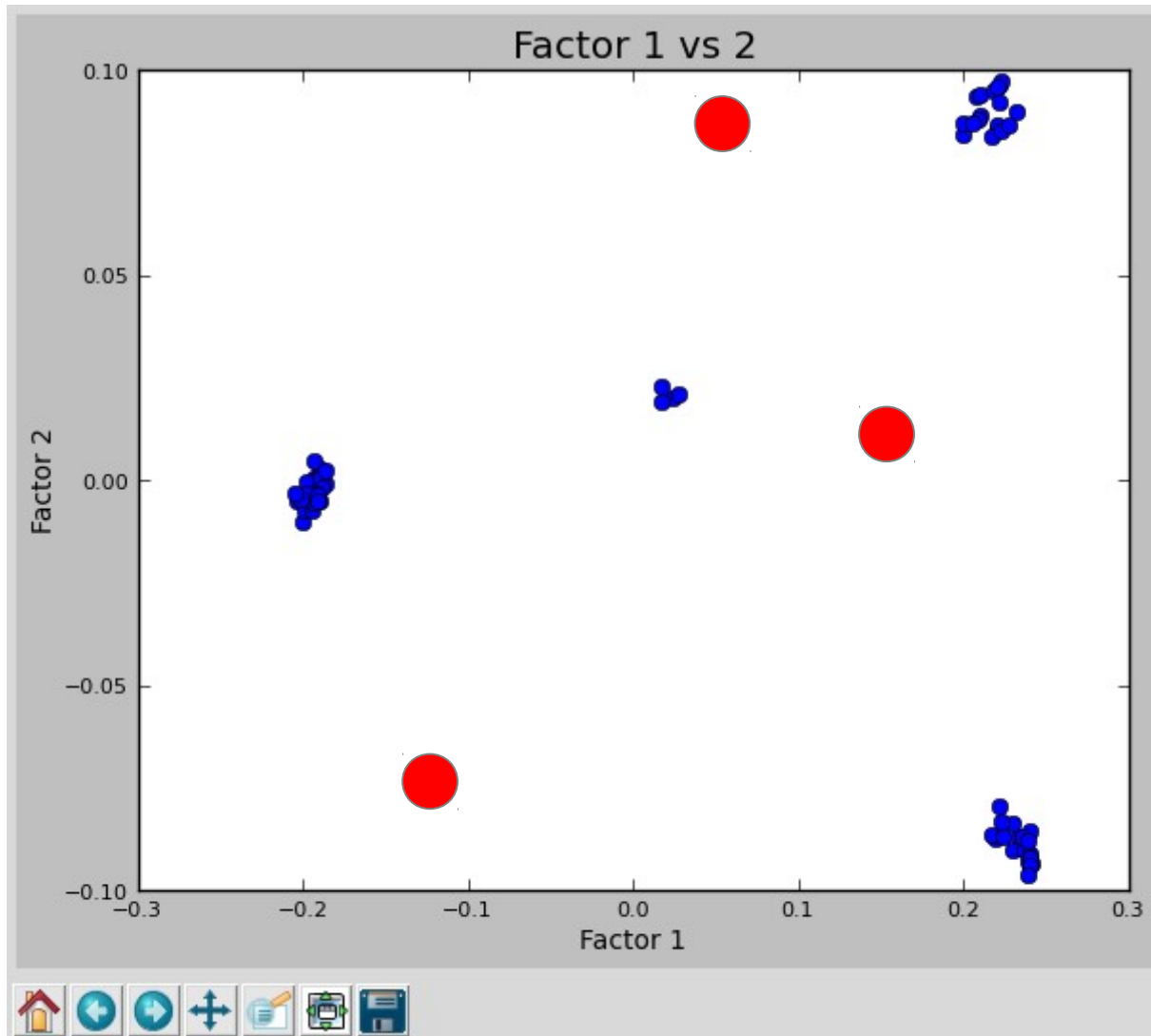


How do we categorize/classify the images?

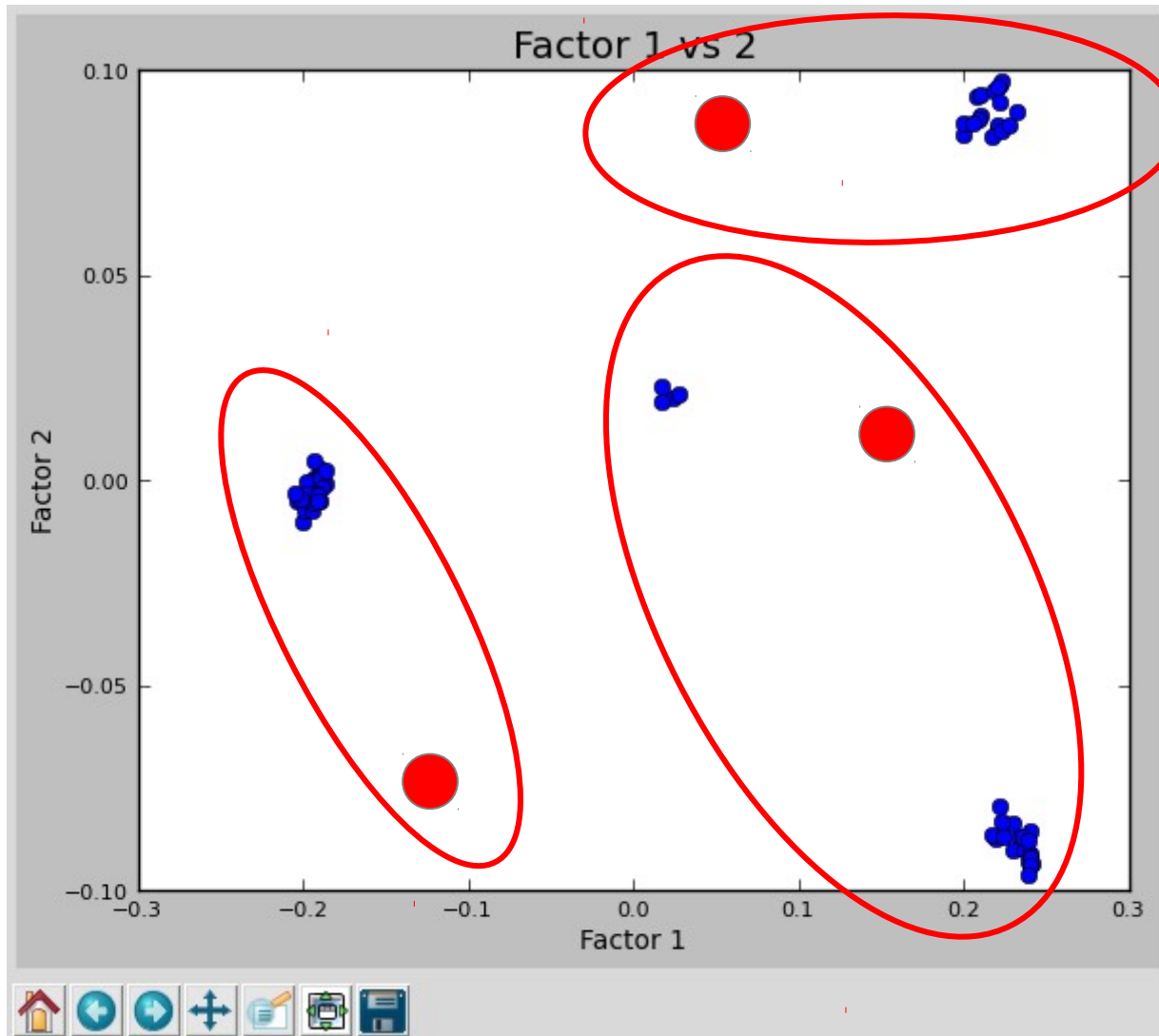
K-means classification



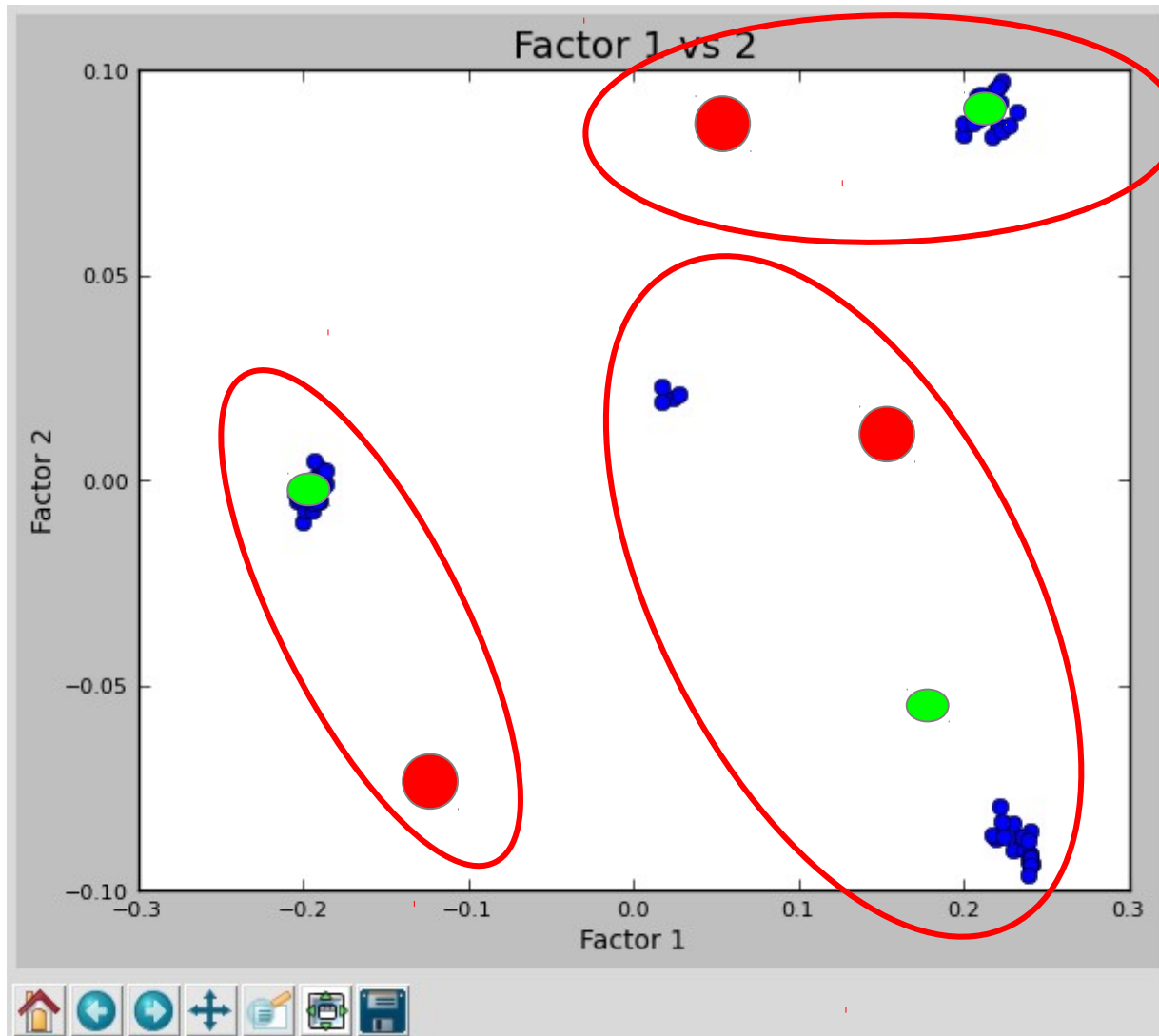
Diday's method of moving centers



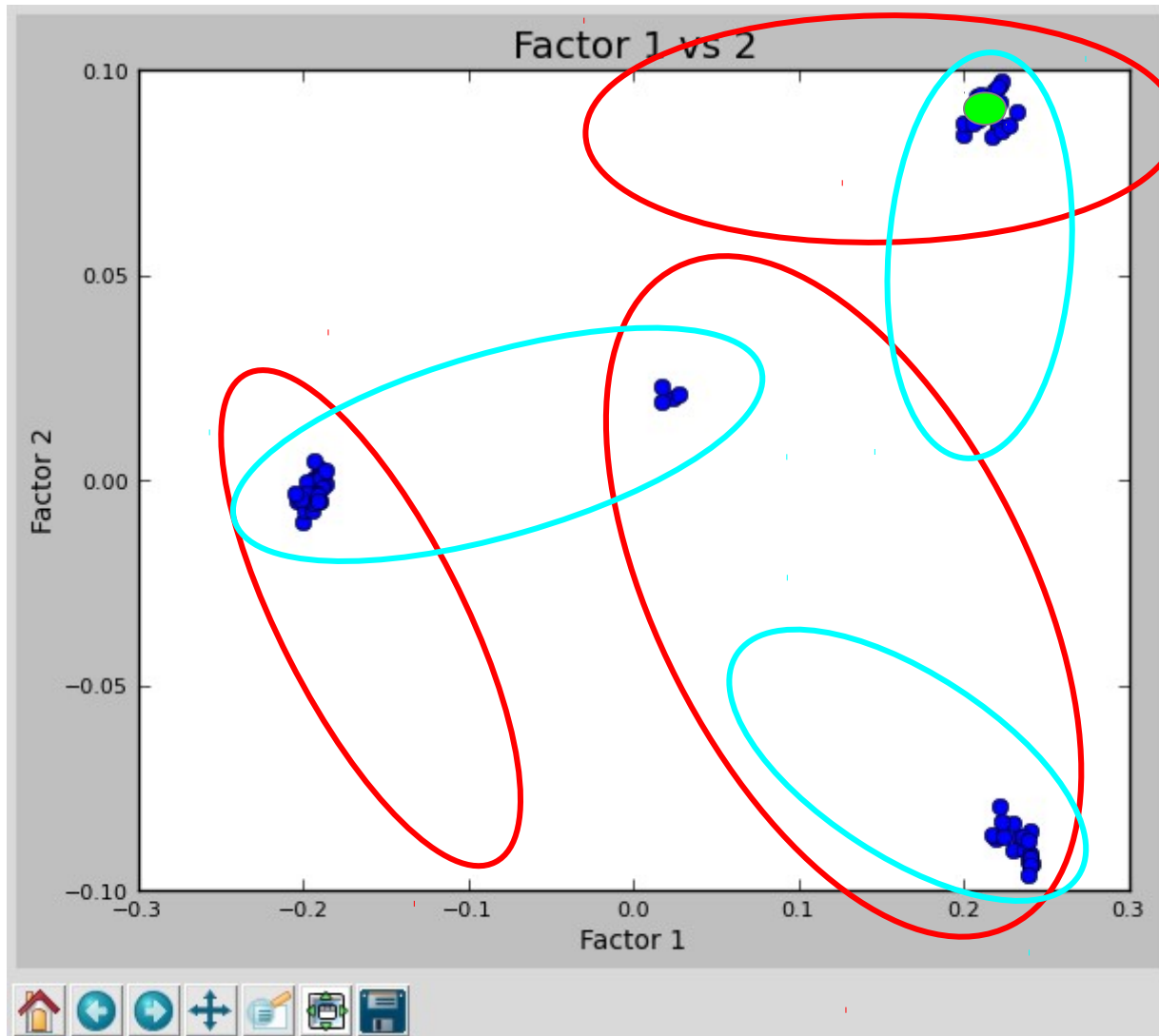
Diday's method of moving centers



Diday's method of moving centers

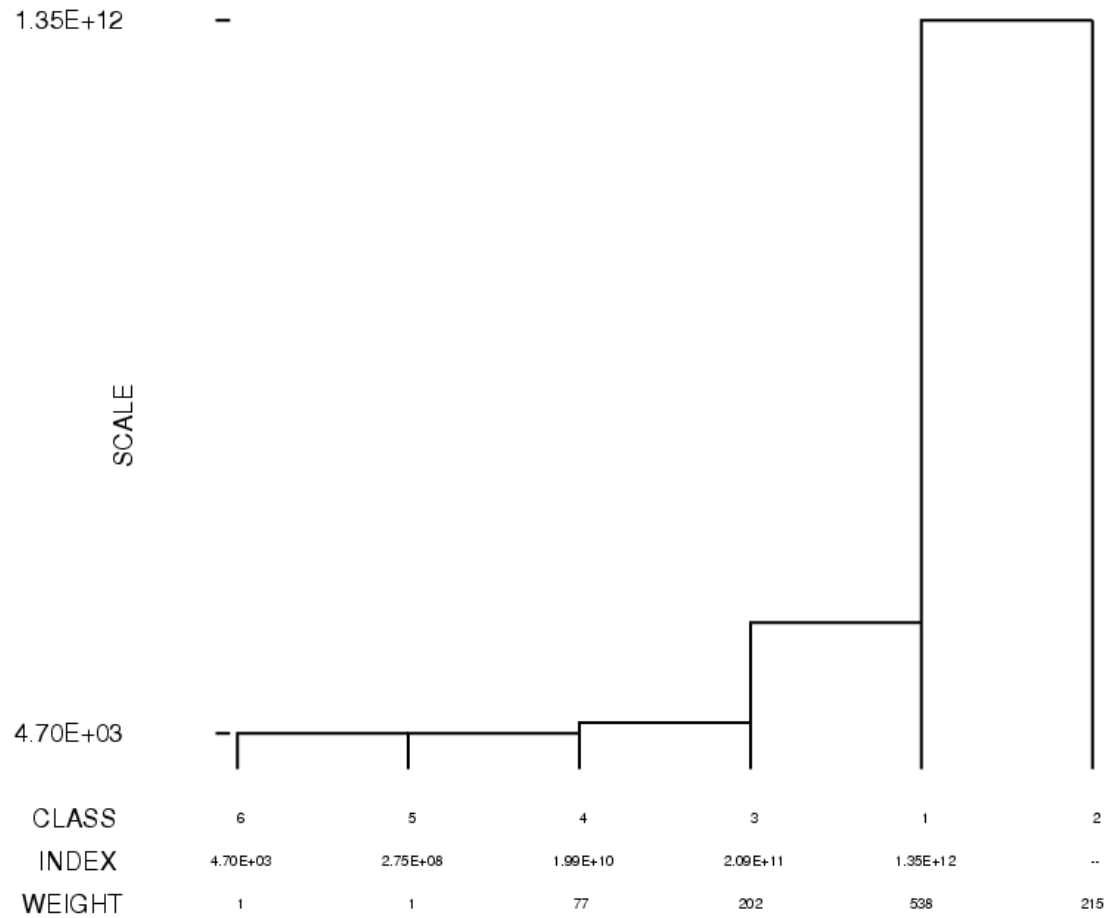


Diday's method of moving centers

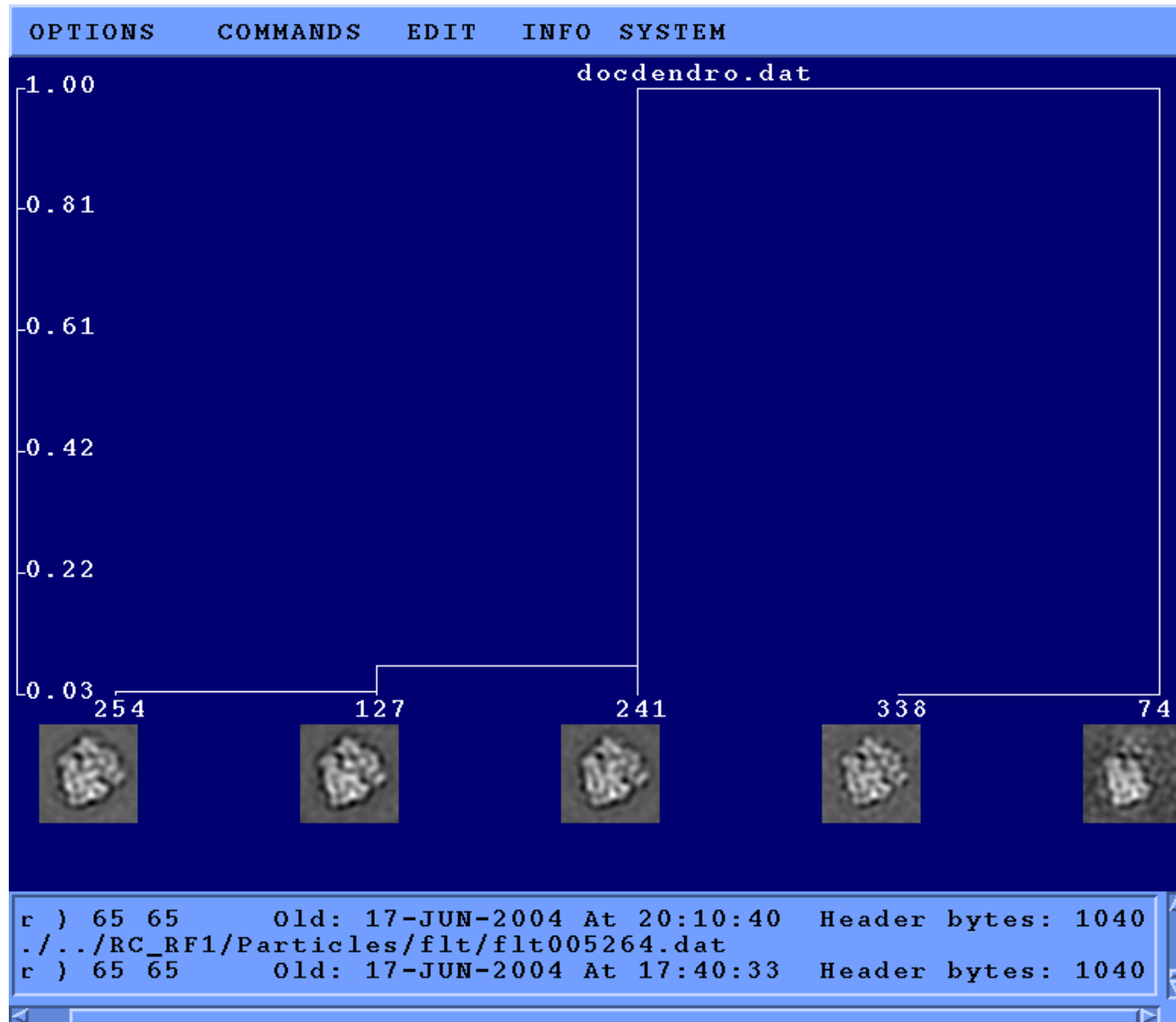


Dendrogram

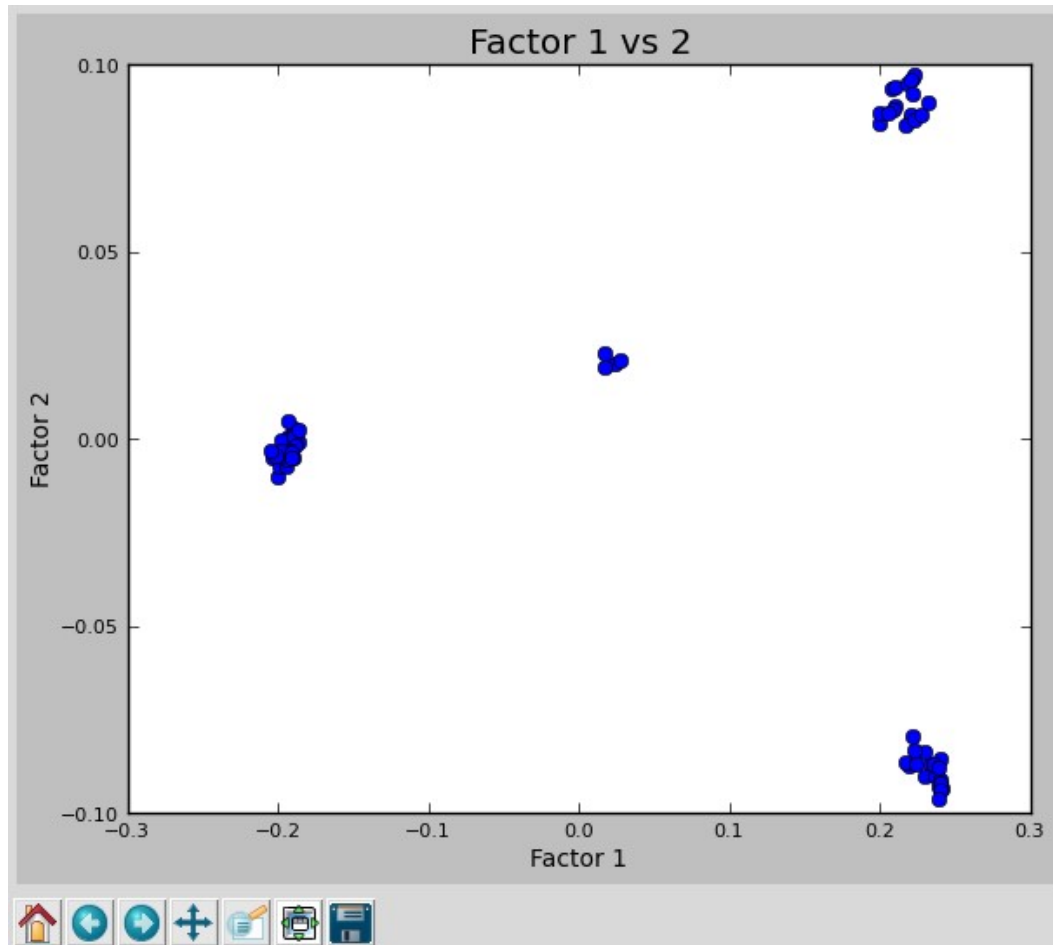
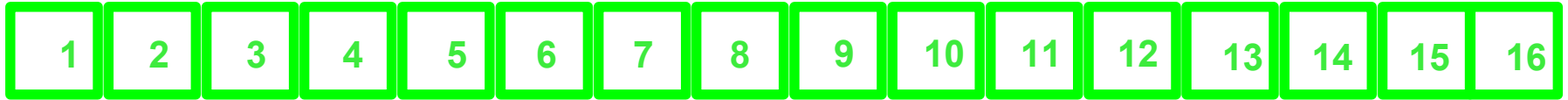
CLA/dendrogram.ps



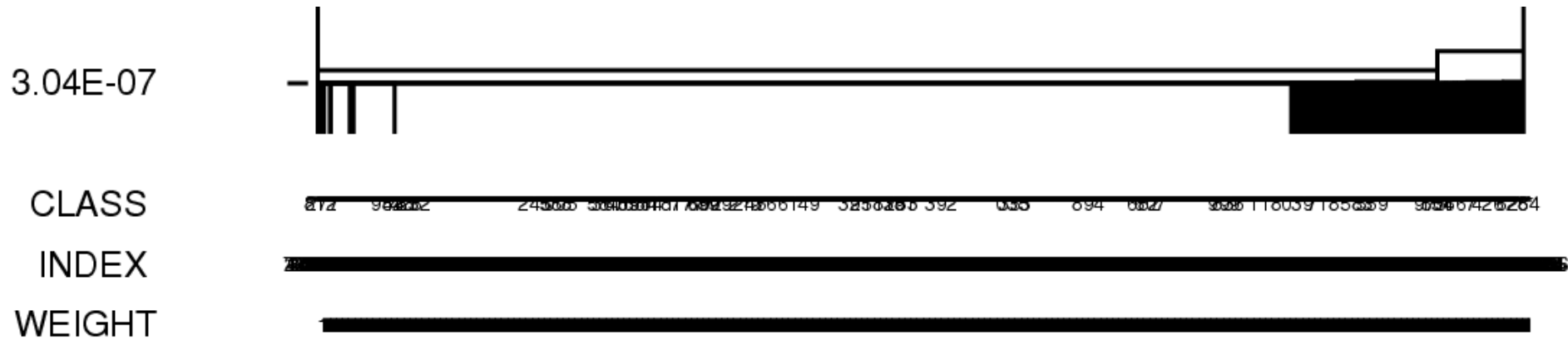
Dendrogram



Hierarchical Ascendant Classification



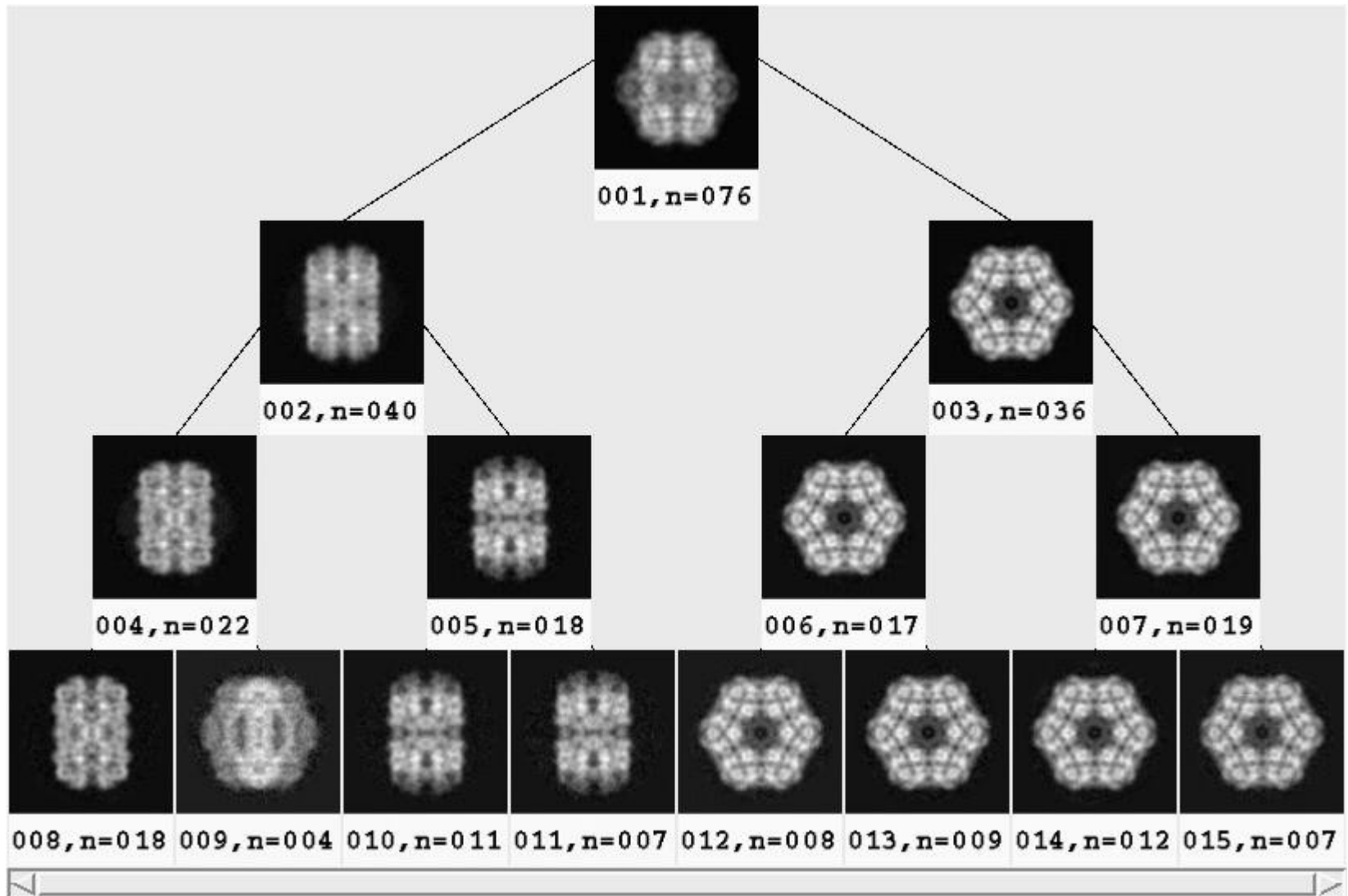
Hierarchical Ascendant Classification



All images are represented.

The dendrogram will be too heavily branched to interpret without truncation.

Binary-tree viewer

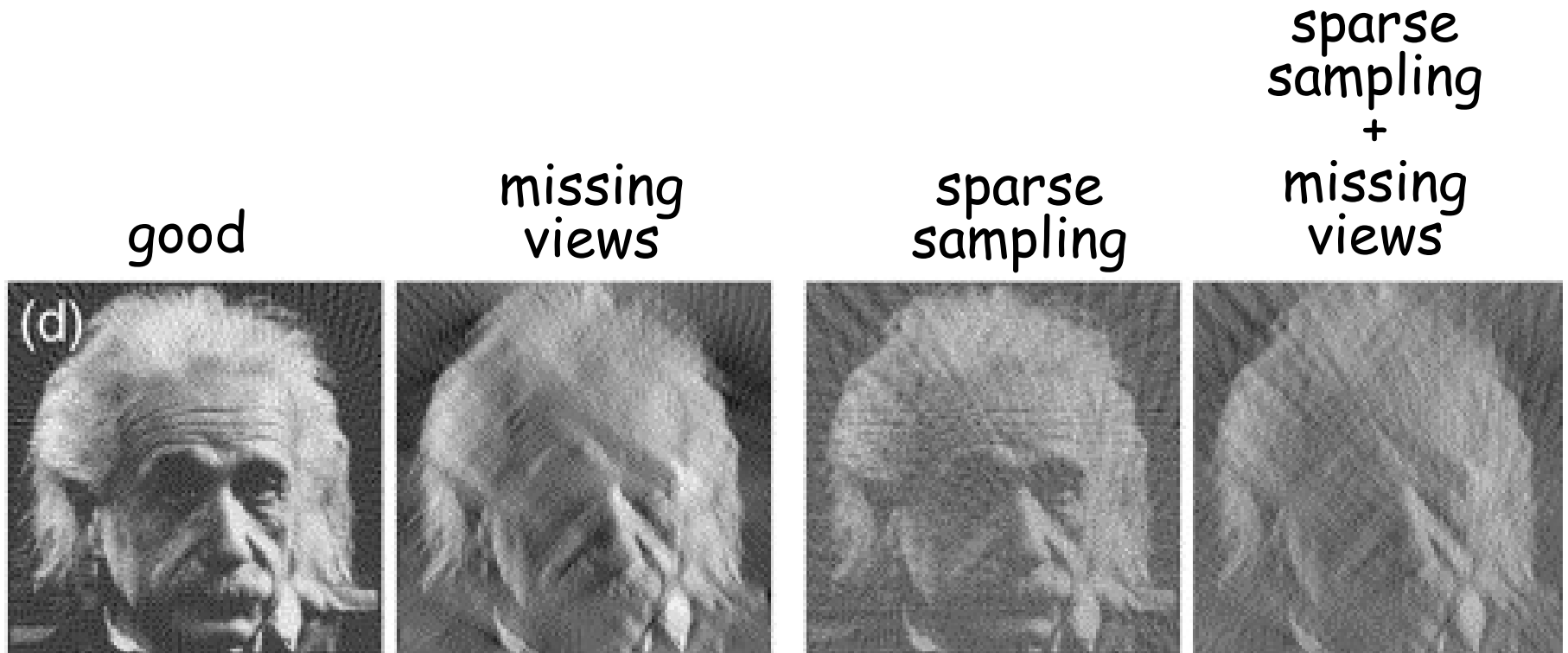


3D Reconstruction

What information do we need for 3D reconstruction?

1. different orientations
2. known orientations
3. many particles

What happens when we're missing views?

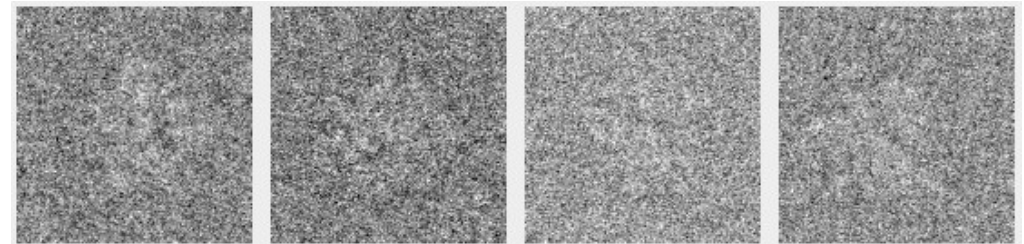
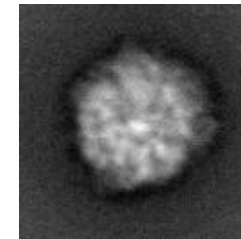
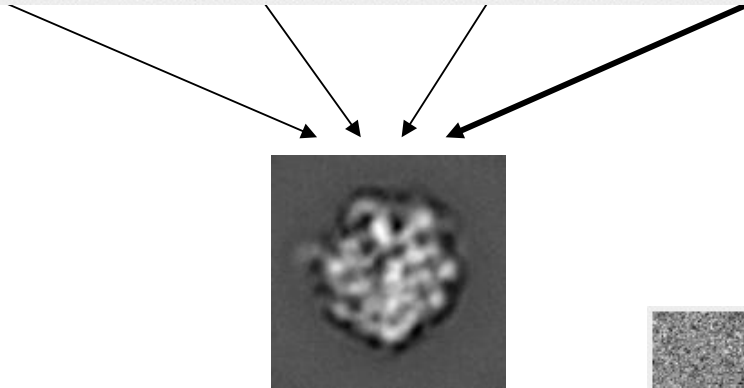
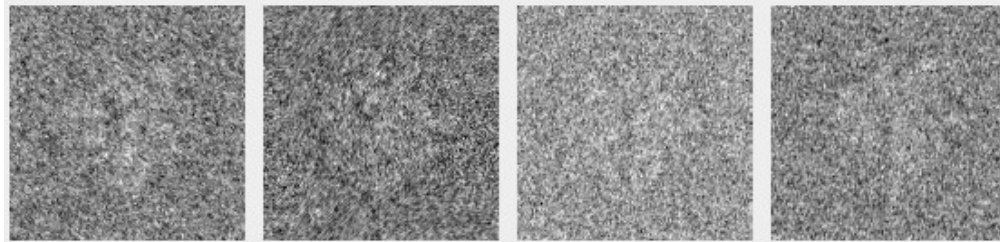


Baumeister et al. (1999), *Trends in Cell Biol.*, **9**: 81-5.

Your sample isn't guaranteed to adopt different orientations, in which case you may need to explicitly tilt the microscope stage.
(more later...)

Why do we need orientation?

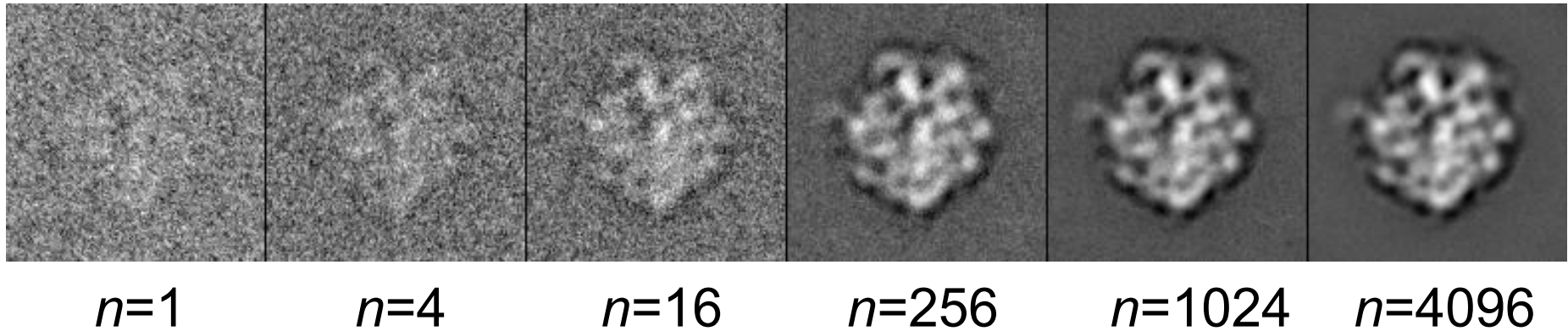
aligned images 1-4 of 4096 total



unaligned images 1-4 of 4096 total

This is a simple 2D case, but the effects are analogous in 3D.

What happens as we include more particles?



Signal-to-noise ratio increases with \sqrt{n}

What information do we need for 3D reconstruction?

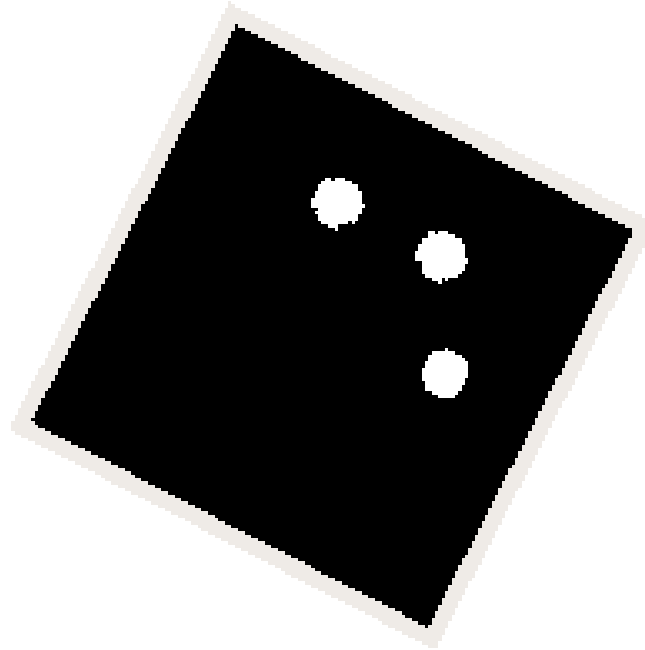
1. different orientations
2. known orientations
3. many particles

*I have all of this information.
Now what?*

There are two general categories of 3D reconstruction

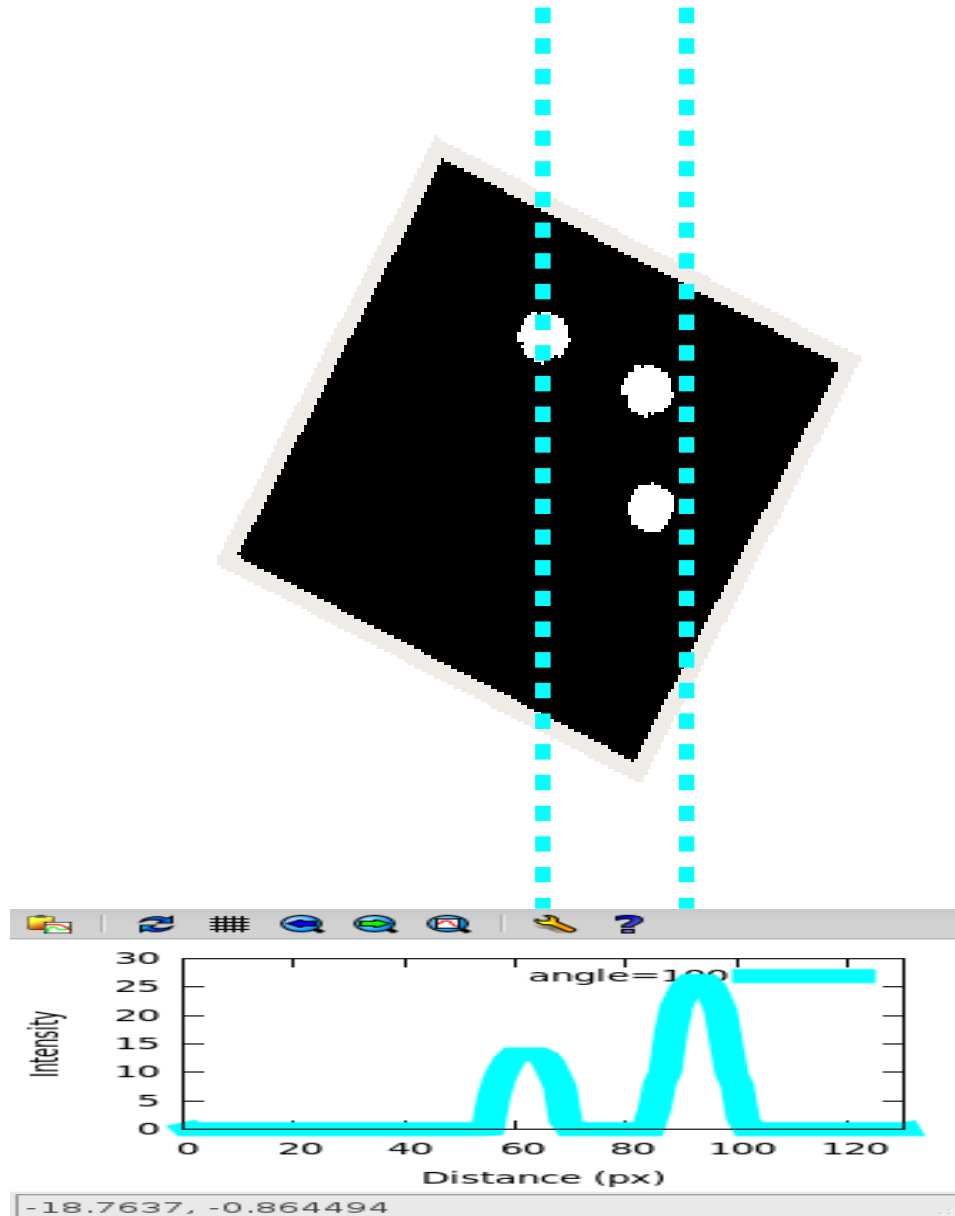
1. Real space
2. Fourier space

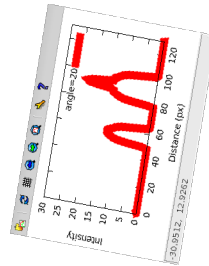
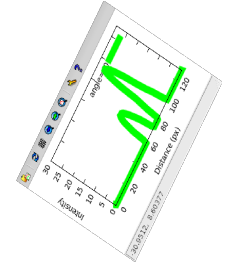
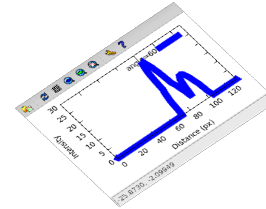
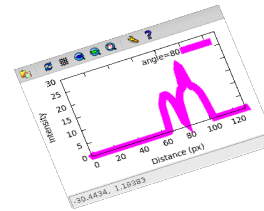
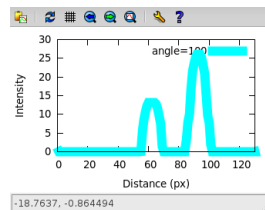
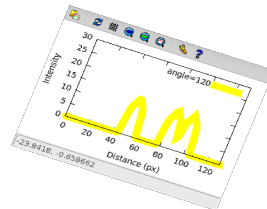
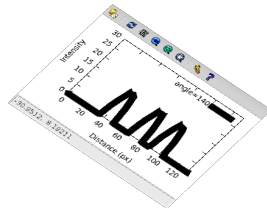
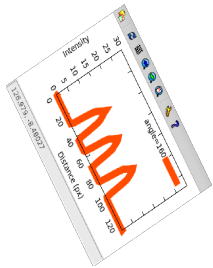
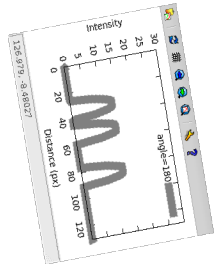
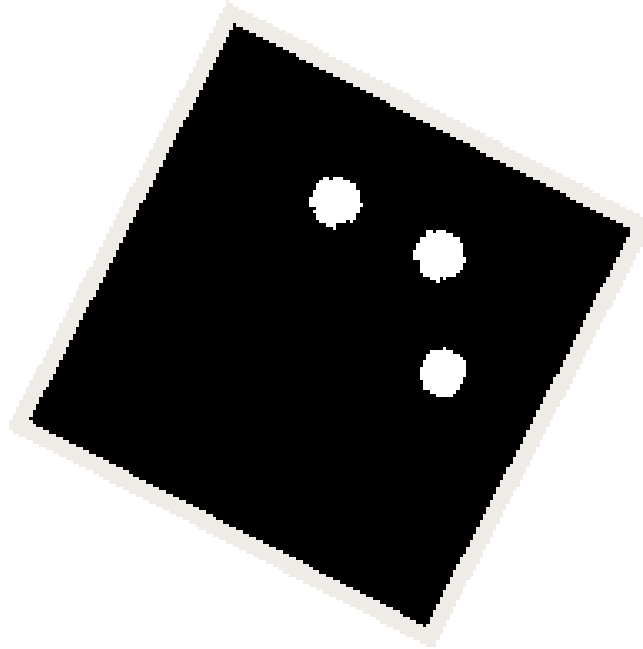
Reconstruction in real space



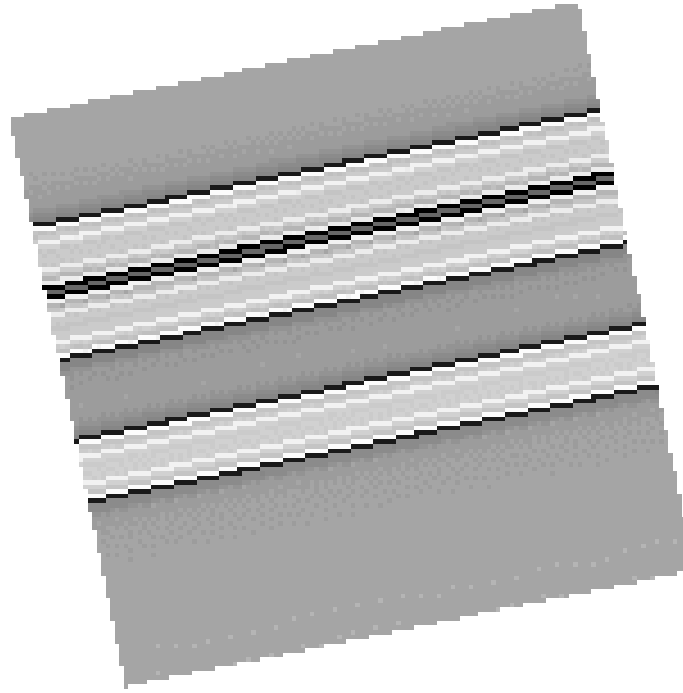
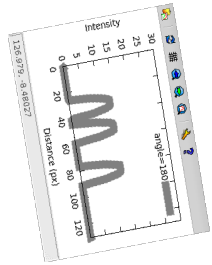
We are going to reconstruct a 2D object from 1D projections. The principle is the similar to, but simpler than, reconstructing a 3D object from 2D projections.

Projection of our 2D object

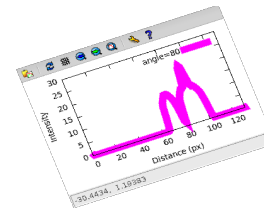
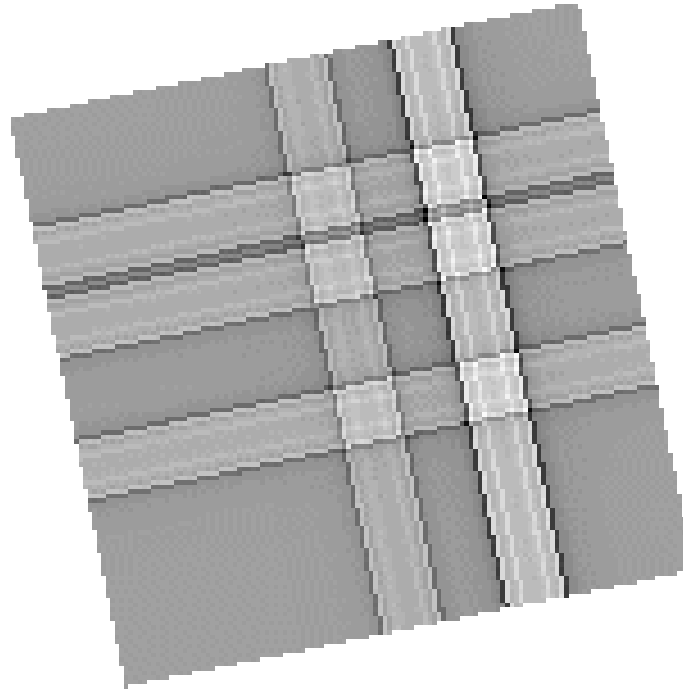
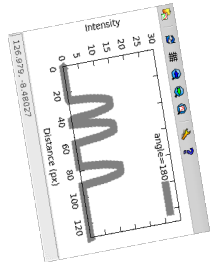




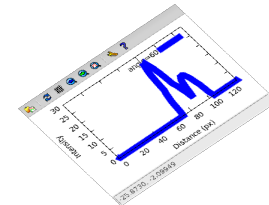
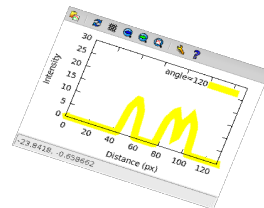
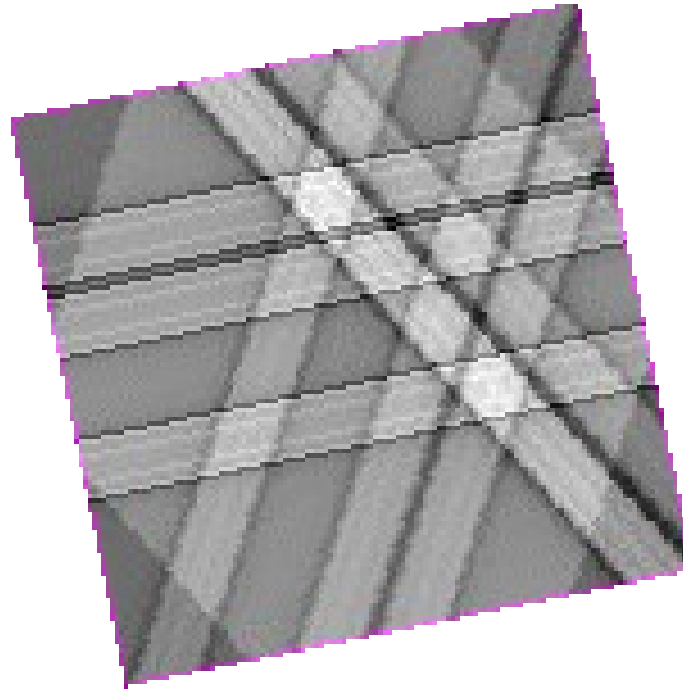
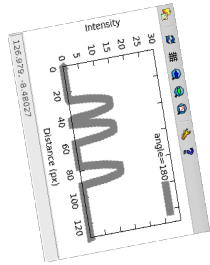
Reconstruction is the inversion of projection



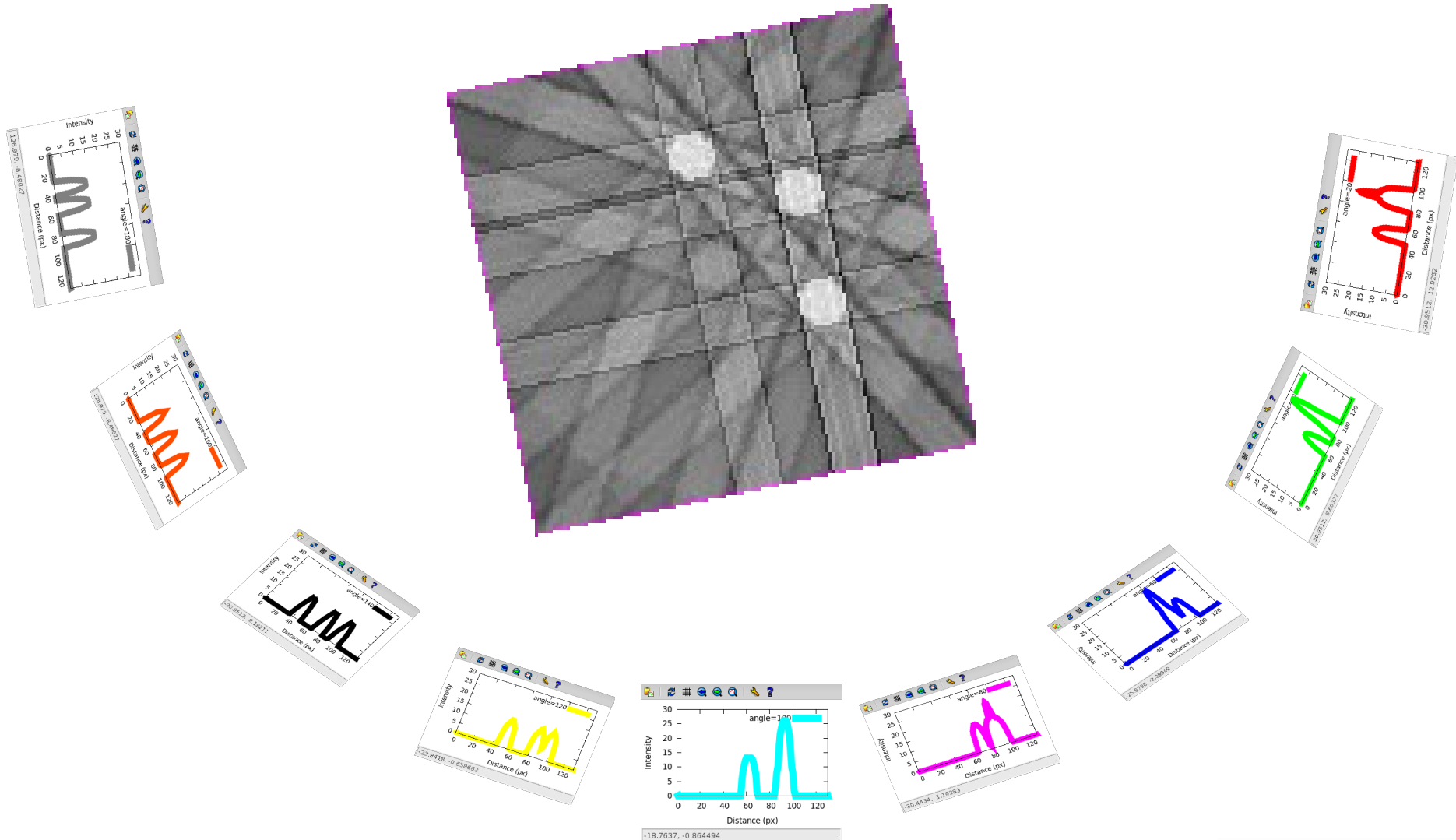
Reconstruction is the inversion of projection



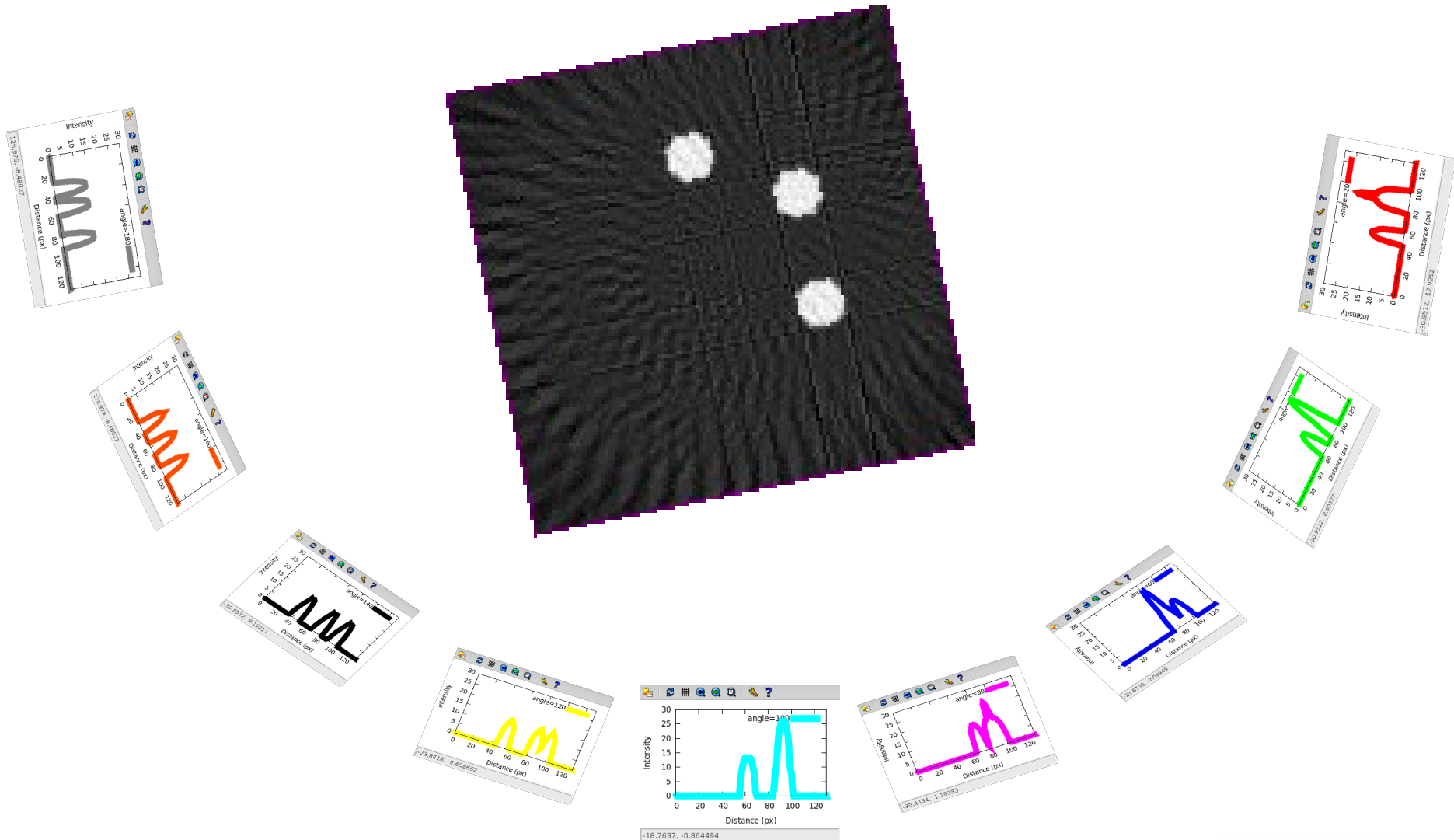
Reconstruction is the inversion of projection



Reconstruction is the inversion of projection

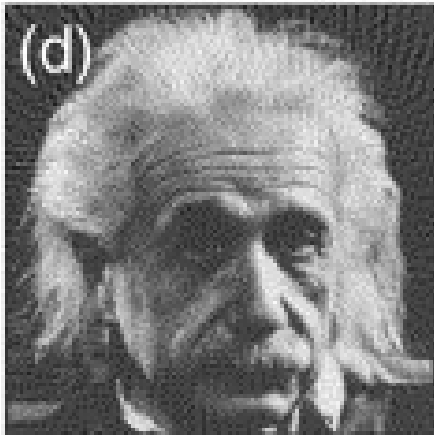


Reconstruction is the inversion of projection



What happens when we're missing views?

good



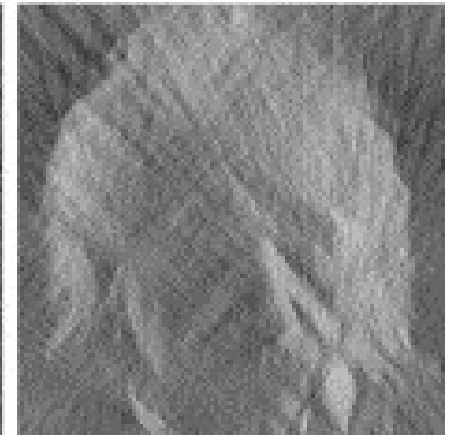
missing
views



sparse
sampling

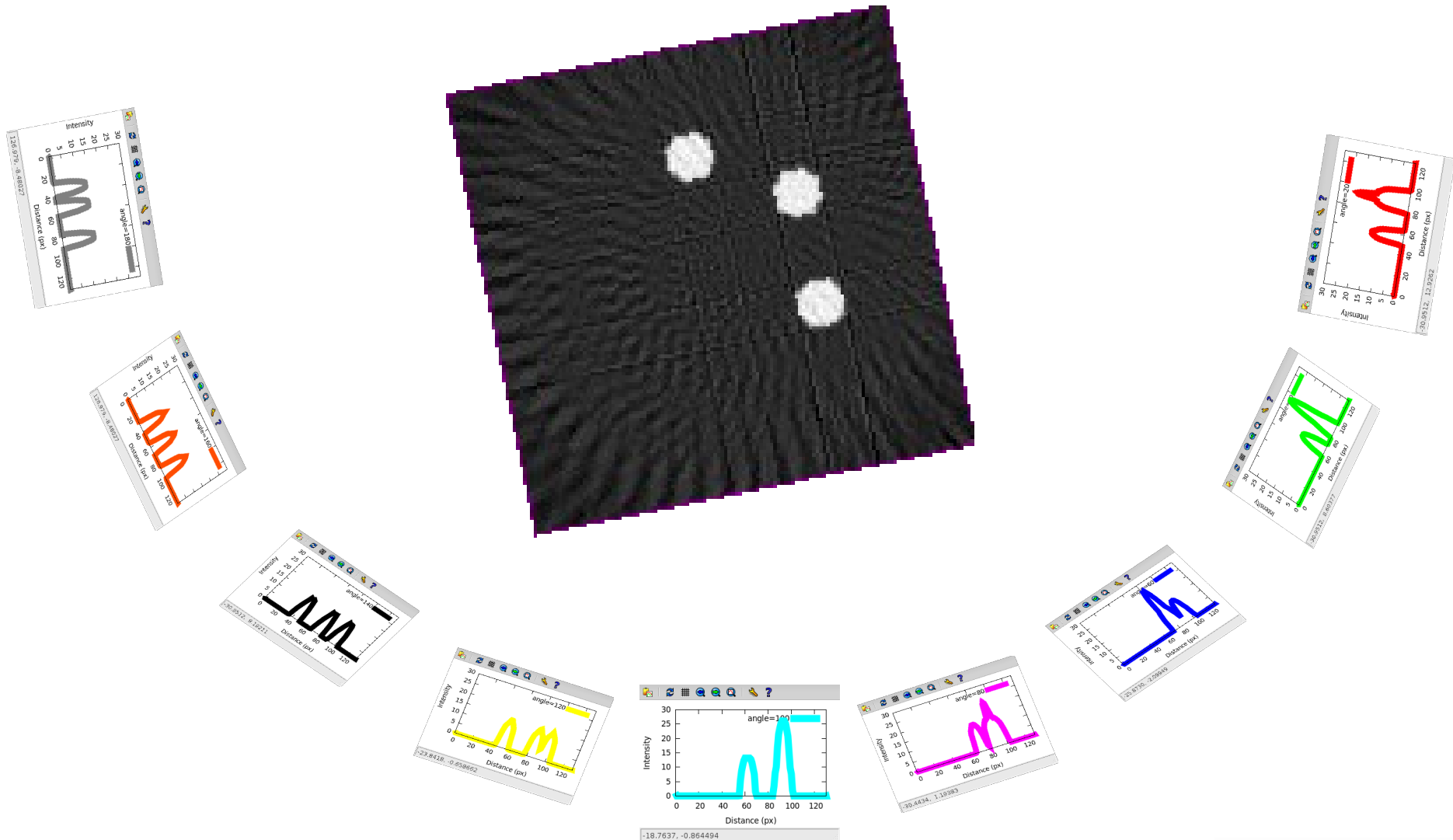


sparse
sampling
+
missing
views



Baumeister et al. (1999), *Trends in Cell Biol.*, **9**: 81-5.

Simultaneous Iterative Reconstruction Technique



*The reconstruction doesn't agree well with the projections.
What can we do?*

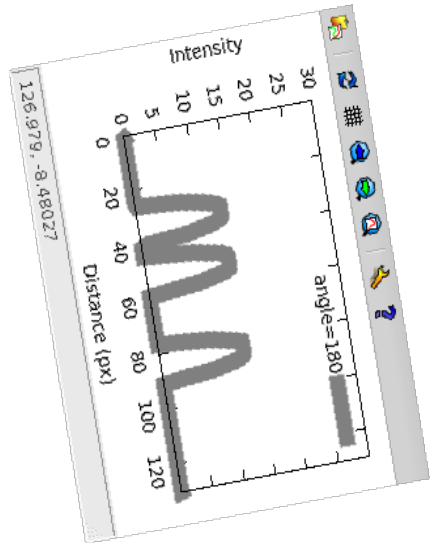
(one) ANSWER:
Simultaneous Iterative Reconstruction Technique

Simultaneous Iterative Reconstruction Technique

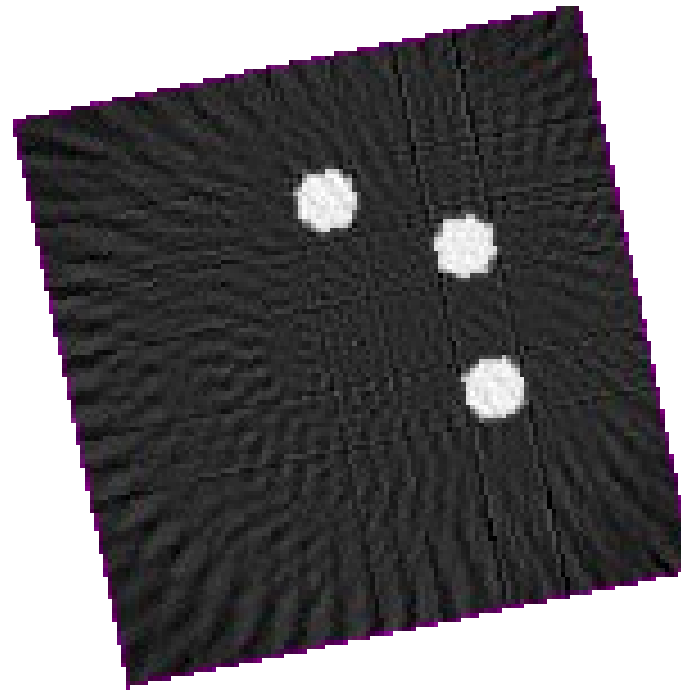
The idea:

- ◆ You compute re-projections of your model.
- ◆ Compare the re-projections to your experimental data.
 - There will be differences.
- ◆ You weight the differences by a fudge factor, λ .
- ◆ You adjust the model by the difference weighted by λ .
- ◆ Repeat.

Simultaneous Iterative Reconstruction Technique



Experimental projection

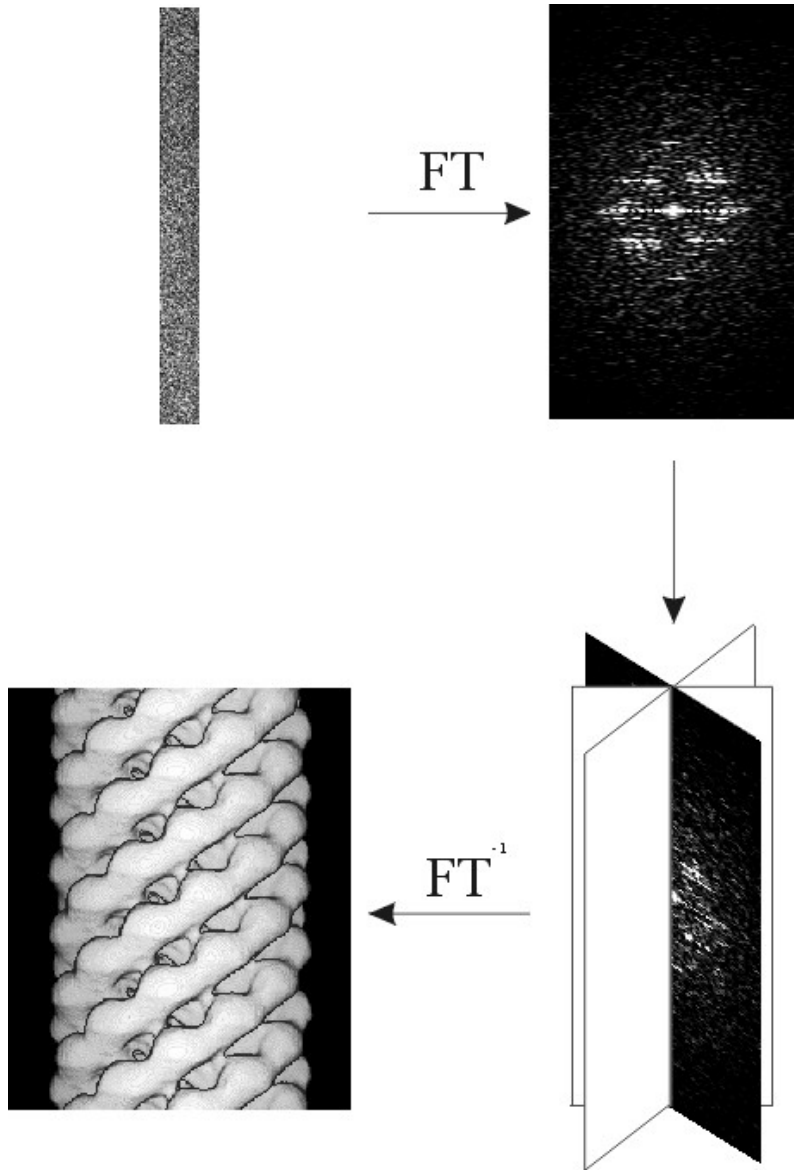


Model

Here, the differences (which will be down-weighted by λ) are the ripples in the background.

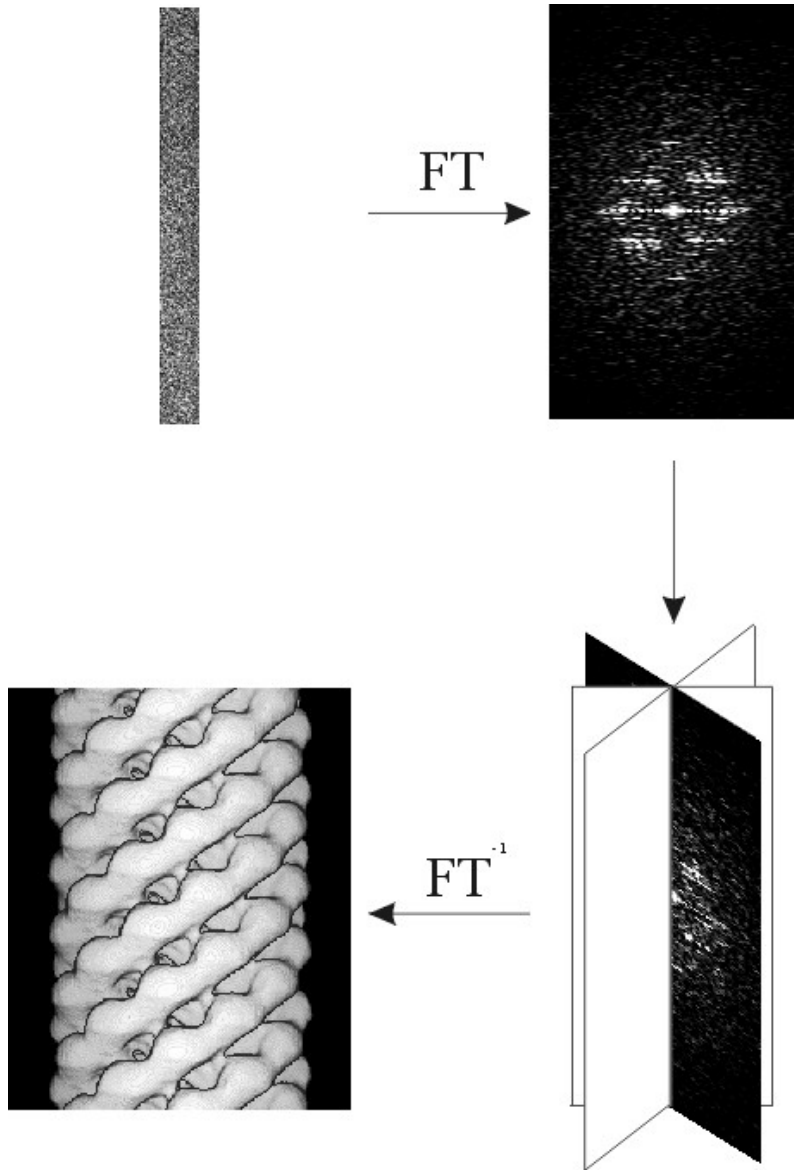
If we didn't down-weight by λ , we would over compensate, and would amplify noise.

Reconstruction in Fourier space



Projection theorem (or Central Section Theorem)

A central section through the 3D Fourier transform is the Fourier transform of the projection in that direction.



Projection theorem (or Central Section Theorem)

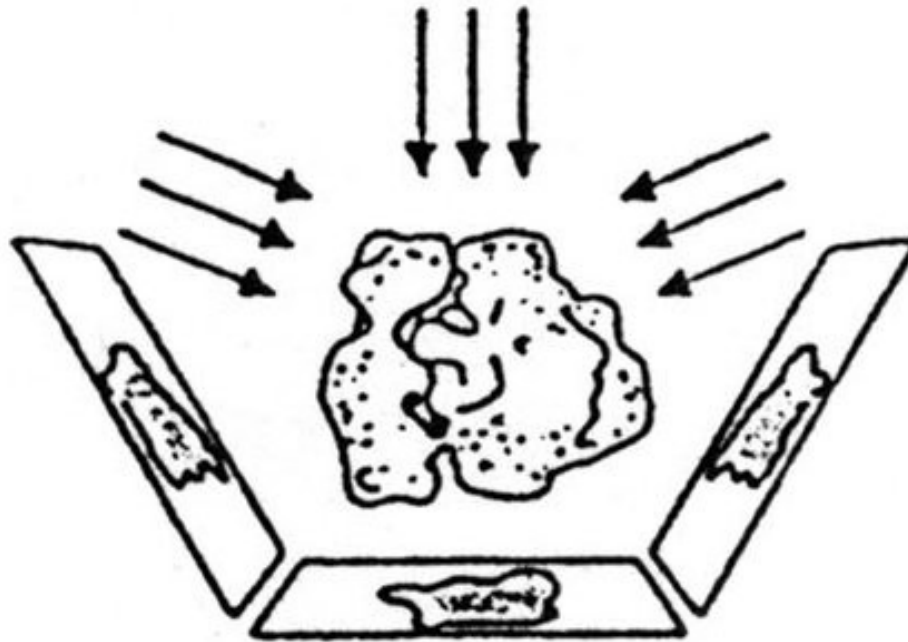
The disadvantage is that you have to resample your central sections from polar coordinates to Cartesian space, i.e. interpolate. There are new methods to better interpolate in Fourier space.

*Reference-based alignment
(or projection-matching)*

Reference-based alignment

You will record the direction of projection (the Euler angles), such that if you encounter an experimental image that resembles a reference projection, you will assign that reference projection's Euler angles to the experimental image.

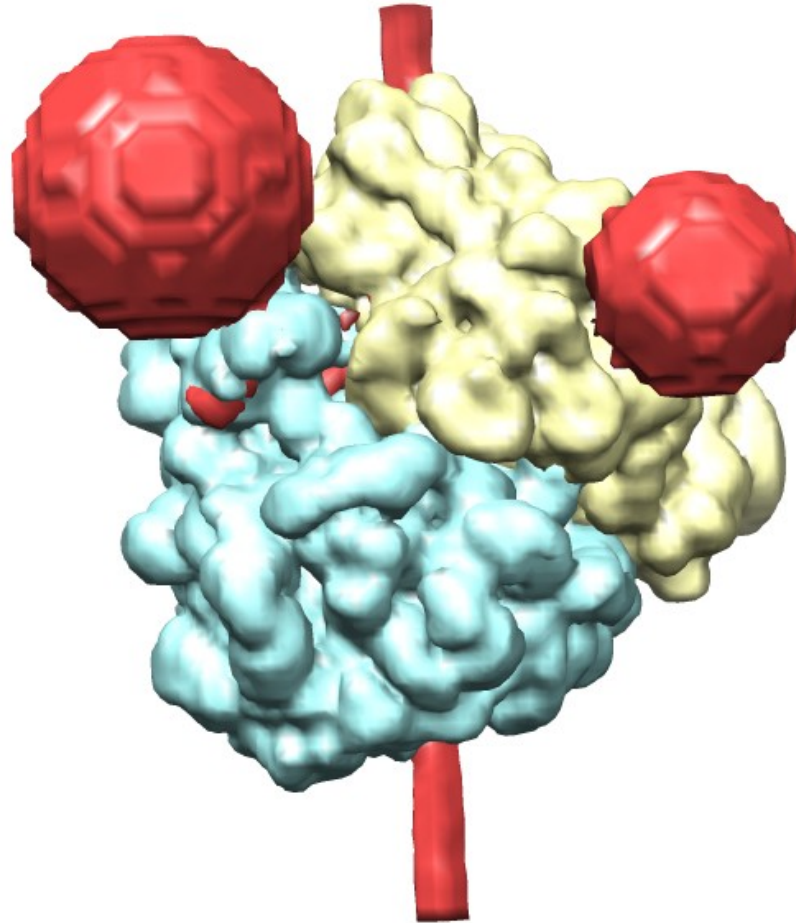
Step 1: Generation of projections of the reference.



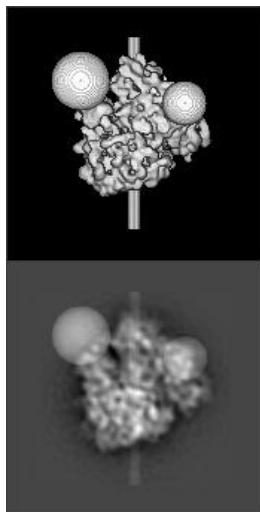
From Penczek *et al.* (1994), *Ultramicroscopy* **53**: 251-70.

Assumption: reference is similar enough to the sample that it can be used to determine orientation.

The model



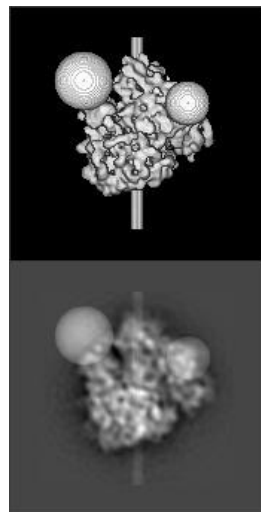
(The extra features helped determine handedness in noisy reconstructions.)



phi=000

theta=000

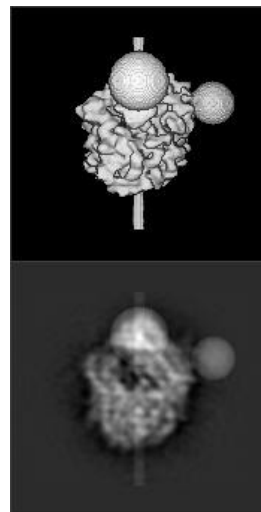
psi=000



phi=000

theta=000

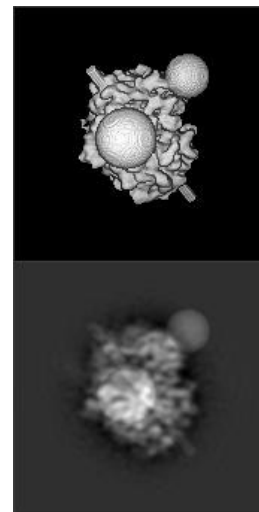
psi=000



phi=000

theta=045

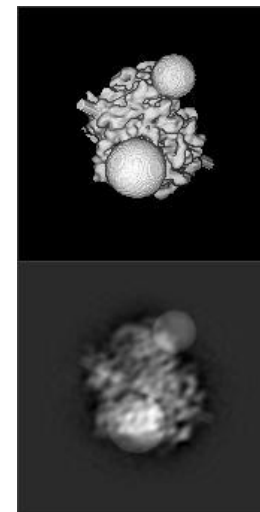
psi=000



phi=048

theta=045

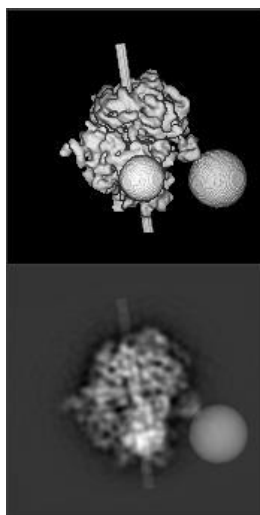
psi=000



phi=072

theta=045

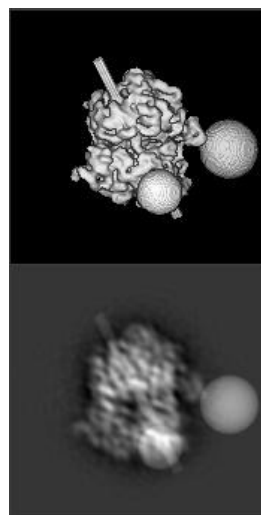
psi=000



phi=192

theta=045

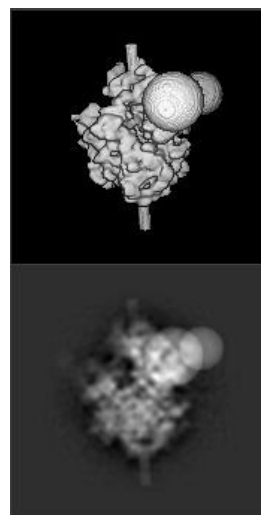
psi=000



phi=216

theta=045

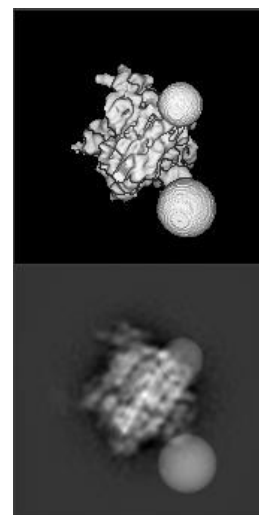
psi=000



phi=016

theta=075

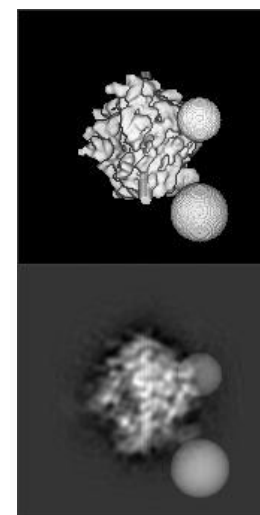
psi=000



phi=115

theta=075

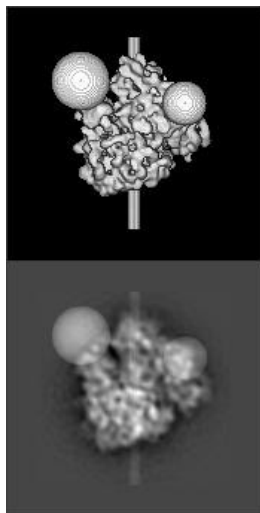
psi=000



phi=131

theta=090

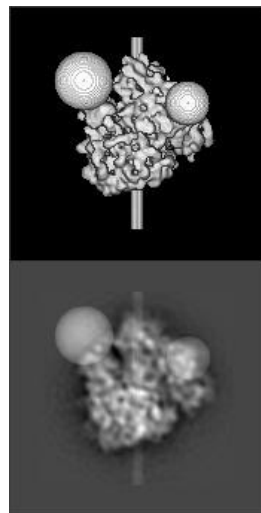
psi=000



phi=000

theta=000

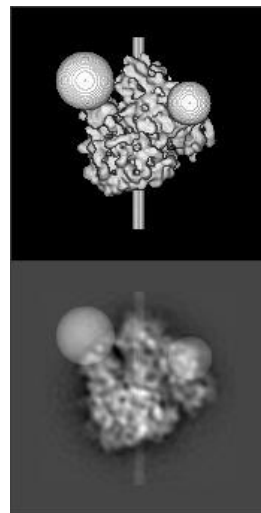
psi=000



phi=000

theta=000

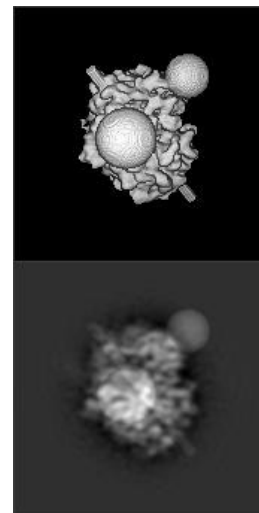
psi=000



phi=000

theta=000

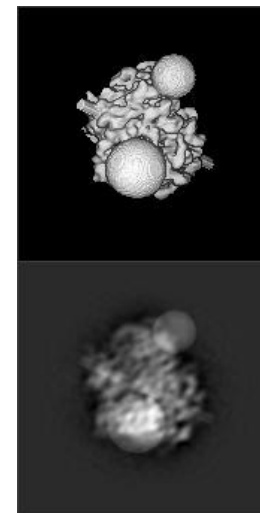
psi=000



phi=048

theta=045

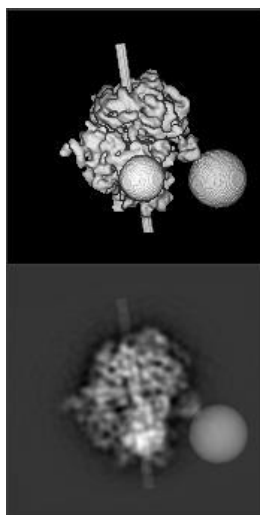
psi=000



phi=072

theta=045

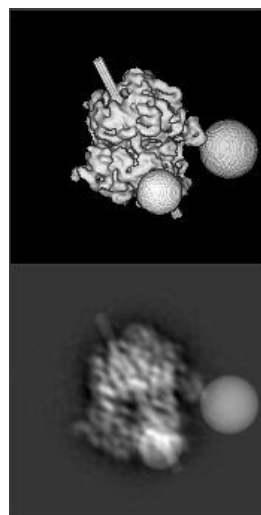
psi=000



phi=192

theta=045

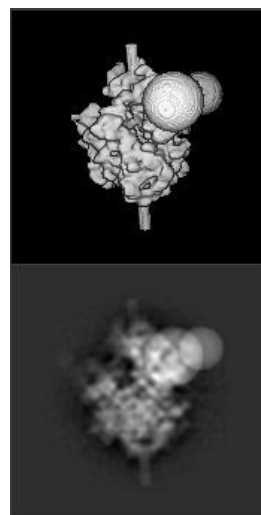
psi=000



phi=216

theta=045

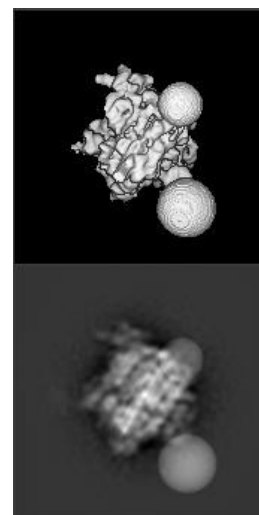
psi=000



phi=016

theta=075

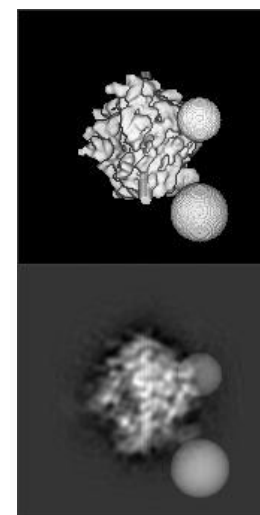
psi=000



phi=115

theta=075

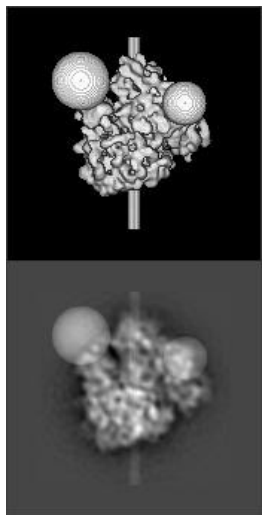
psi=000



phi=131

theta=090

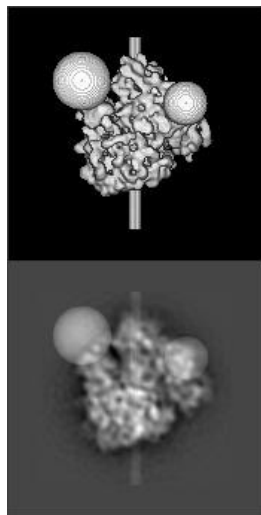
psi=000



phi=000

theta=000

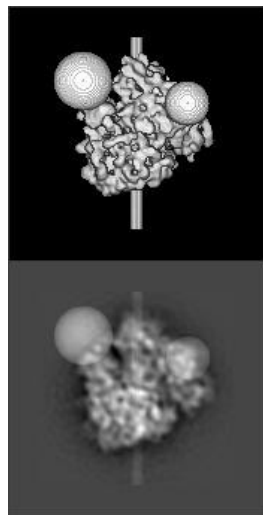
psi=000



phi=000

theta=000

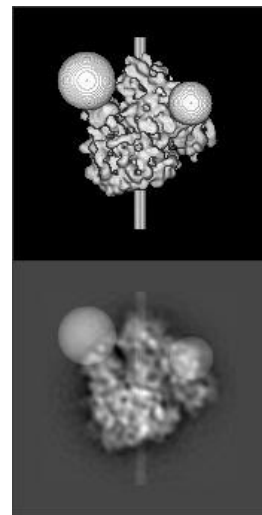
psi=000



phi=000

theta=000

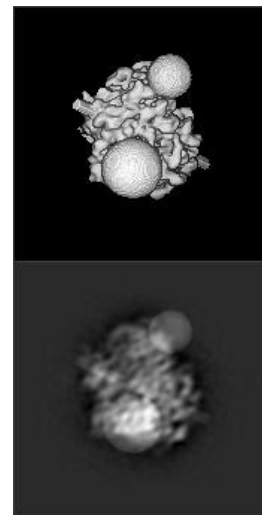
psi=000



phi=000

theta=000

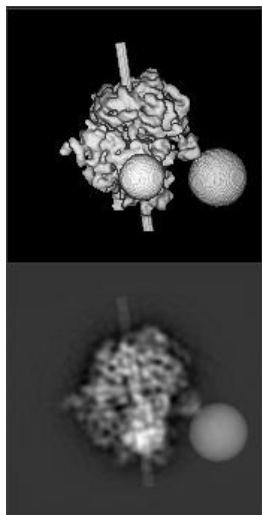
psi=000



phi=072

theta=045

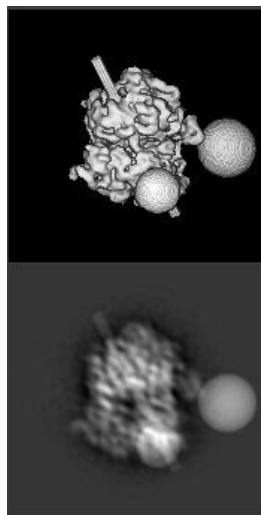
psi=000



phi=192

theta=045

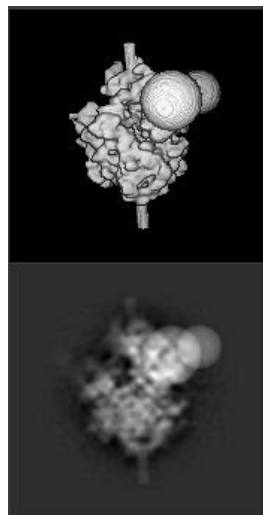
psi=000



phi=216

theta=045

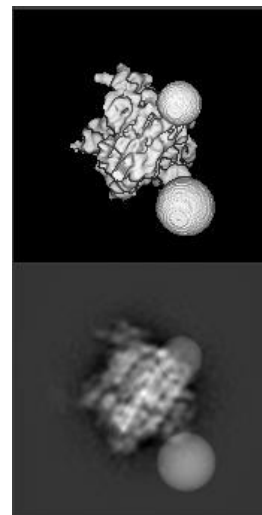
psi=000



phi=016

theta=075

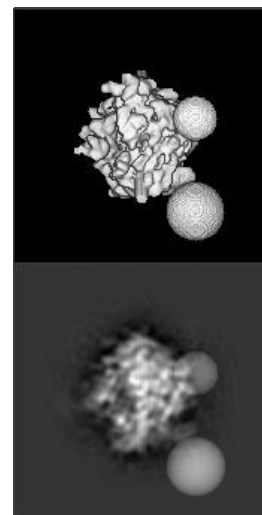
psi=000



phi=115

theta=075

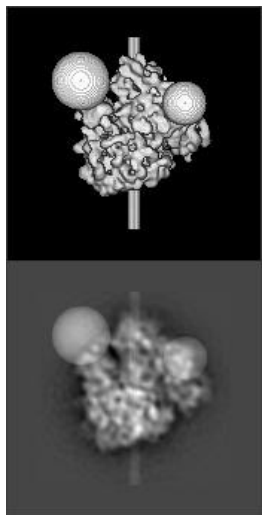
psi=000



phi=131

theta=090

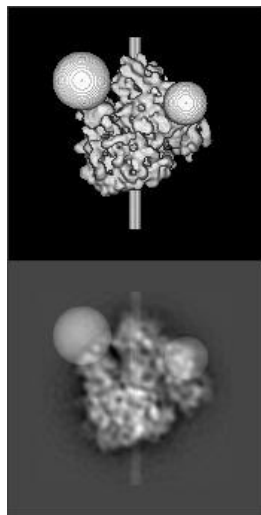
psi=000



phi=000

theta=000

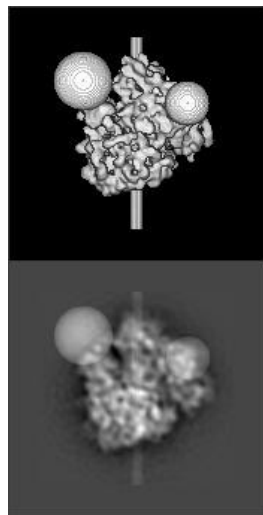
psi=000



phi=000

theta=000

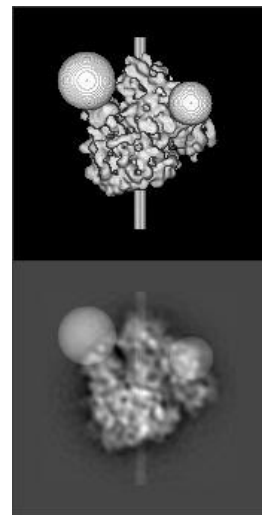
psi=000



phi=000

theta=000

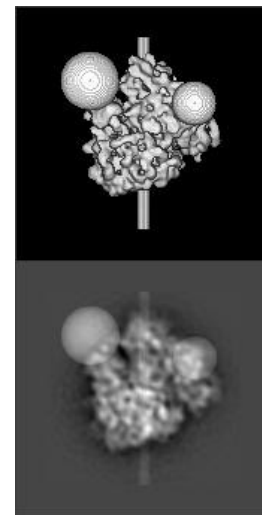
psi=000



phi=000

theta=000

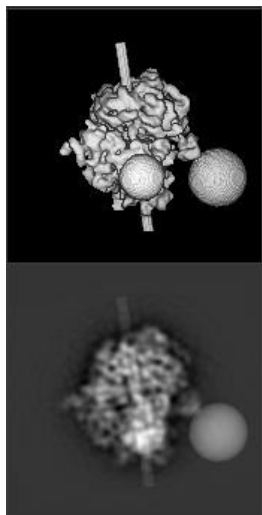
psi=000



phi=000

theta=000

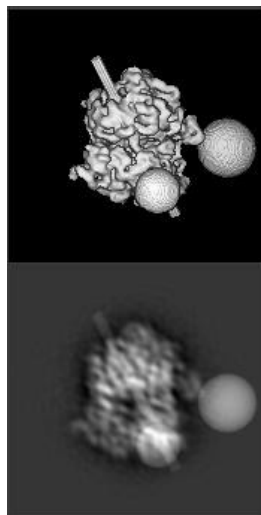
psi=000



phi=192

theta=045

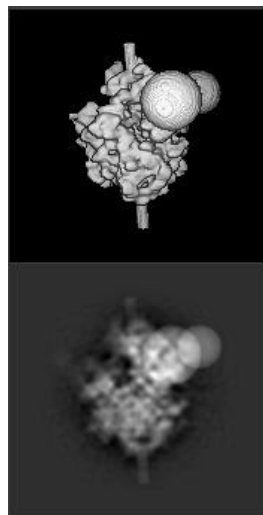
psi=000



phi=216

theta=045

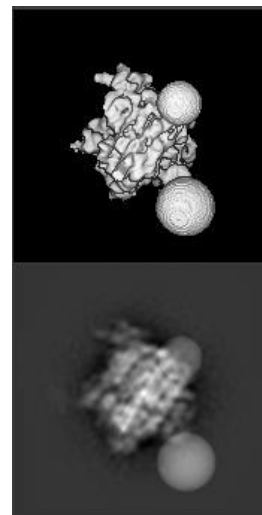
psi=000



phi=016

theta=075

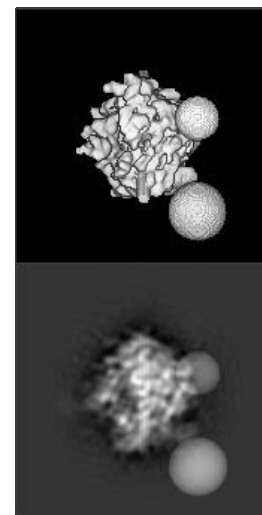
psi=000



phi=115

theta=075

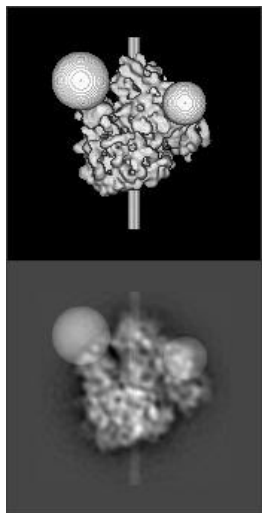
psi=000



phi=131

theta=090

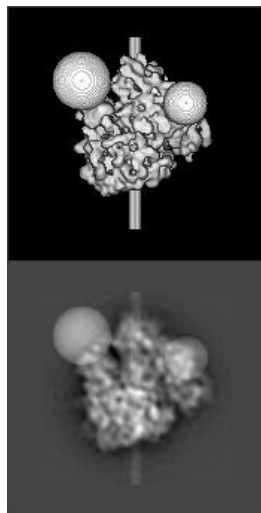
psi=000



phi=000

theta=000

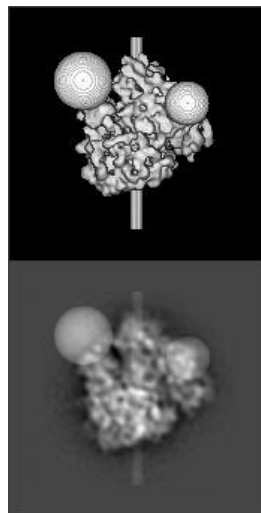
psi=000



phi=000

theta=000

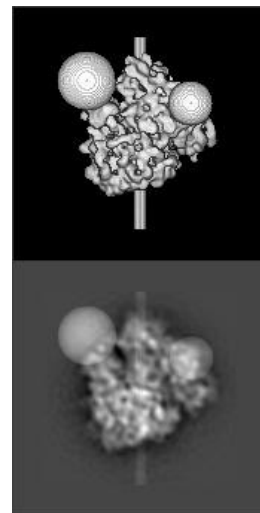
psi=000



phi=000

theta=000

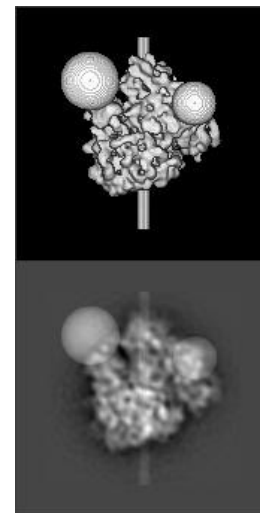
psi=000



phi=000

theta=000

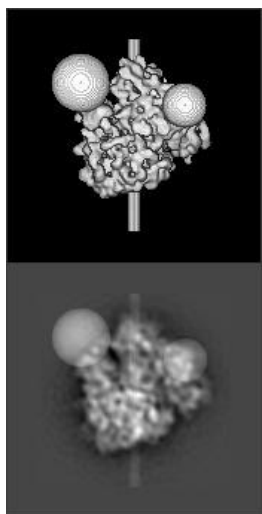
psi=000



phi=000

theta=000

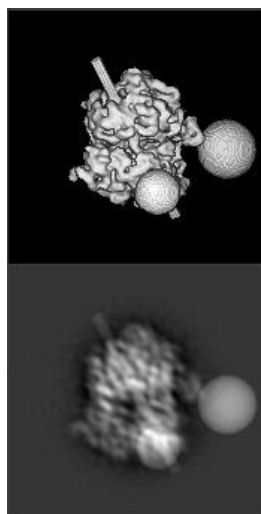
psi=000



phi=000

theta=000

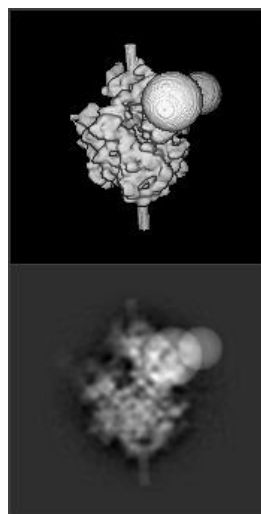
psi=000



phi=216

theta=045

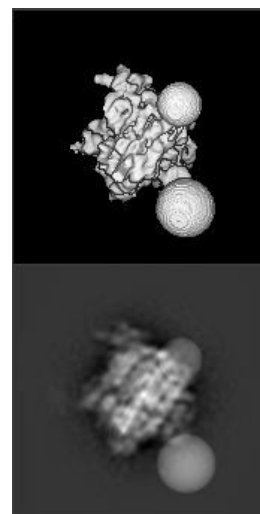
psi=000



phi=016

theta=075

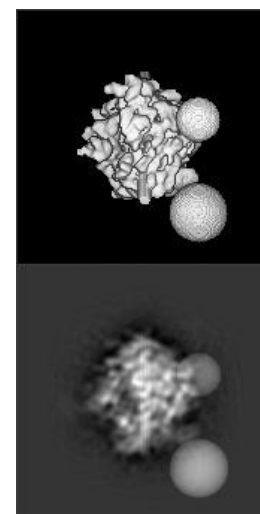
psi=000



phi=115

theta=075

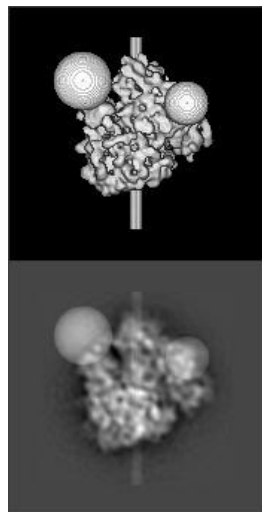
psi=000



phi=131

theta=090

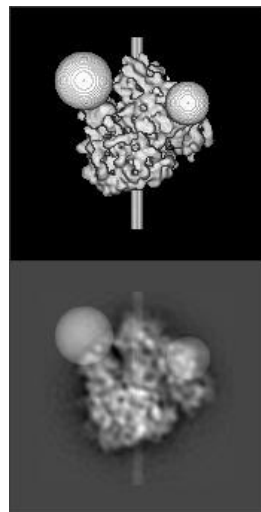
psi=000



phi=000

theta=000

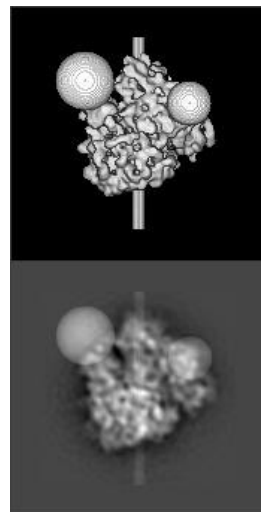
psi=000



phi=000

theta=000

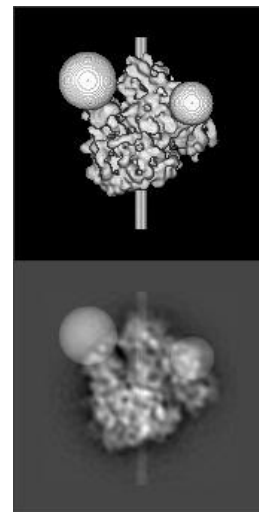
psi=000



phi=000

theta=000

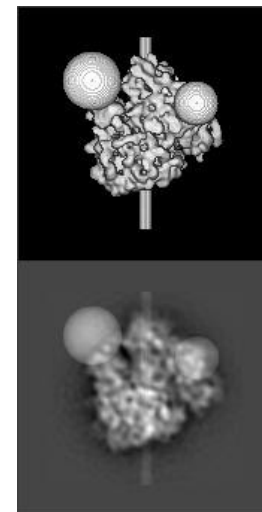
psi=000



phi=000

theta=000

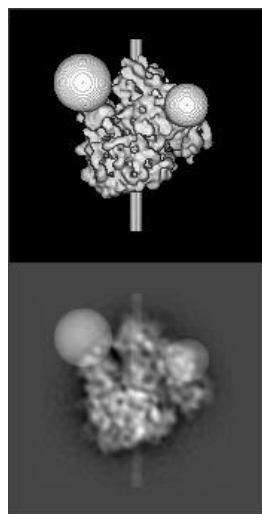
psi=000



phi=000

theta=000

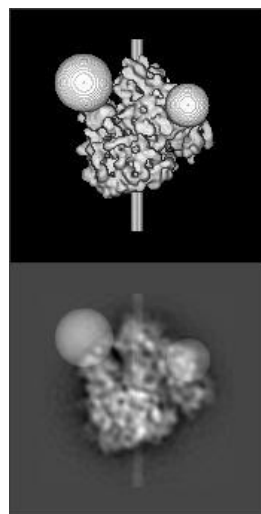
psi=000



phi=000

theta=000

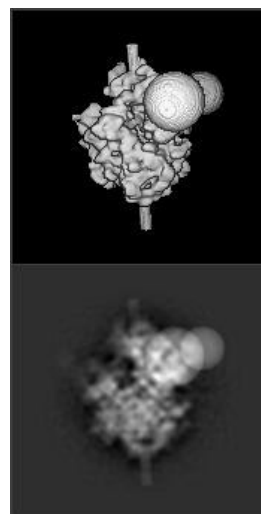
psi=000



phi=000

theta=000

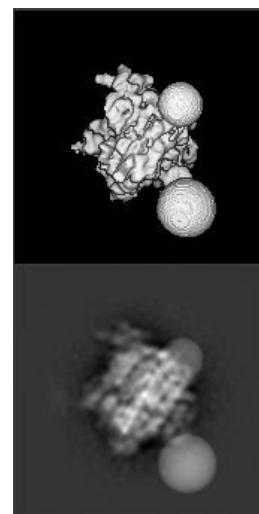
psi=000



phi=016

theta=075

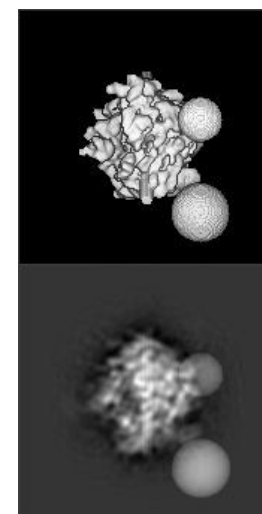
psi=000



phi=115

theta=075

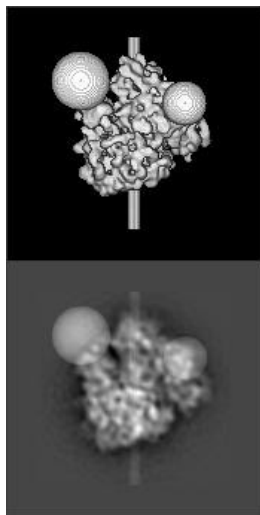
psi=000



phi=131

theta=090

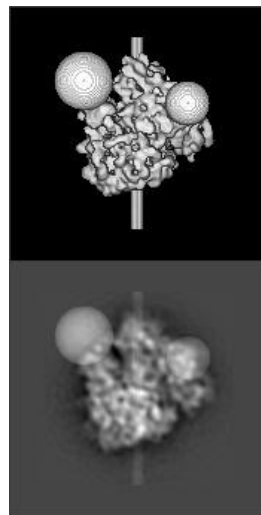
psi=000



phi=000

theta=000

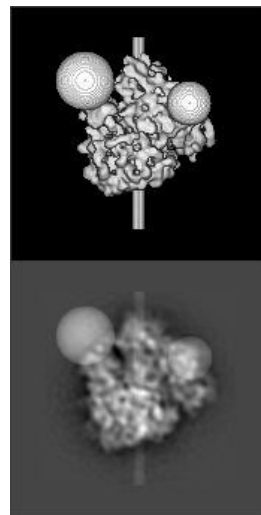
psi=000



phi=000

theta=000

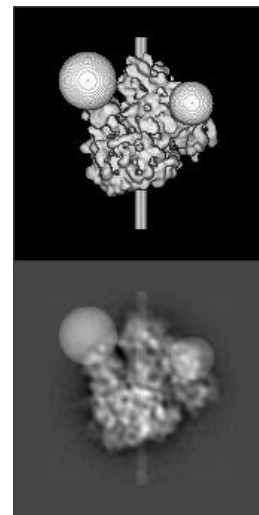
psi=000



phi=000

theta=000

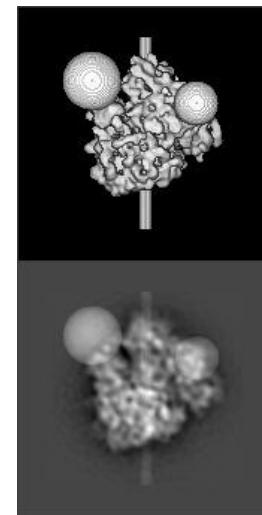
psi=000



phi=000

theta=000

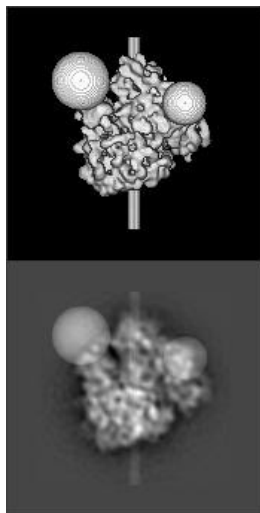
psi=000



phi=000

theta=000

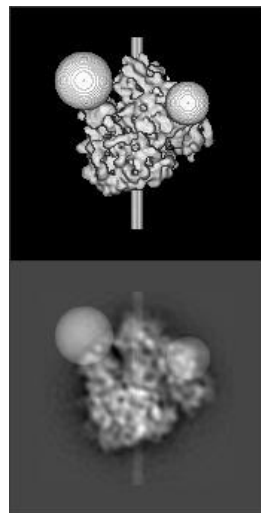
psi=000



phi=000

theta=000

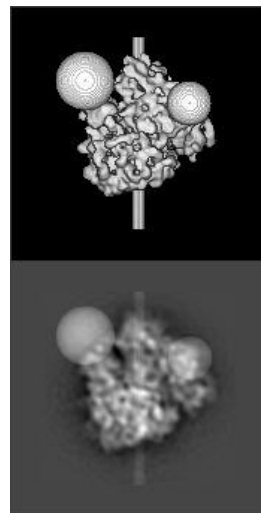
psi=000



phi=000

theta=000

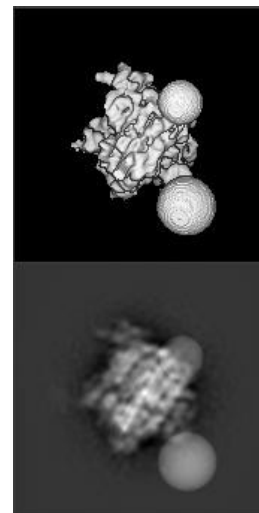
psi=000



phi=000

theta=000

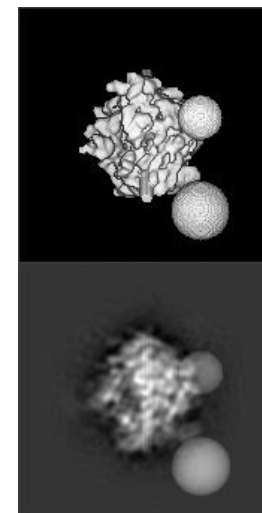
psi=000



phi=115

theta=075

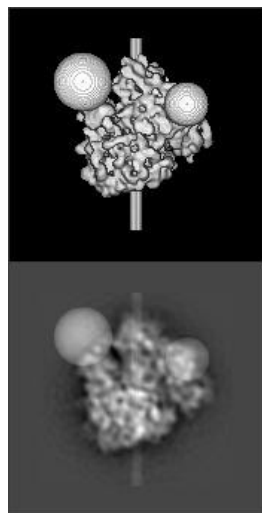
psi=000



phi=131

theta=090

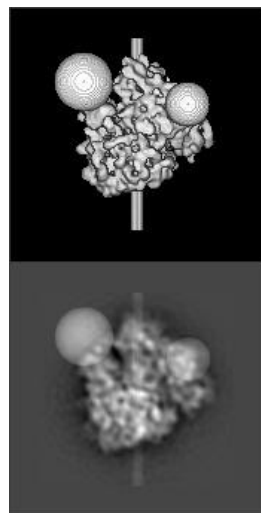
psi=000



phi=000

theta=000

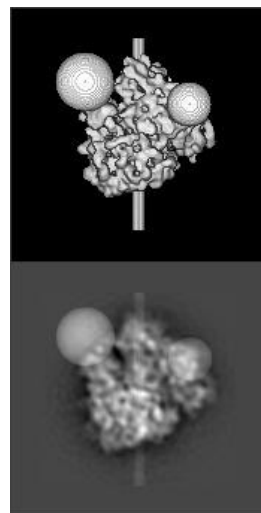
psi=000



phi=000

theta=000

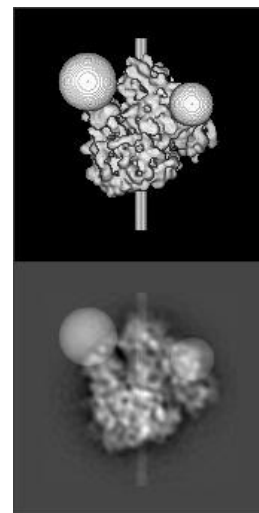
psi=000



phi=000

theta=000

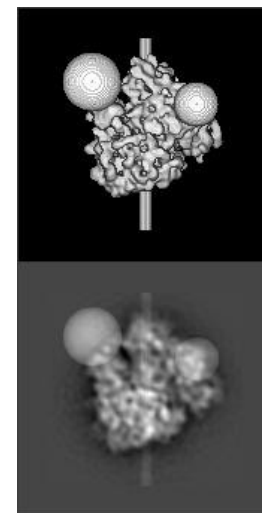
psi=000



phi=000

theta=000

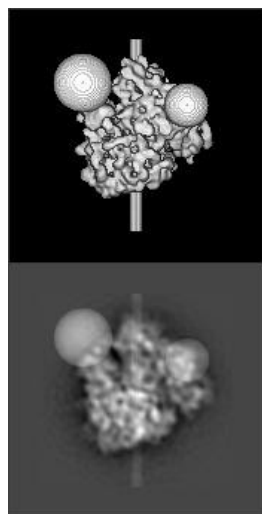
psi=000



phi=000

theta=000

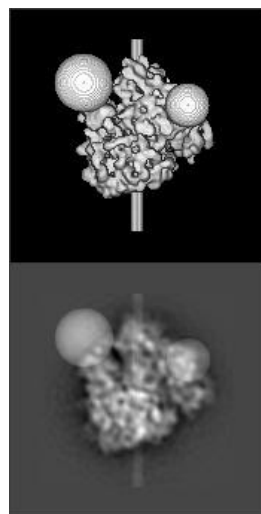
psi=000



phi=000

theta=000

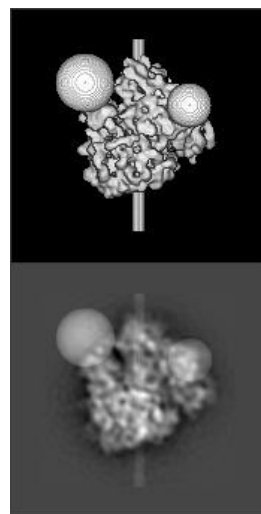
psi=000



phi=000

theta=000

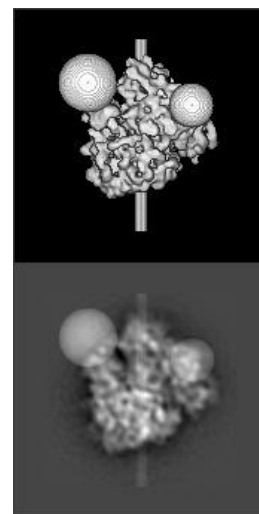
psi=000



phi=000

theta=000

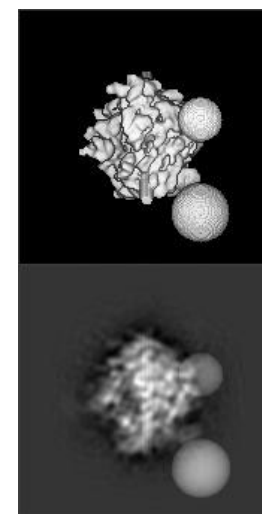
psi=000



phi=000

theta=000

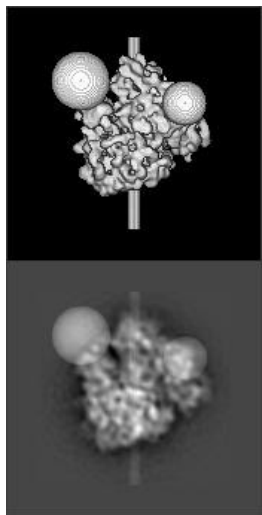
psi=000



phi=131

theta=090

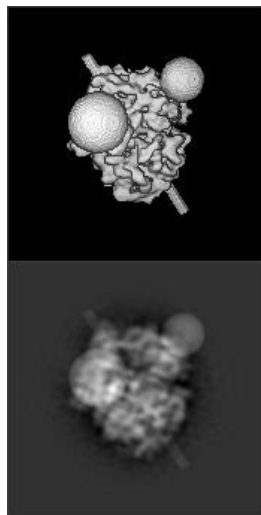
psi=000



phi=000

theta=000

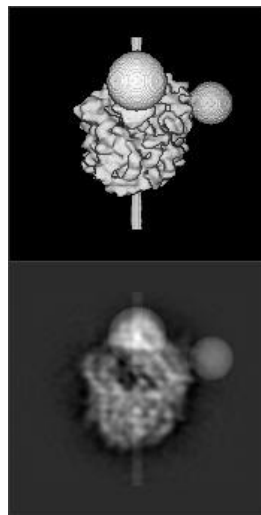
psi=000



phi=036

theta=030

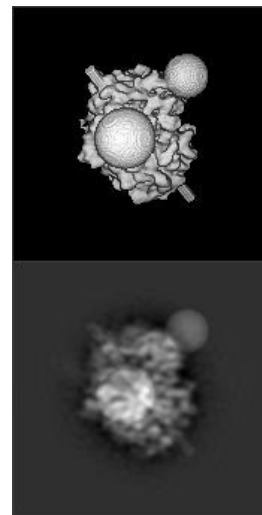
psi=000



phi=000

theta=045

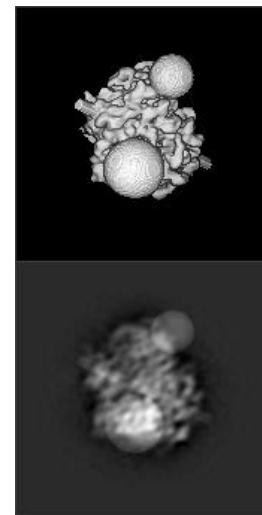
psi=000



phi=048

theta=045

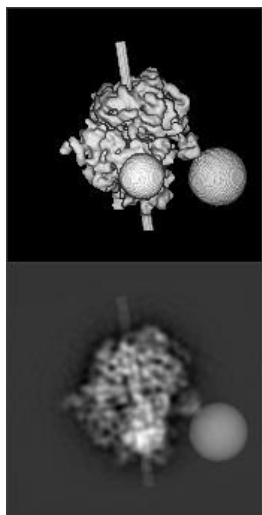
psi=000



phi=072

theta=045

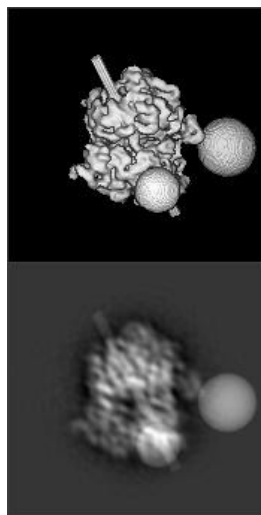
psi=000



phi=192

theta=045

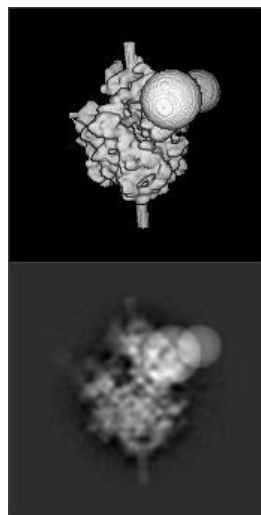
psi=000



phi=216

theta=045

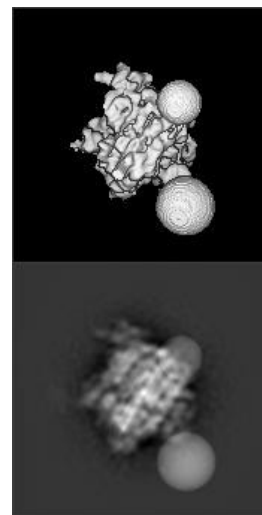
psi=000



phi=016

theta=075

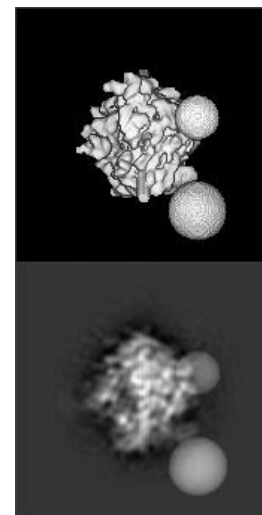
psi=000



phi=115

theta=075

psi=000

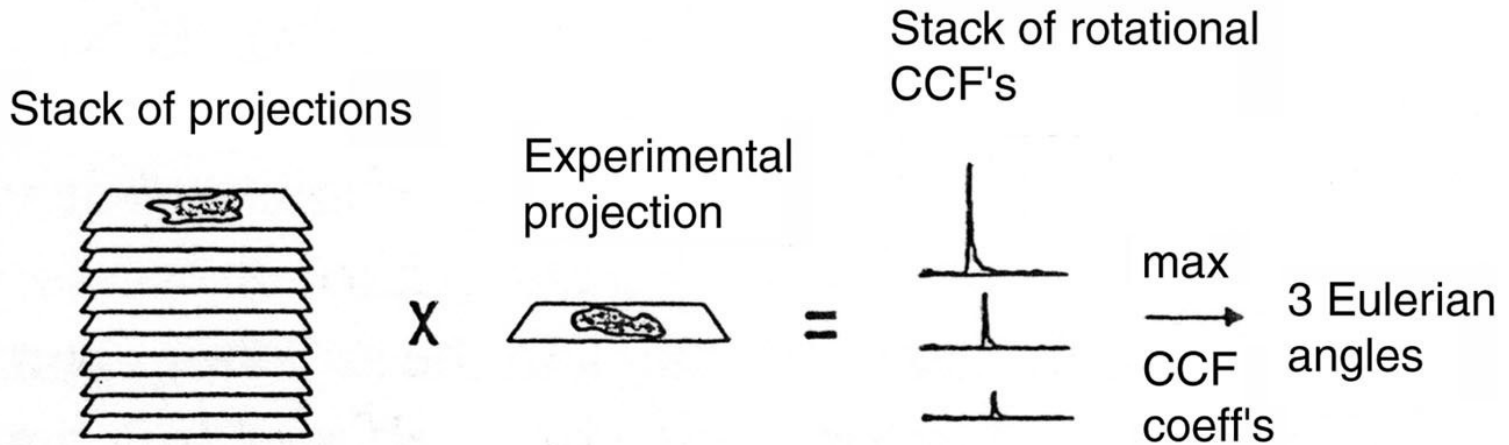


phi=131

theta=090

psi=000

Reference-based alignment

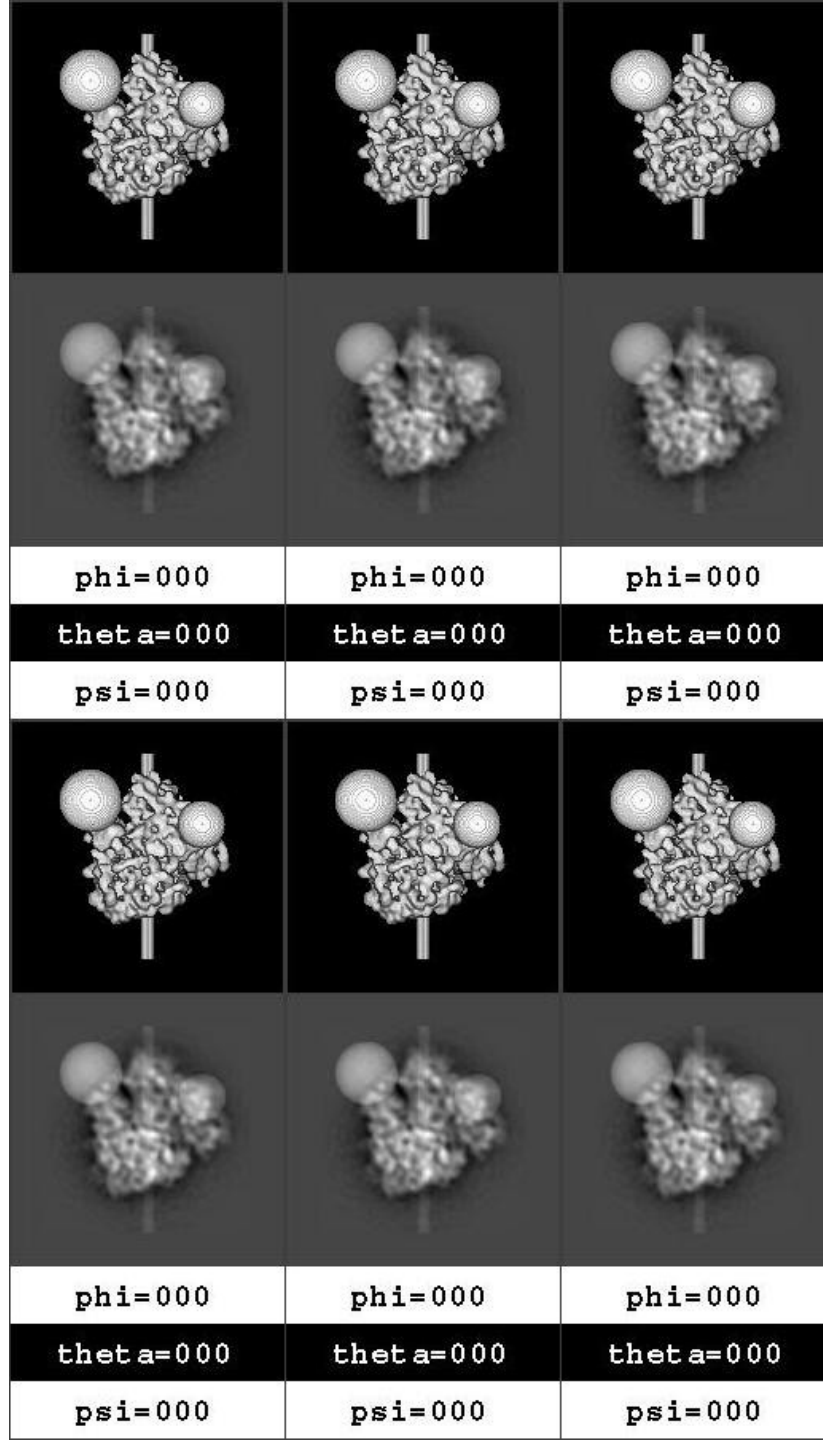


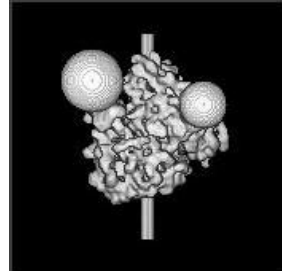
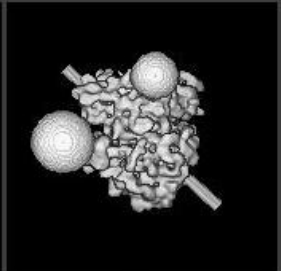
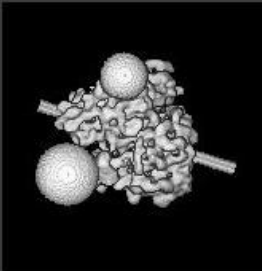
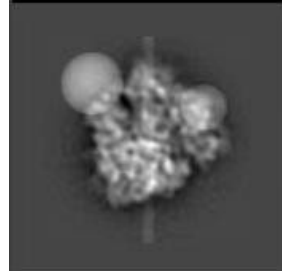
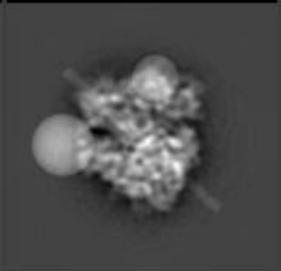
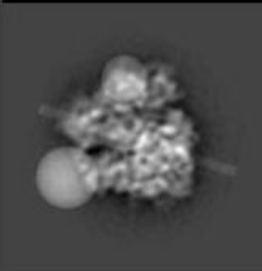
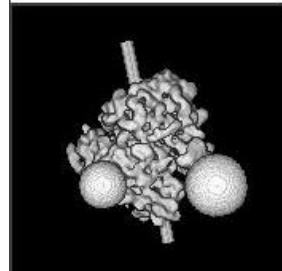
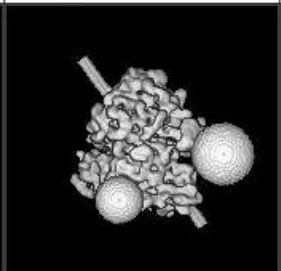
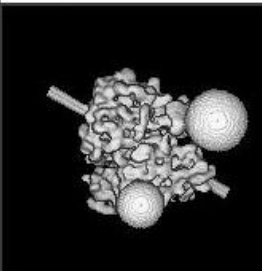
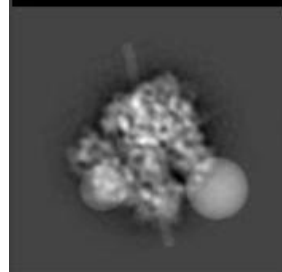
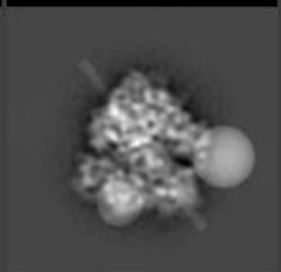
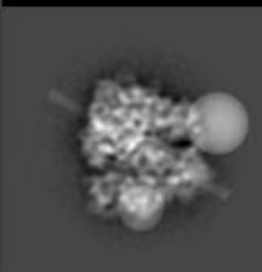
From Penczek *et al.* (1994), *Ultramicroscopy* **53**: 251-70.

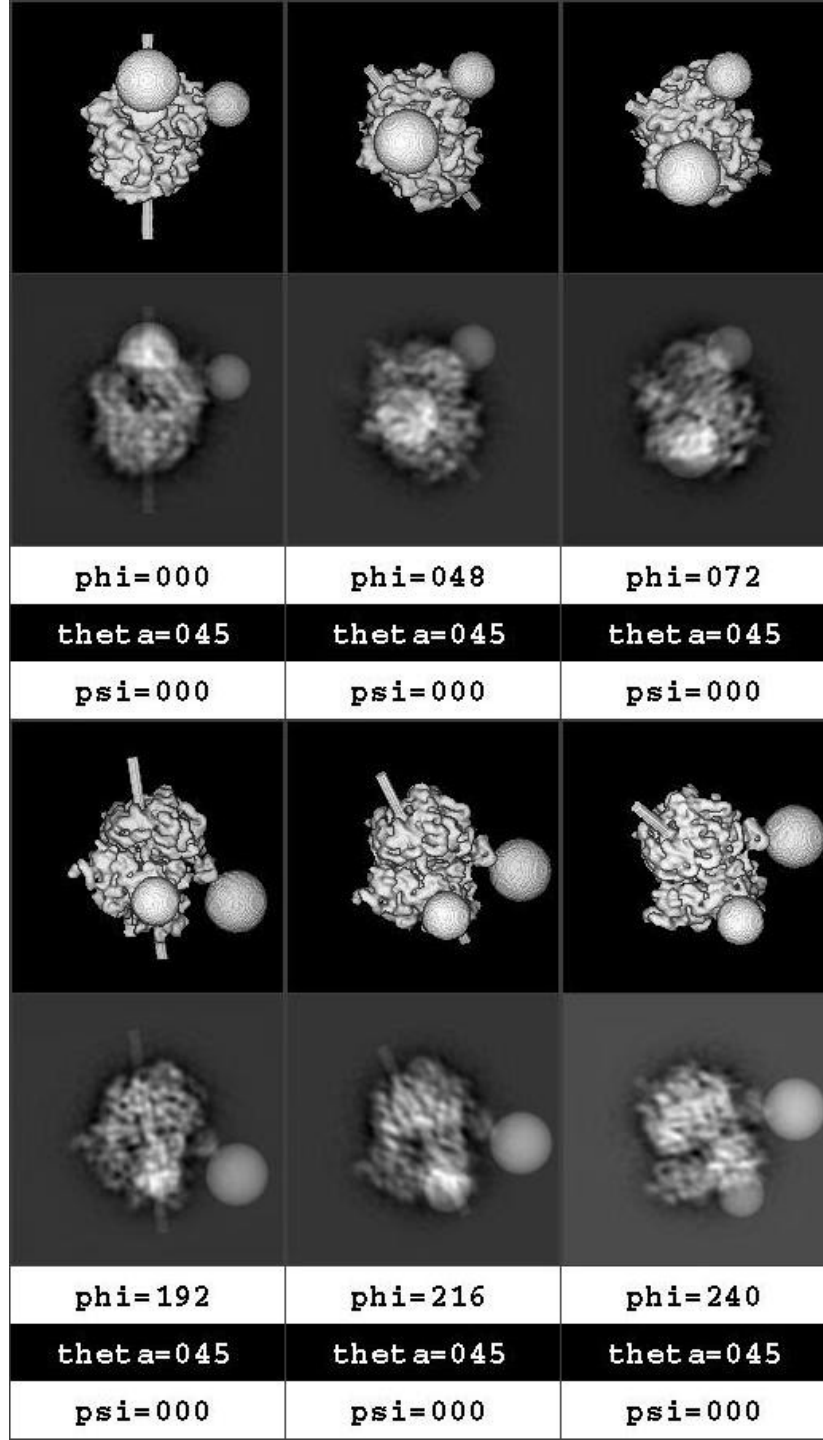
Steps:

1. Compare the experimental image to all of the reference projections.
2. Find the reference projection with which the experimental image matches best.
3. Assign the Euler angles of that reference projection to the experimental image.

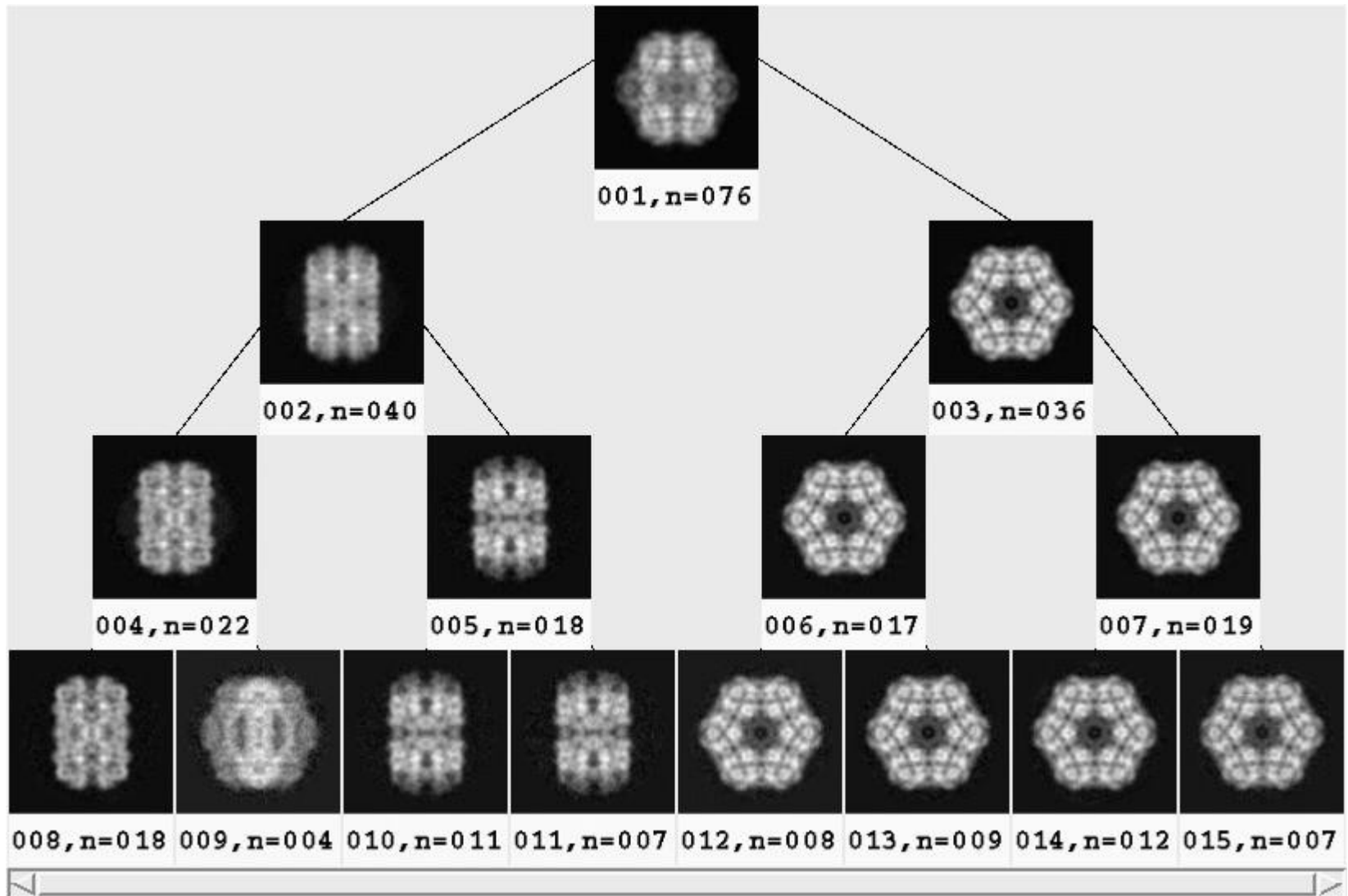
Random conical tilt



		
		
<code>phi=000</code>	<code>phi=048</code>	<code>phi=072</code>
<code>theta=001</code>	<code>theta=001</code>	<code>theta=001</code>
<code>psi=000</code>	<code>psi=000</code>	<code>psi=000</code>
		
		
<code>phi=192</code>	<code>phi=216</code>	<code>phi=240</code>
<code>theta=001</code>	<code>theta=001</code>	<code>theta=001</code>
<code>psi=000</code>	<code>psi=000</code>	<code>psi=000</code>

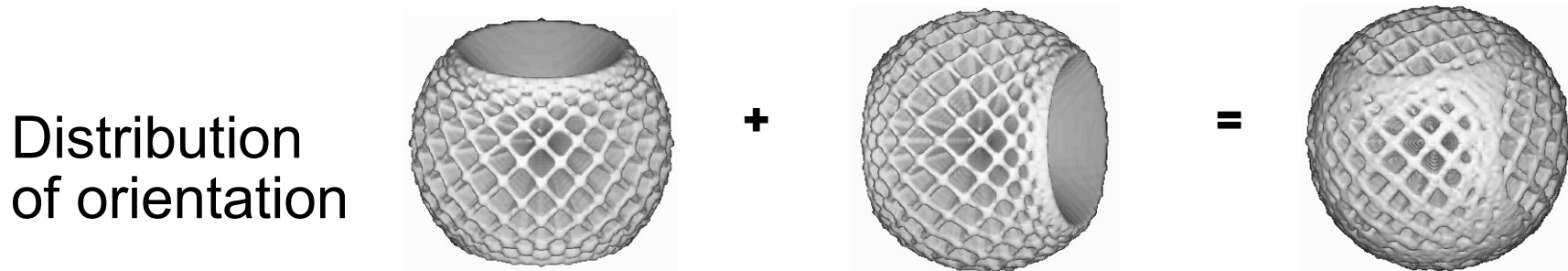
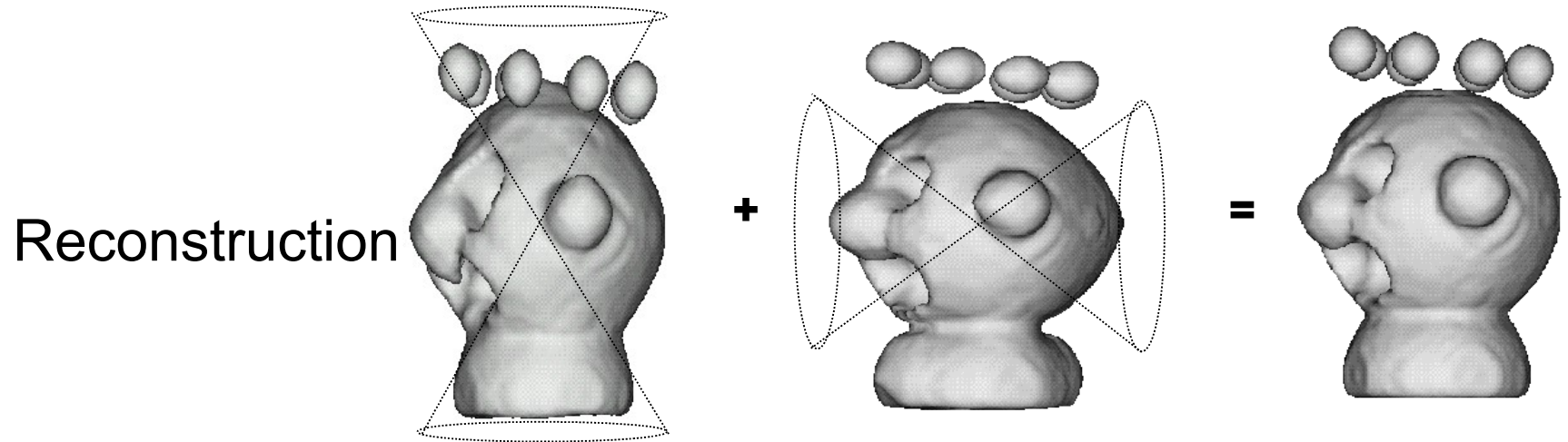


Binary-tree viewer

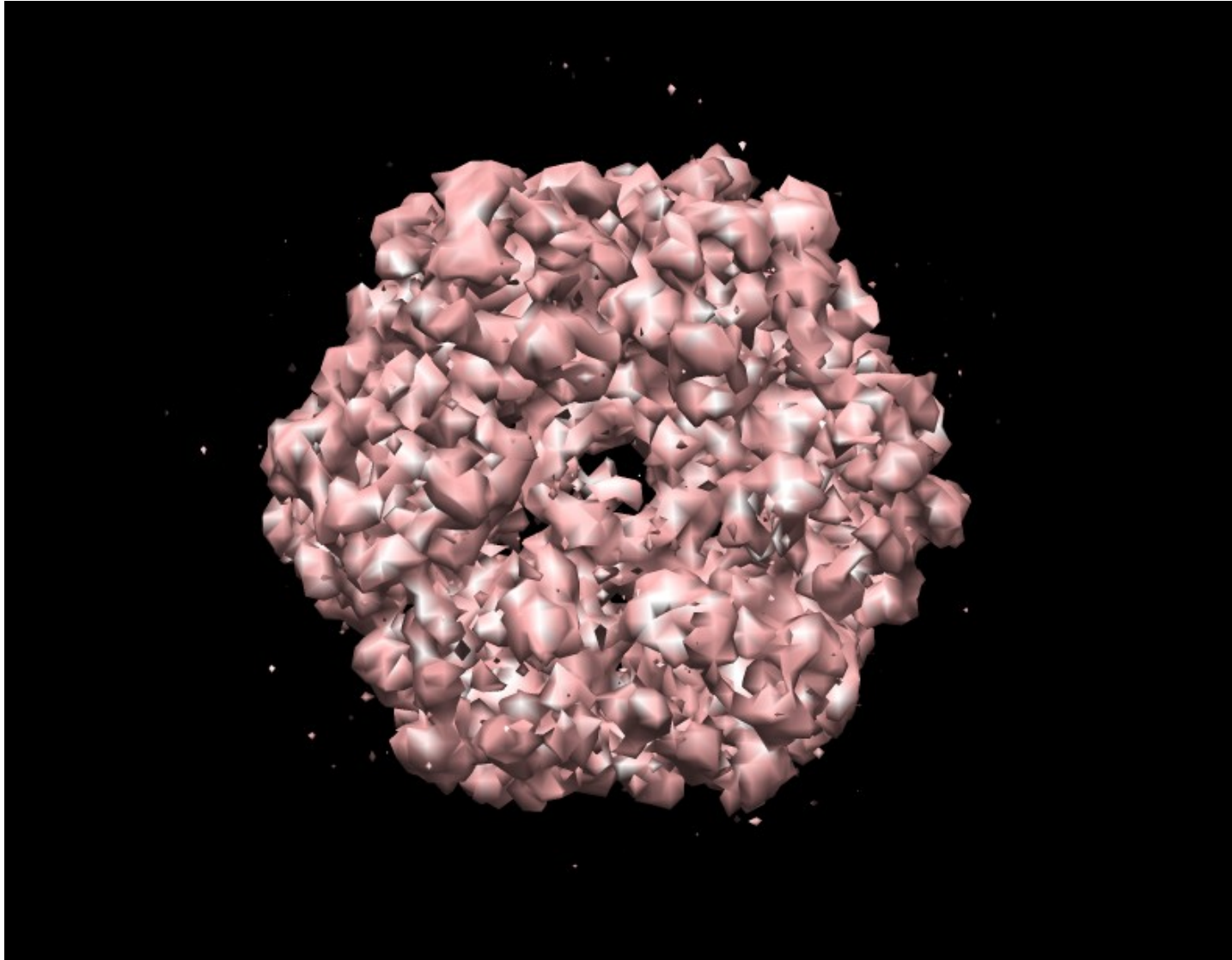


Random-conical tilt: Filling the missing cone Filling the missing cone

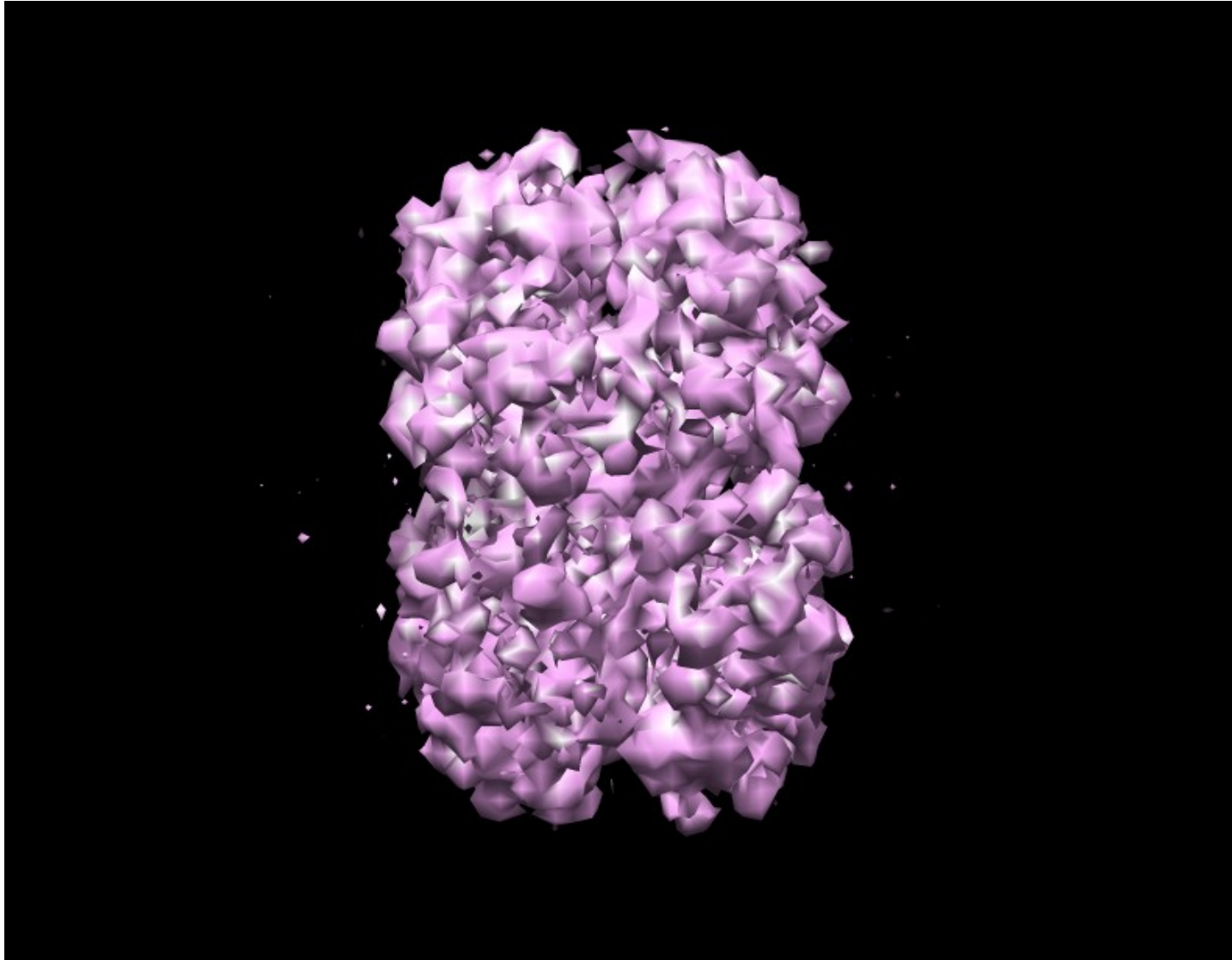
If there are multiple preferred orientations, or if there is symmetry that fills the missing cone, you can cover all orientations.



Top view



Side view



3D classification

Classification:

Multi-reference alignment vs. Maximum likelihood (ML3D)

Multi-reference alignment:

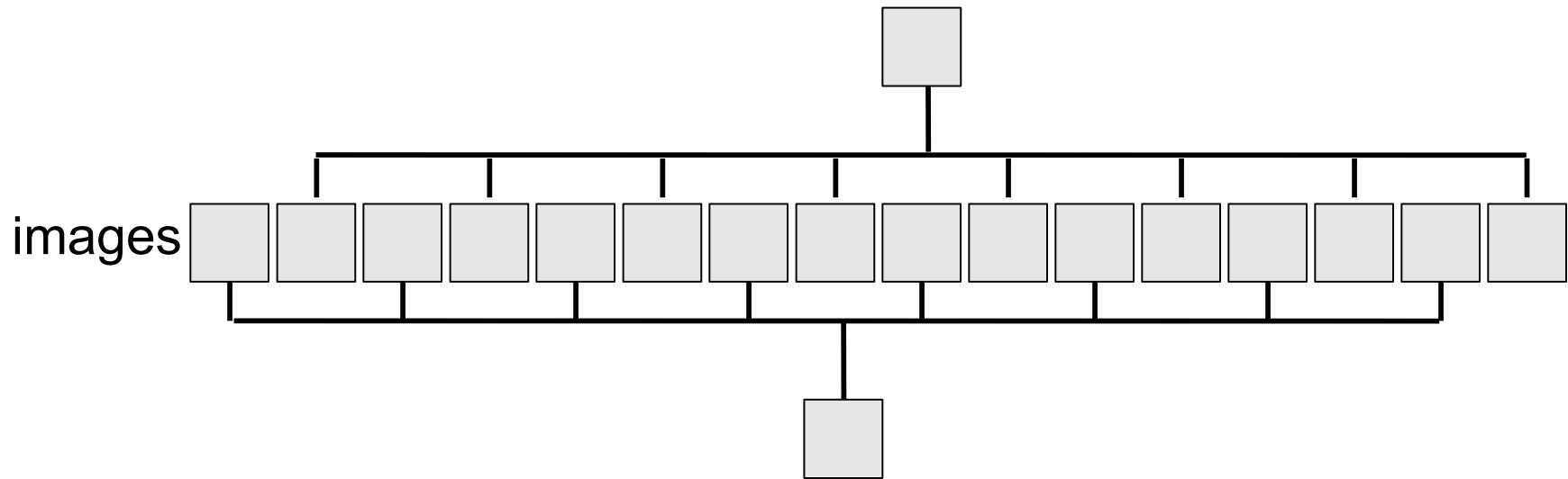
- Possible conformations must be known.
- The combination of parameters (shift, rotation, class) is chosen from the highest correlation value.

ML3D

- Possible conformations are not known.
- The probability of the occurrence of the parameters (shift, rotation, class) is maximized.

Seeding ML3D classification

We split the data set into K classes at random.



There will be slight differences in the reconstructions.
We will iteratively maximize the likelihood of a particle belonging to a particular class.

Thank you for your attention



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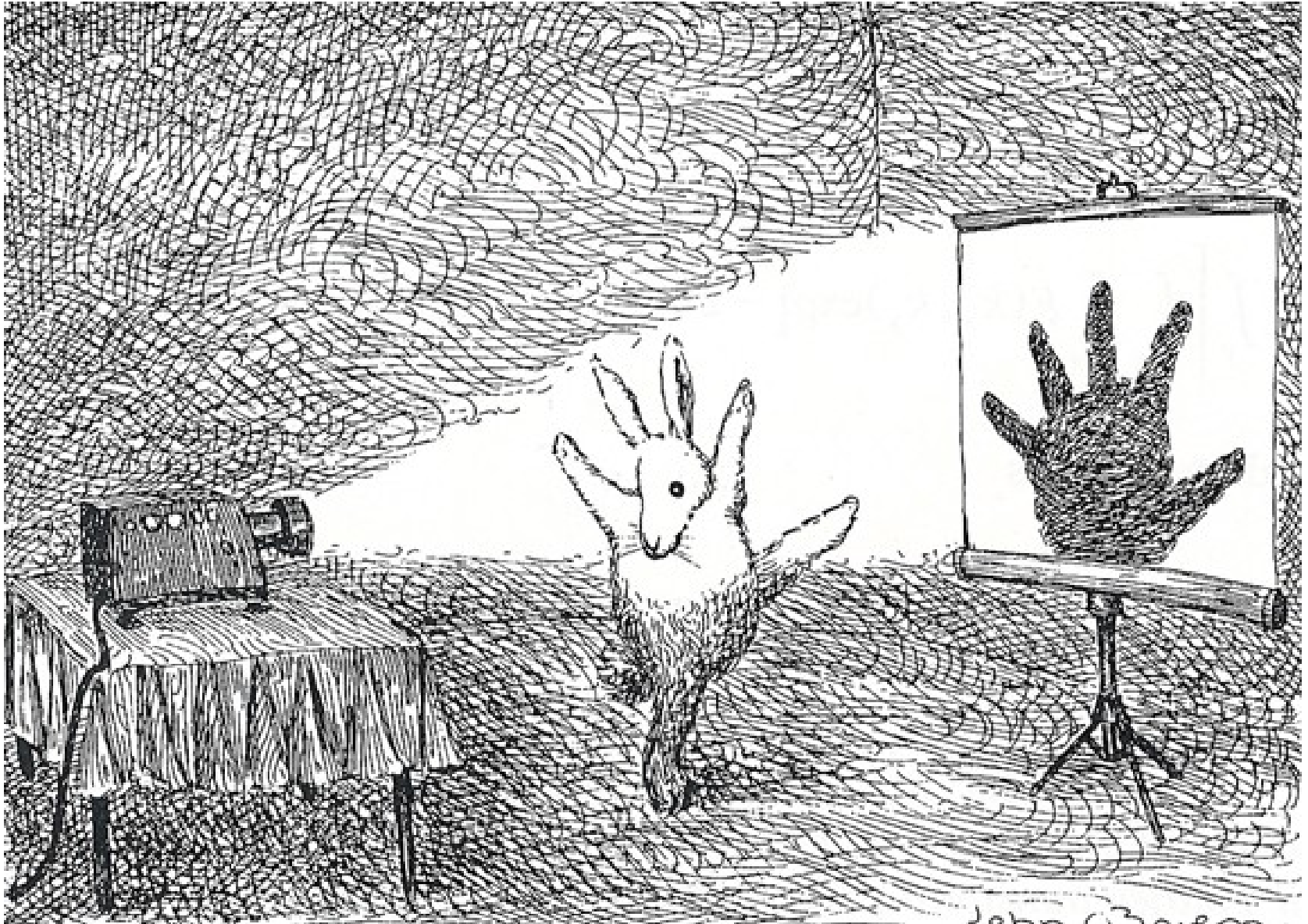
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Development for Innovation



There isn't an unambiguous 3D structure if there's only one



John O'Brien, 1991, *The New Yorker*

What information do we need for 3D reconstruction?

1. different orientations
2. known orientations
3. many particles
4. identical particles