Charged Particle Optics

Radovan Vašina

Application of charged particle optics

Charged particle optics is/was used in the following areas:

- Cathode ray tubes television, oscilloscopes, radars – obsolete
- **Electron and ion microscopes wavelength reduction to enhance resolution**
- Electron and ion lithographes
- Particle accelerators
- Plasma coating, microwave magnetron

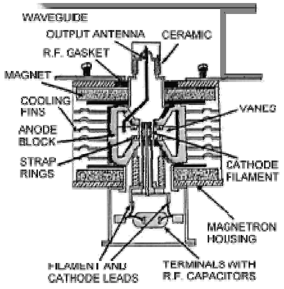


Figure 1 Sectional view of a typical magnetron (Courtesy of Michael S. Wagner)

Comparison: Charged Particle and Light Optics

Feature	Charged Particle Optics	Analogy in Light Optics
Optical elements	Electrostatic and/or magnetic field	Glass and transparent materials
Optical axis must be	Vacuum	Transparent
Lenses	Variable focus	Fixed focus
Focusing	Changing field strength	Moving lens or object Exchange lens
Deflection and scanning	Electrostatic and/or magnetic	Mechanical
Aberrations	Not correctable at round lenses	Easily correctable
Wavelength	~ 2 – 200 pm (electrons)	~ 200 – 1000 nm
Depth of focus	High	Low
Magnification	2x – 1000 000x	1x – 2000x
Maximum resolution	1 nm – 0.1 nm	500 nm

Overview of optical elements

Optical element	Charged particle optics	Analogy in Light Optics
Sources	Hairpin, Schottky emitter, CFEG	Arc lamp, laser, LEDs etc.
„Round“ lenses	Magnetostatic, electrostatic lens	Convergent, divergent lenses
Apertures	Round, annular, arrays	Many types
Deflectors	Magnetostatic, electrostatic	~ Prisms, gratings, mirrors
Multipoles	Magnetostatic, electrostatic	~ Cylindric lenses
Mirrors	Electrostatic	Concave, convex mirrors
Grids	Electrostatic	-
Immersion lens	Immersion lenses	Immersion objectives
Exotic	Wien filter, RF cavities	-

Creation of a charged particle optics device

Mechanical design

Calculation of the fields

Calculations of the beam trajectories and optical properties

Optimization iterations to get the performance

Tolerancing

Models

Controlling software

Basic facts

Electron charge	$e=1.602\text{x}10^{-19}\text{C}$
Electron mass	$m_0=9.109\text{x}10^{-31}\text{kg}$
Proton/electron mass ratio	1826
Relativistic energy	$E = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$
Non-relativistic	$E = \lvert e\Phi \rvert = \frac{1}{2} m v^2$
Electron wavelength	$\lambda = \frac{h}{p} = \frac{h}{\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}}$
Non-relativistically	$\lambda[nm] = \frac{1.2}{\sqrt{\lvert e\Phi \rvert}}$

Device	Energy [keV]	Wavelength [m]
TEM	50 ÷ 1000	$5.46 \div 1.22 \text{ x } 10^{-12}$
SEM	1 ÷ 30	$38.6 \div 7.04 \text{ x } 10^{-12}$

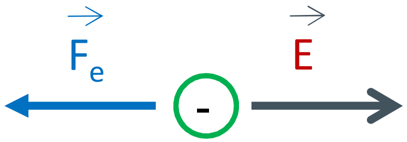
Lorentz force

Force acting on a single particle in a electrostatic and magnetostatic fields

$$\frac{d}{dt}\left(\frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \mathrm{e}\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

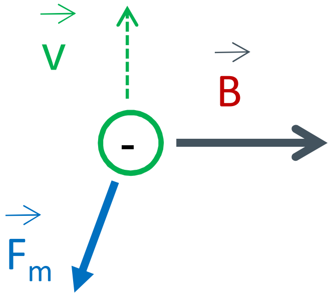
Electro*static* field - conservative

- Acts always on the particle
- Changes the particle energy and velocity
- Usage up to ~30kV
- Coulomb interaction among particles



Magnetostatic field – not conservative

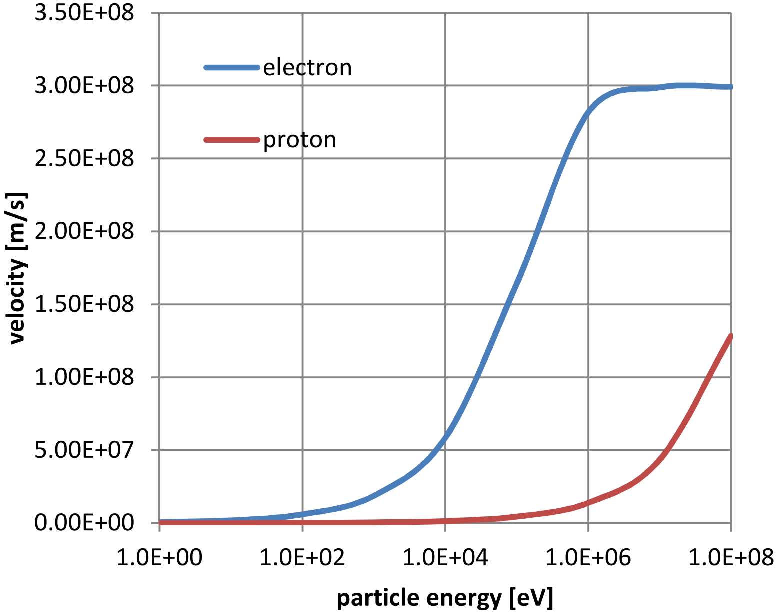
- Acts only on a moving particle
- Does not change energy of the particle
- Usage up to energies of GeV and more

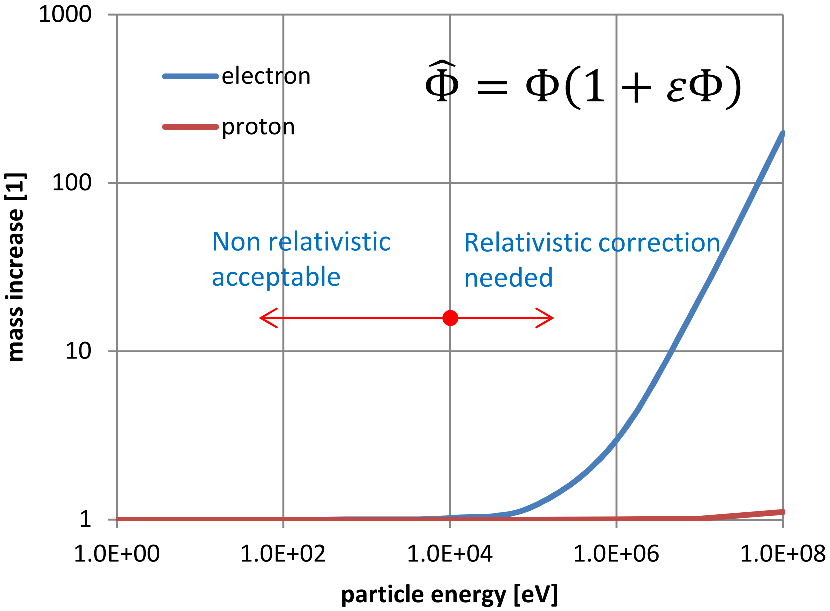


Relativistic effects

The relativistic effects can be neglected at ions

Usage of relativistic potential beneficial for systems with no electrostatic lenses



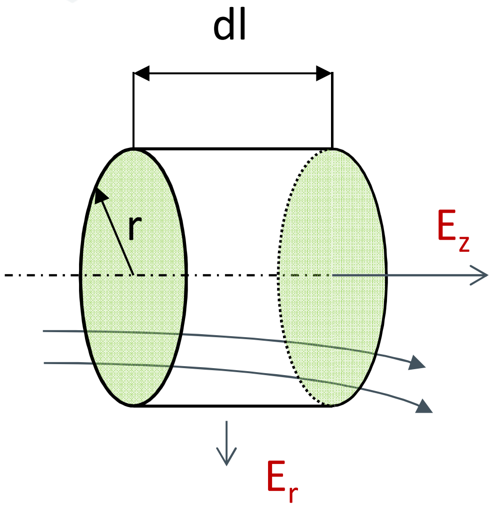


$$\hat{\Phi} = \Phi(1 + \varepsilon\Phi)$$

Non relativistic acceptable

Relativistic correction needed

Paraxial approximation



The paradigm of the charged particle optics

Angles and lateral distances from the axis are small

Energy width is small

All higher orders in the potential and field series are neglected

$$div\, \vec{E} = 0 \qquad \oint d\vec{S}\vec{E} = 0$$

$$2\pi r E_r + \pi r^2 \left(E_z + \frac{\partial E_z}{\partial z} dl \right) - \pi r^2 E_z = 0$$

$$E_r = -\frac{r}{2}\frac{\partial E_z}{\partial z}$$

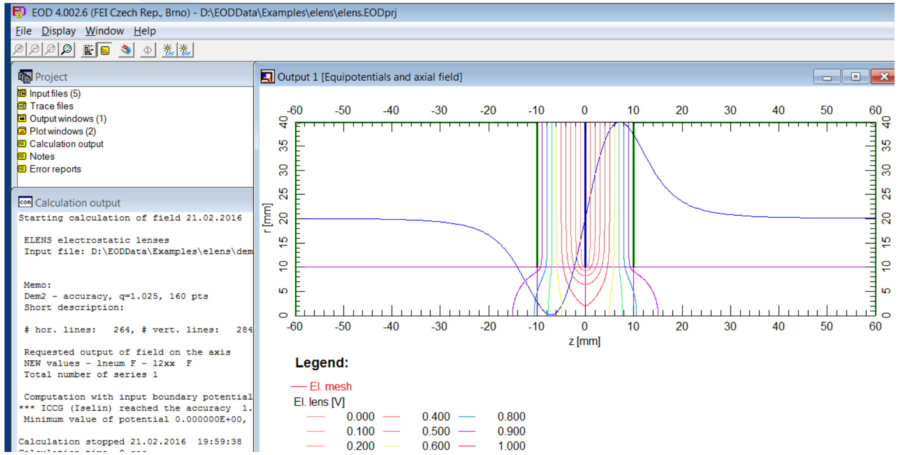
In an analogous way

$$B_r = -\frac{r}{2}\frac{\partial B_z}{\partial z}$$

Linearity of the field only near to the optical axis

Determination of the field (in cylindrical CS)

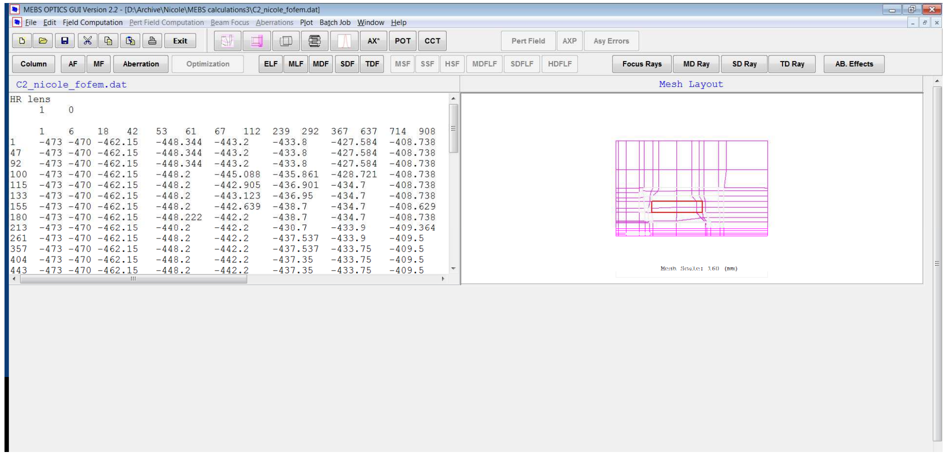
EOD J.Zlámal&B.Lencová



```
Starting calculation of field 21.02.2016

 ELENS electrostatic lenses
 Input file: D:\EODData\Examples\elens\dem


 Memo:
 Dem2 - accuracy, q=1.025, 160 pts
 Short description:

 # hor. lines:   264, # vert. lines:   284

 Requested output of field on the axis
 NEW values - lneum F - 12xx  F
 Total number of series 1

 Computation with input boundary potential
*** ICCG (Iselin) reached the accuracy  1.
 Minimum value of potential 0.000000E+00,

Calculation stopped 21.02.2016  19:59:38
```

FEM with linear elements

MEBS E.Munro&J.Rouse



```
HR lens
     1     0


     1     6    18    42    53    61    67   112   239   292   367   637   714   908
1    -473 -470 -462.15    -448.344   -443.2     -433.8     -427.584   -408.738
47   -473 -470 -462.15    -448.344   -443.2     -433.8     -427.584   -408.738
92   -473 -470 -462.15    -448.344   -443.2     -433.8     -427.584   -408.738
100  -473 -470 -462.15    -448.2     -445.088   -435.861   -428.721   -408.738
115  -473 -470 -462.15    -448.2     -442.905   -436.901   -434.7     -408.738
133  -473 -470 -462.15    -448.2     -443.123   -436.95    -434.7     -408.738
155  -473 -470 -462.15    -448.2     -442.639   -438.7     -434.7     -408.629
180  -473 -470 -462.15    -448.222   -442.2     -438.7     -434.7     -408.738
213  -473 -470 -462.15    -448.2     -442.2     -438.7     -433.9     -409.364
261  -473 -470 -462.15    -448.2     -442.2     -437.537   -433.9     -409.5
357  -473 -470 -462.15    -448.2     -442.2     -437.537   -433.75    -409.5
404  -473 -470 -462.15    -448.2     -442.2     -437.35    -433.75    -409.5
443  -473 -470 -462.15    -448.2     -442.2     -437.35    -433.75    -409.5
```

FEM with quadratic elements

Finite difference methods   Simion

FEM 3D Ansys, Comsol, CST

Boundary element methods

Symmetry of the fields and OEs

Symmetry $\boldsymbol{n}$ of the field with optical elements: $\boldsymbol{2n}$ multipoles

n = 0 Round lenses - focusing

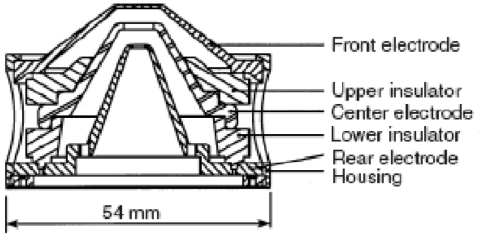
n = 1 Dipoles – deflectors

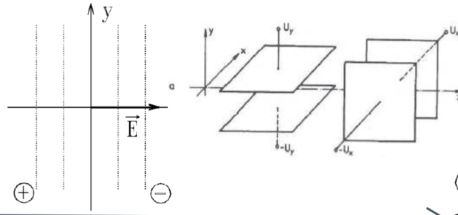
n = 2 Quadrupoles - stigmators

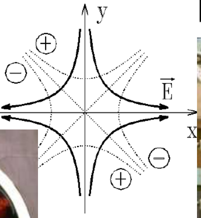
n = 3 Hexapoles - correctors

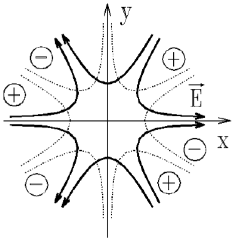
n = 4 Octopoles – universal $n \in \langle 1,4 \rangle$

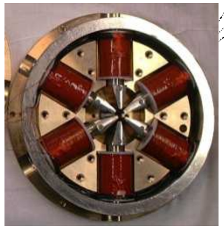
n > 4 Multipoles - correctors

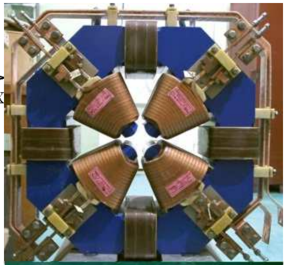






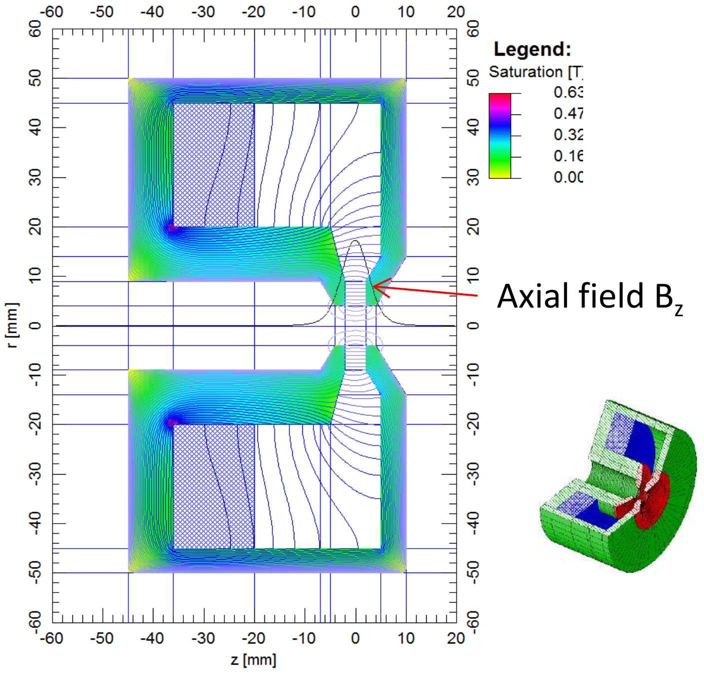




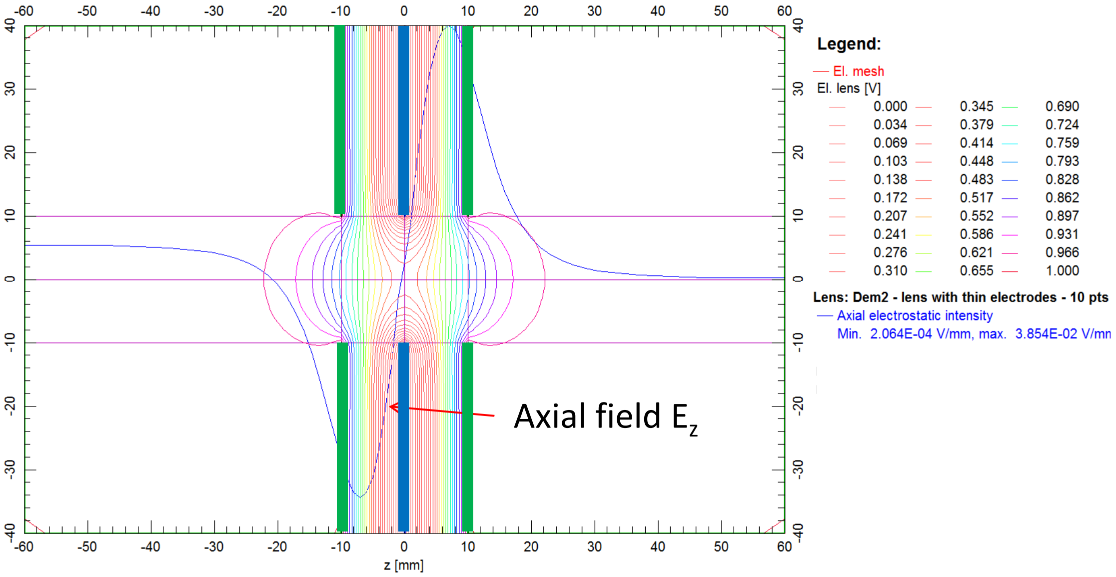


Axisymmetrical fields (*n*=0)

Magnetic field of a round lens

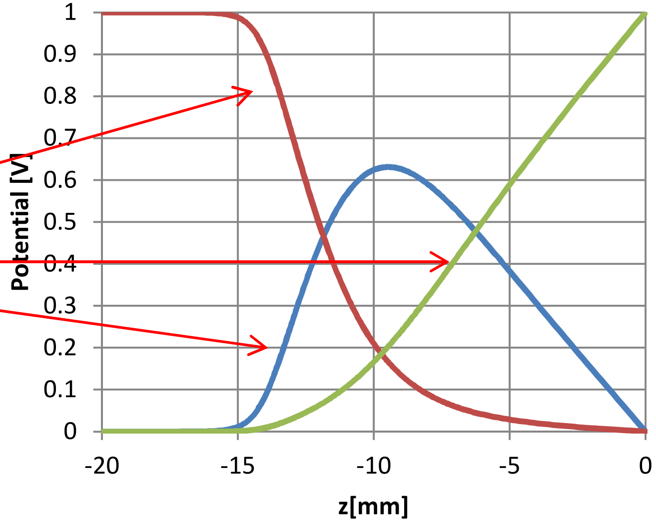


Electrostatic field of a round unipotential lens



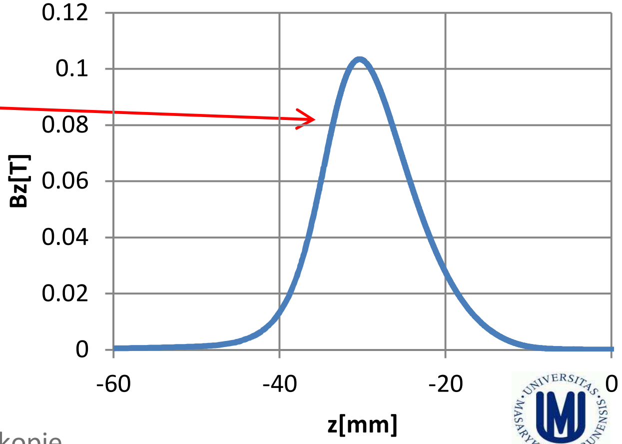
Axial fields

Total electrostatic field can summed up as a weighted sum of the contribution of particular electrodes
Assigning the voltages to the respective electrodes



- Magnetostatic field
  - Axial field
- Deflector magnetostatic field
  - Axial field

Assigning the currents to the respective coils calculated with excitation 1A*Number of turns



Equation of motion

Time dependent

$$m\vec{a} = \mathrm{q}\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$m_0 \frac{d}{dt}\left(\frac{\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}}\right) = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Relativistically corrected potential for electrons

$$\widehat{\Phi} = \Phi(1 + \varepsilon\Phi) \qquad \varepsilon = \frac{|e|}{2mc^2} \qquad \eta = \sqrt{\frac{|e|}{2m}}$$

Time independent

$$\frac{d}{dz}\left[\sqrt{\frac{\widehat{\Phi}}{1 + x'^2 + y'^2}}\,x'\right] = -\left(\frac{1}{2} + \varepsilon\Phi\right)\sqrt{\frac{1 + x'^2 + y'^2}{\widehat{\Phi}}}\,E_x + \eta\left(B_y - y'B_z\right)$$

$$\frac{d}{dz}\left[\sqrt{\frac{\widehat{\Phi}}{1 + x'^2 + y'^2}}\,y'\right] = -\left(\frac{1}{2} + \varepsilon\Phi\right)\sqrt{\frac{1 + x'^2 + y'^2}{\widehat{\Phi}}}\,E_y + \eta(-B_x + x'B_z)$$

Integration methods

The initial value problem $\dot{y} = f(t, y)\ y(t_0) = y_0$

Runge Kutta

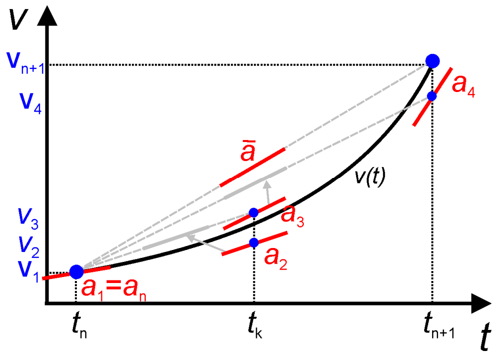
$$k_{1i} = h\, f(t_n, y_{ni})$$

$$k_{2i} = h\, f(t_n + a_2 h, y_{ni} + b_{21} k_{1i})$$

$$\ldots$$

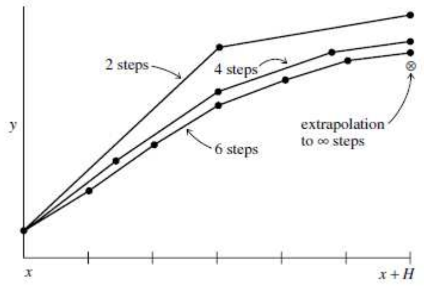
$$k_{6i} = h\, f(t_n + a_6 h, y_{ni} + b_{61} k_{1i} + \ldots + b_{65} k_{5i})$$

$$y_{(n+1)i} = y_{ni} + c_1 k_{1i} + c_2 k_{2i} + c_3 k_{3i} + c_4 k_{4i} + c_5 k_{5i} + c_6 k_{6i} + O(h^6)$$



Bulirsch Stoer

- Richardson extrapolation $h \to 0$
- Using rational functions for extrapolation
- Error function of the integrator dependent on h^2



Interpolation

A general problem: The value of the field known only at discreete points (FEM not BEM). The integration routine needs to obtain the accurate field values at a general place.

Interpolation of the **axial** fields: paraxial approximation

- Cubic or quintic splines: cubic or quintic polynomials with continuous 2nd or 4th derivative
- Fourier Bessel series

$$\boxed{B_z = \sum_{m=1}^{M} C_m \sin\left(\frac{m\pi z}{L}\right) I_0\left(\frac{m\pi r}{L}\right),} \qquad \boxed{B_r = -\sum_{m=1}^{M} C_m \cos\left(\frac{m\pi z}{L}\right) I_1\left(\frac{m\pi r}{L}\right)}$$

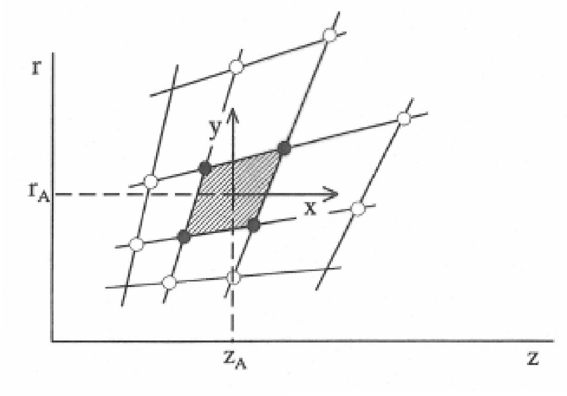
- Hermite series

$$h_0(x) = \pi^{-0.25} \exp\left(-\tfrac{1}{2}x^2\right)$$

$$h_1(x) = \sqrt{2}\, x h_0(x)$$

$$h_i(x) = \left\{\sqrt{2}\, x h_{i-1}(x) - \sqrt{i-1}\, h_{i-2}(x)\right\} / \sqrt{i}$$

Interpolation



Interpolation of **general** data:

- Bicubic spline
- ZRP method – interpolation using Laplacian base functions (author J.Chmelík)

Using a local coordinate system

$$x = z - z_A, \qquad y = r - r_a$$

$$\Phi(x,y) = \sum_{i=1}^{N} C_i g_i(x,y)$$

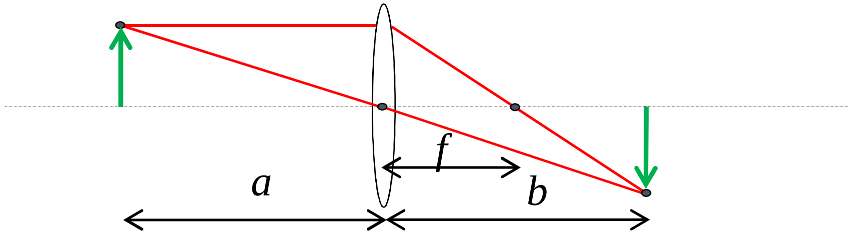
$$\begin{aligned}
g_1 &= 1\\
g_2 &= x\\
g_3 &= -2r_A y + (2x^2-y^2)\\
g_4 &= -6r_A xy + x(2x^2-3y^2)\\
g_5 &= -12r_A^2(x^2-y^2) + 12r_A y(-4x^2+y^2) + (8x^4-24x^2y^2+3y^4)\\
g_6 &= -20r_A^2(x^3-3xy^2) + 20r_A xy(-4x^2+3y^2) + x(8x^4-40x^2y^2+15y^4)\\
g_7 &= -40r_A^3(y^3-3yx^2) + 20r_A^2(-4x^4+21x^2y^2-3y^4) +\\
&\quad + 30r_A y(-8x^4+12x^2y^2-y^4) + (16x^6-120x^4y^2+90x^2y^4-5y^6).
\end{aligned}$$

where $g_i(x,y)$ fulfill Laplace equation. The coeficients are fitted from the 4-8 nearest neighbor points with Singular Value Decomposition method. Weight of the points depends on the distance from the element center

Analogy with light optics

Basic phenomena of light optics valid in the charged particle optics

Thin lens approximation



$$\frac{1}{f} = \frac{1}{a} + \frac{1}{b}$$

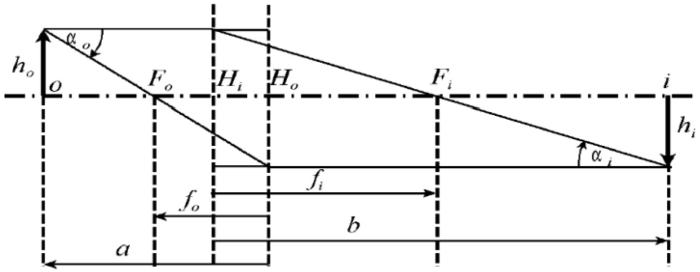
But they are thick lenses

two main planes

in case of immersion lenses two focal lenghts

direct magnification $M = \frac{h_i}{h_o}$

angular magnification $M_\alpha = \frac{\alpha_i}{\alpha_o}$



Solution of the equation of motion

Ordinary differential equation of the second order

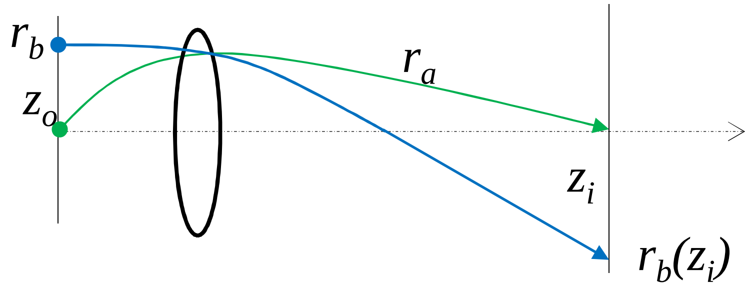
2 independent solutions for the initial conditions

$$r_a(z_o) = 0, r'_a(z_o) = 1 \quad \Rightarrow \quad r_a(z_i) = 0, \quad r'_a(z_i) = \boxed{M_a}$$

$$r_b(z_o) = 1, r'_b(z_o) = 0 \quad \Rightarrow \quad r_b(z_i) = \boxed{M} \quad r'_b(z_i) = \boxed{-\frac{1}{f_i}}$$

Relationship between direct and angular magnification

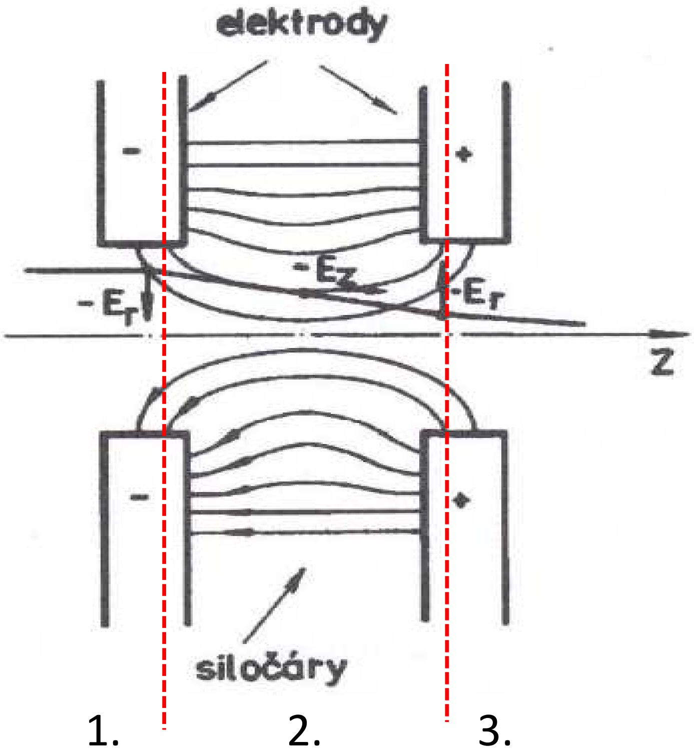
$$\frac{\hat{\Phi}^{1/2}(z_i)}{\hat{\Phi}^{1/2}(z_o)} M M_a = 1$$



Electrostatic lens: how it focuses

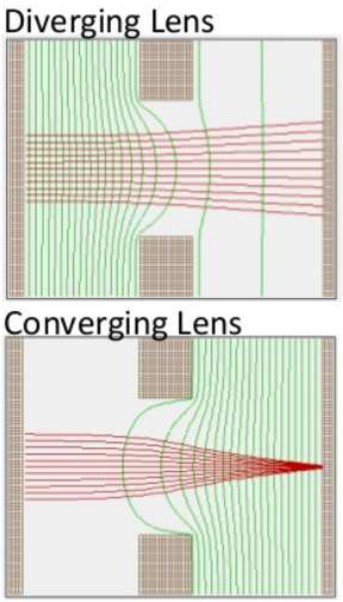
1. Intensity E_r changes quickly, change of the radial velocity
2. Intensity E_z is constant, change of axial velocity
3. Smaller intensity -E_r change of the radial velocity

Axial symmetry of the fringe field causes linear focusing effect

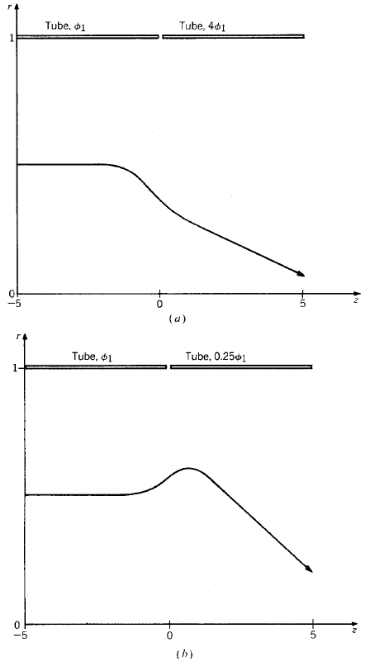


Electrostatic lenses

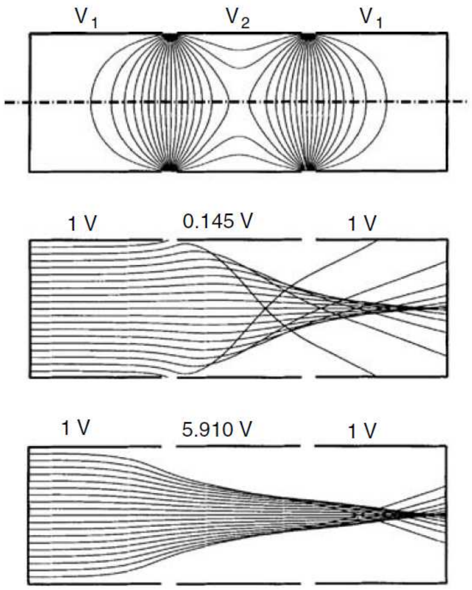
1 electrode lenses:
aperture lens



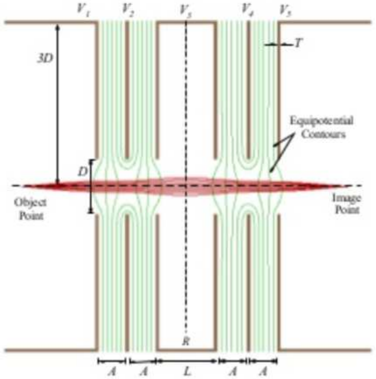
2 electrode lenses:
accelerating, decelerating



3 electrode lenses:
unipotential



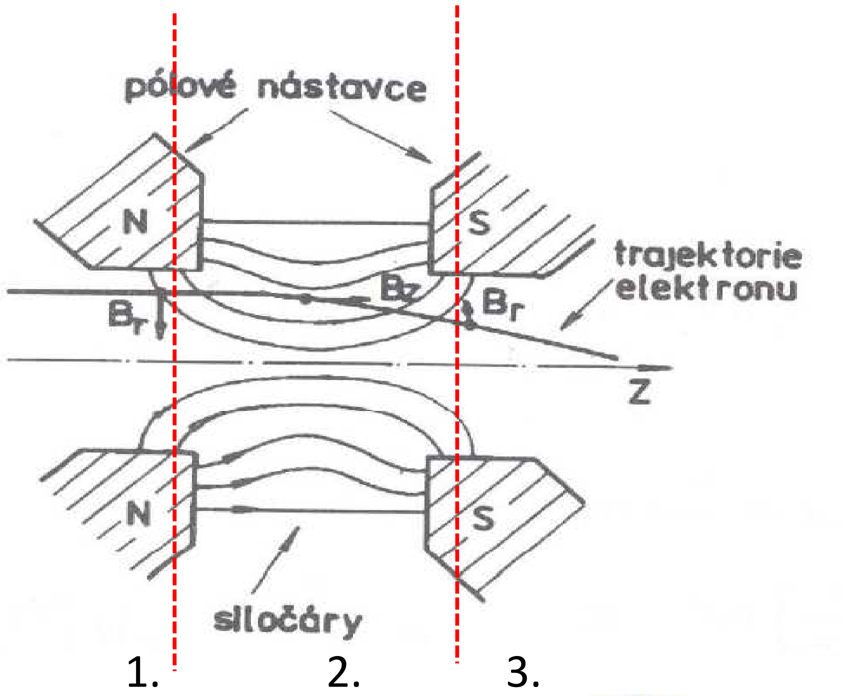
4 and more
zoom lenses



Magnetostatic lens: how it focuses

1. Magnetic induction B_r changes quickly, a change of the tangential velocity
2. Magnetic induction B_z is constant, change of radial velocity
3. Magnetic induction B_r changes quickly, the tangential velocity stops

Axial symmetry of the fringe field causes linear focusing effect



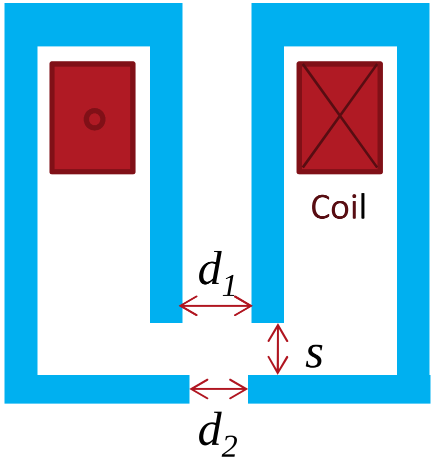
Properties of magnetostatic lenses

First order properties

$$f = \frac{(s+d)k}{10\,P\,(1-P)} \qquad P = \frac{(NI)^2}{300\,\hat{\Phi}} \qquad d = \frac{d_1 + d_2}{2}$$

Excitation parameter

$exc = \frac{NI}{\sqrt{\hat{\Phi}}}$ scaling laws → trajectories for same exc same



Rotation

$$\theta = \sqrt{\frac{|e|}{2m}} \int \frac{B_z(z)}{\sqrt{\hat{\Phi}}} dz$$

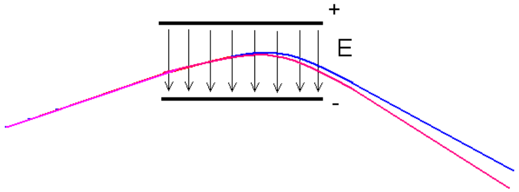
Axial aberrations

$$C_{si} \propto \frac{(1+|M|^4)f^3}{(s+d)d}$$

$$C_{ci} \propto (1+|M|^2)\mathrm{f}$$

Properties of deflectors

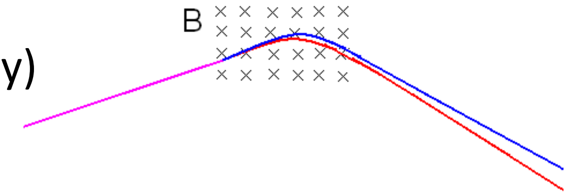
Electrostatic (weak) perpendicular field: (parabolic trajectory)



Derivative of the movement $$\frac{dx}{dz} = \frac{E_x l}{2\Phi}$$

Deflection does not depend mass and charge – usage at focusing elements ion devices FIB, SIMS

Magnetostatic (weak) perpendicular field: (circular trajectory)



Derivative of the movement $$\frac{dy}{dz} = \frac{l}{R} = \frac{B_x}{\sqrt{\frac{2m\Phi}{e}}}$$

Deflection does depend on the mass and charge – filtering elements at ion devices
Weaker dependency on the particle energy – usage of transversal fields at accelerators

Deflectors

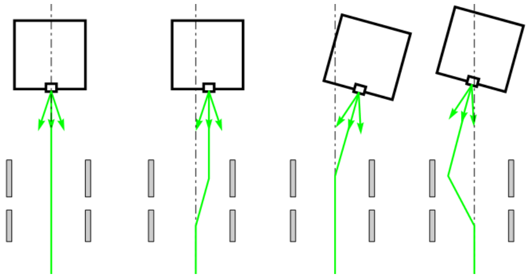
A pair of deflectors can manipulate with the beam

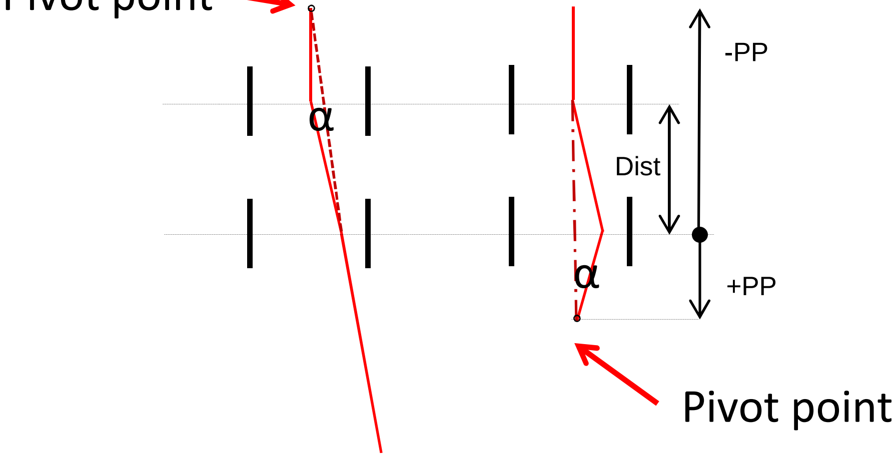
In the lateral position and angularly

$$Exc_{upper} \approx -\frac{\alpha.PP}{Dist}$$

$$Exc_{lower} \approx \alpha\left(1+\frac{PP}{Dist}\right)$$

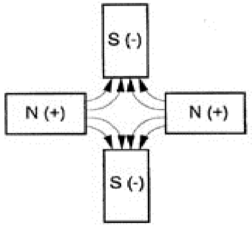
Pivot point is the position in the column where the beam rocks about

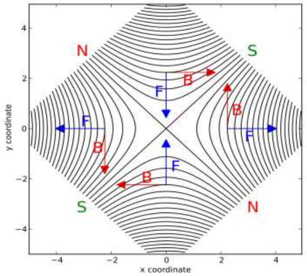




Properties of quadrupoles

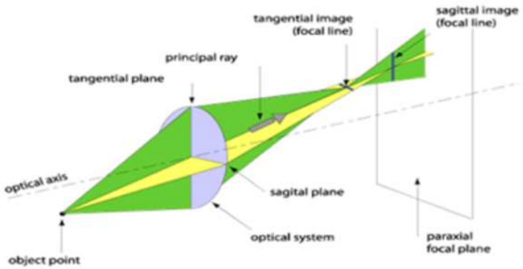
Quadrupole multipole with n=2



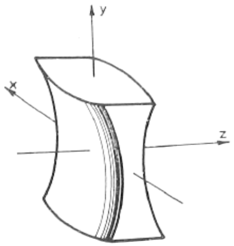


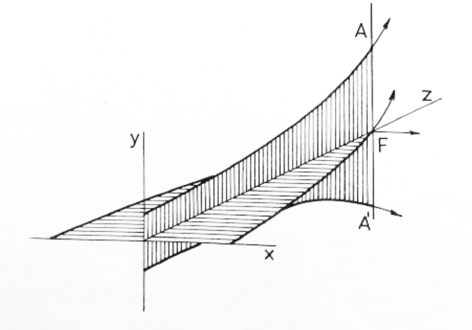
Forces on the charged particle
Fx independent of y position and proportional to x position
Fy independent of x position and proportional to y position
Field/forces on the axis is/are zero

Usage as stigmator



Light



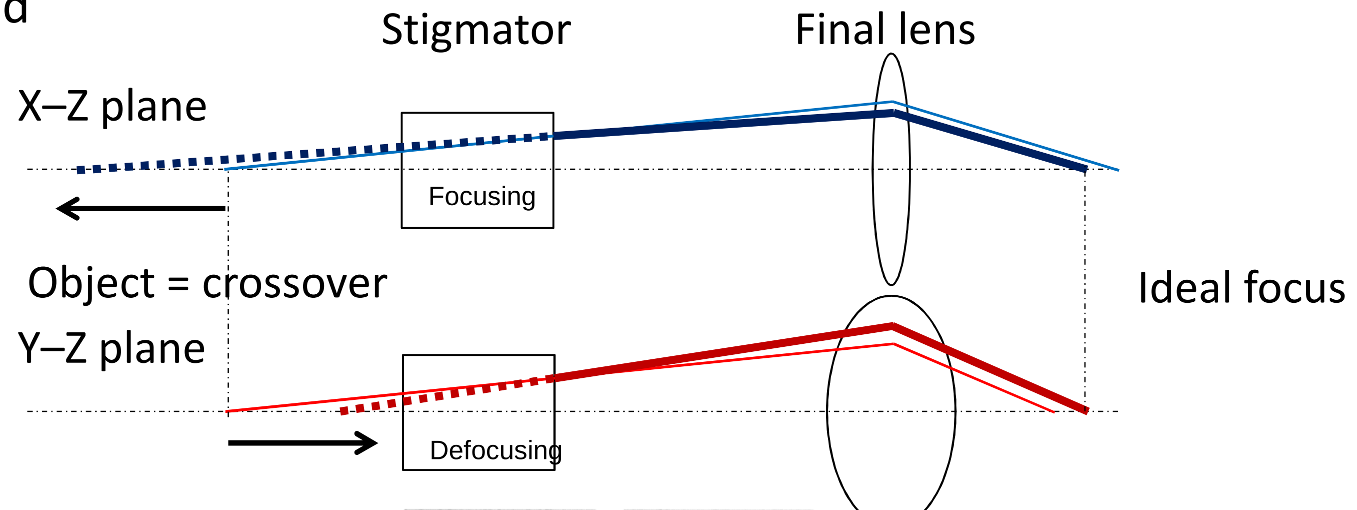


How stigmation works

The final lens is not perfect: the focusing ability is varies along the azimuthal angle with a π period

$$\frac{1}{f_x} = \sqrt{\kappa}\,\sin\sqrt{\kappa}\,L$$

$$\frac{1}{f_y} = -\sqrt{\kappa}\,\sinh\sqrt{\kappa}\,L$$


Stigmator

Final lens

X–Z plane

Focusing

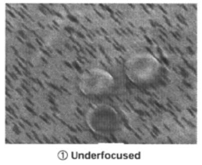
Object = crossover

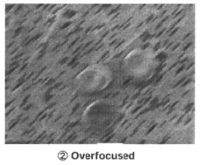
Ideal focus

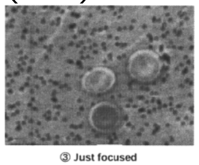
Y–Z plane

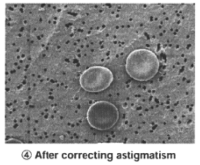
Defocusing

The stigmator shifts the position of the virtual object differently in the orthogonal planes,
depending linearly in each plane

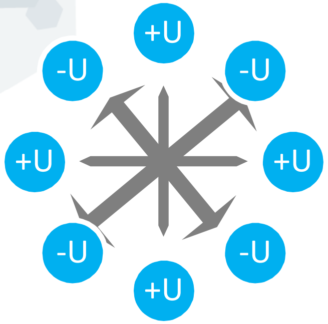

① Underfocused


② Overfocused

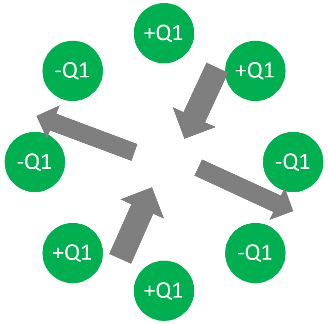

③ Just focused


④ After correcting astigmatism

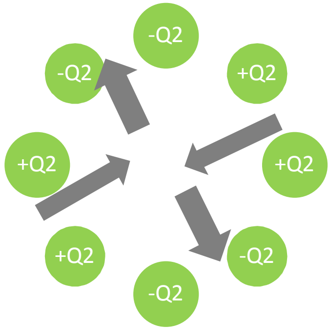
Multipoles



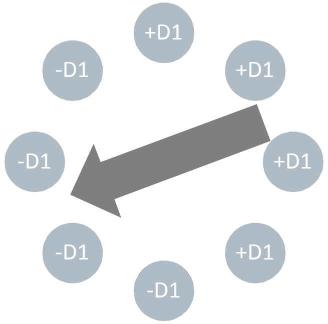
octupole



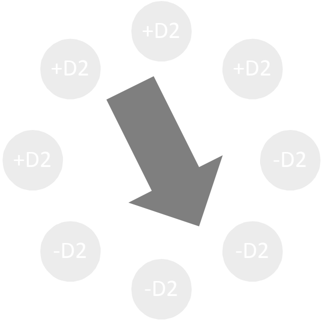
quadrupole 1



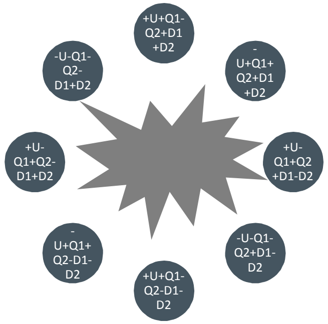
quadrupole 2



dipole 1



dipole 2



Superposition of all

Scaling with Accelerating Voltage

- Equal trajectories if $\frac{E_z}{\Phi} = const$
- Equal trajectories if $\frac{B_z}{\sqrt{\hat{\Phi}}} = const$
- Particle mass depends on Φ! Use relativistic corrected $\hat{\Phi}$

Consequences:

- Electrostatic field does not and magnetic field does split particles according mass and charge
- Permanent magnet effect $\propto \frac{1}{\sqrt{\hat{\Phi}}}$
- Charging effects $\propto \frac{1}{\Phi}$
- Geometrical misalignment effects do not depend on Φ
- Magnetic saturation effects at high Φ

Aberrations

Aberrations cause blurring of an point source image

-Diffraction aberration due to wave character of particles

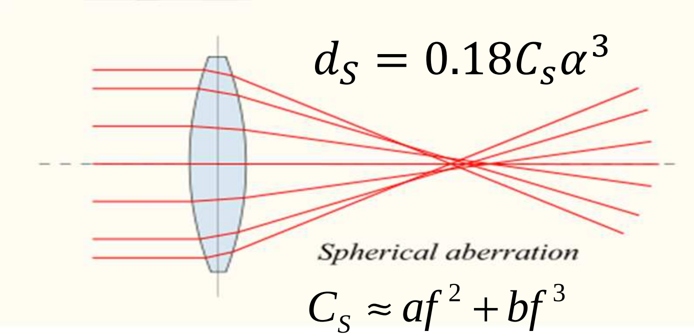
-Geometrical aberrations describing deviation from the linear behavior

-Chromatic aberrations due to finite energy width of the beam

-Parasitic aberrations … stemming from the imperfection in

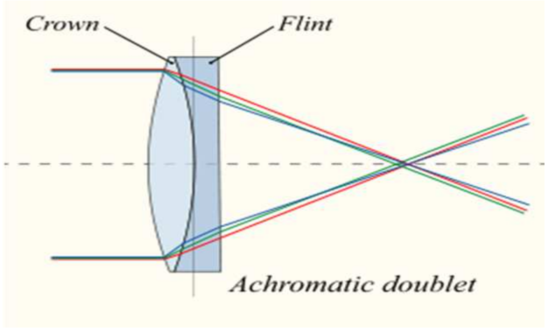
    manufacturing
    bad alignment
    instabilities of HT and power supplies
    thermal drift of the column assemblies
    lack of homogeneity of the lens materials

Axial aberrations

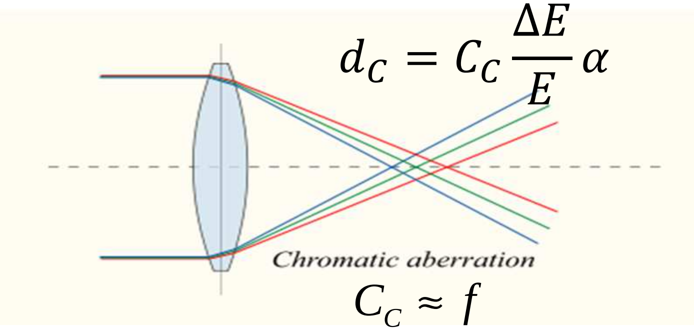


$$d_S = 0.18 C_s \alpha^3$$

$$C_S \approx af^2 + bf^3$$



Correction not possible in
charged particle optics
in such an easy way



$$d_C = C_C \frac{\Delta E}{E} \alpha$$

$$C_C \approx f$$

Scherzer’s theorem (1936)

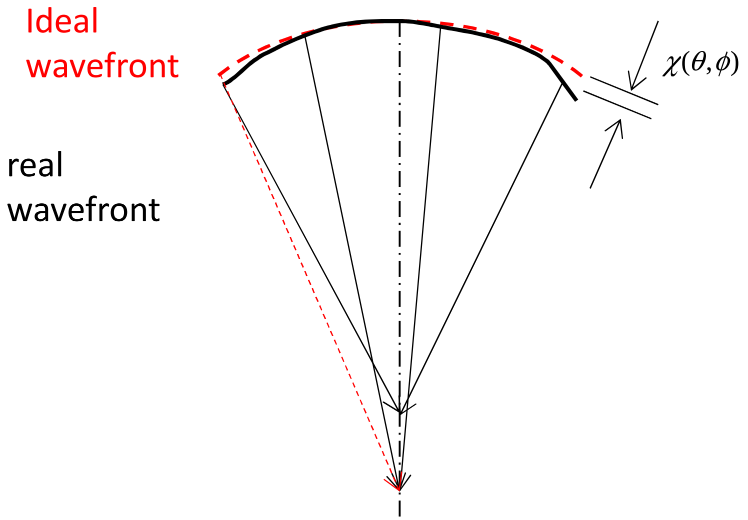
Electromagnetic lenses have unavoidable aberrations (spherical and chromatic) as long as the following conditions are fulfilled:

- Lens fields are rotationally symmetric → multipoles
- The electromagnetic fields are static → RF operation
- There is no space charges → electron mirrors

Aberrations

Deviation of the real wavefront from the ideal one

Aberration function
With the length dimension [m, µm, …]
Dependent on the radial angle θ and azimuthal angle ϕ



Precision needed λ/4 ~ 1pm
Relative precision ~ deviation of the wavefront / diameter of the wavefront in the objective …i.e. 1pm/100µm = $10^{-12}/10^{-4} = 10^{-8}$
Comparable with modern astronomical telescopes

Aberrations

Representation of geometrical aberration function Křivánek, Delby & Lupini

$$\chi(\theta,\phi)=\sum_{n}\sum_{m}\left\{C_{n,m,a}\theta^{n+1}\cos(m\phi)+C_{n,m,b}\theta^{n+1}\sin(m\phi)\right\}/(n+1)$$

Where

θ …semiangle $\in$ 0 ÷ ~50 mrad

Φ …azimuthal angle $\in$ 0 ÷ 2π

n …order $\in$ 0 ÷ 7

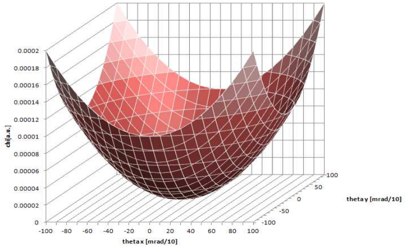
m …multiplicity $\in$ 0 ÷ 6

For odd n even $m = 0, 2, 4 \ldots n+1$ e.g. $C_{1,0}$; $C_{1,2}$; $C_{3,0}$;$C_{5,2}$

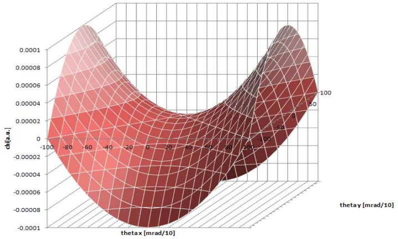
For even n odd $m = 1, 3, 5 \ldots n+1$ e.g. $C_{0,1}$; $C_{2,1}$;$C_{2,3}$;$C_{4,5}$

For $m = 0$ no azimuthal dependence is no term $C_{n,0,b}$ e.g. $C_{3,0}$

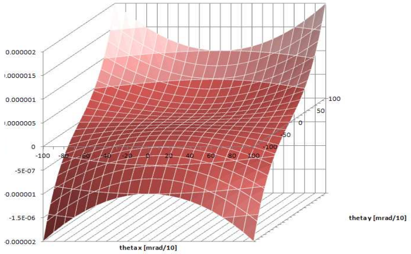
Wavefront errors



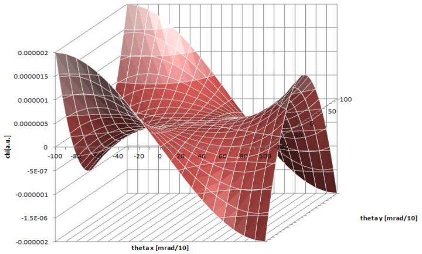
Defocus $C_{1,0}$



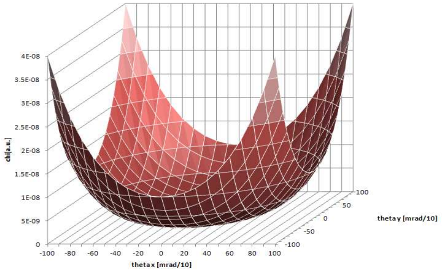
Astigmatism $C_{1,2,a}$



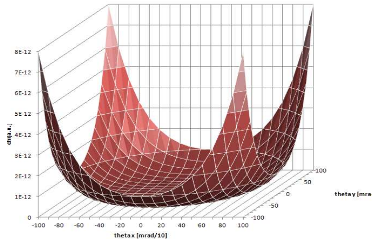
Axial coma $C_{2,1,b}$



3-f Astigmatism $C_{2,3,a}$



Spherical aberr.$C_{3,0}$

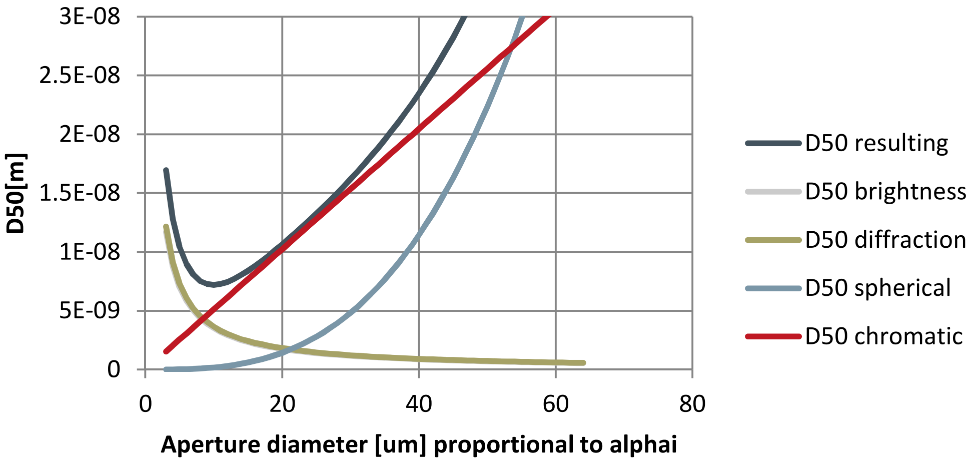


Spherical aberr.$C_{5,0}$

Resolution at scanning probe systems

Resolution depends on:

- Spherical axial aberration
- Chromatic axial aberration
- Diffraction aberration
- Spot size influence by the brightness
- Coulomb interactions
- Many other effects…noise, vibrations, stray magnetic field

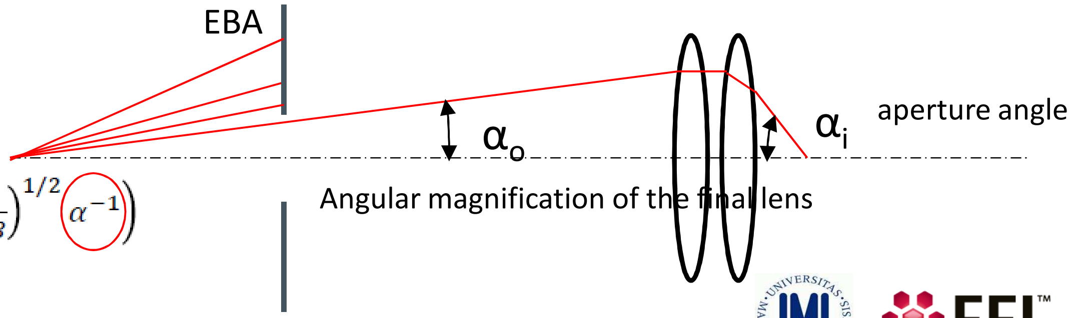


$$d^2 = \left[\left(d_S^4 + d_D^4\right)^{\frac{1.3}{4}} + d_G^{1.3}\right]^{2/1.3} + d_C^2$$

$$d_S = K_S C_S \alpha^3,\ d_C = K_C C_C \frac{d\phi}{\phi}\alpha,\ d_D = K_D \lambda \alpha^{-1},\ d_G = M d_0$$

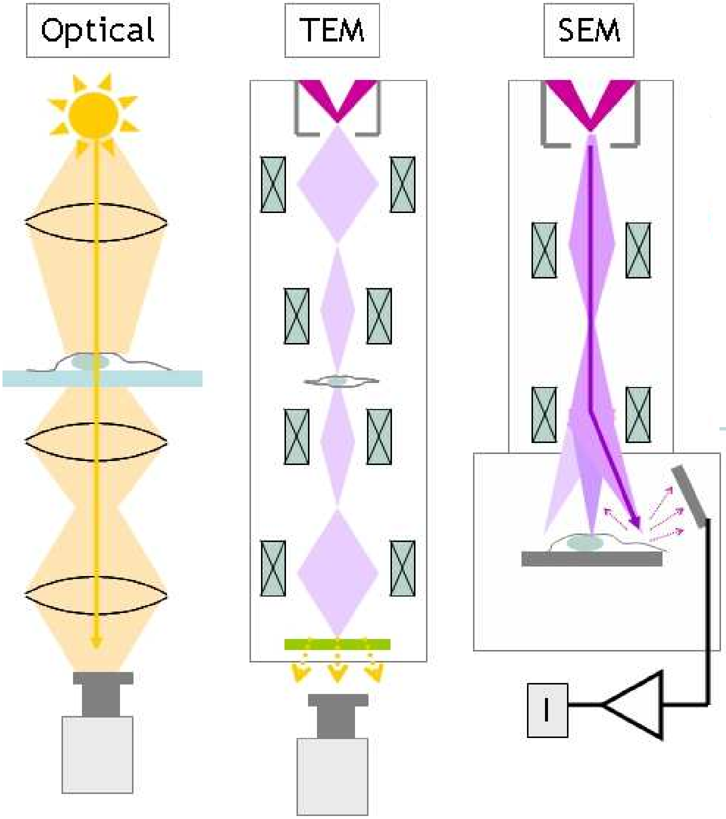
$$K_S = 0.18,\ K_C = 0.34,\ K_D = 0.54$$

$$\left(d_G = \left(\frac{4I_p}{pi^2\beta}\right)^{1/2}\alpha^{-1}\right)$$



Angular magnification of the final lens

Image creation



Full field systems

Advantages:

- traditional approach,
- image instantaneously available
- high resolution due to optics

Disadvantages:

- more complex, more expensive
- pixelated detectors
- complicated specimen preparation

Scanning probe systems:

Advantages:

- simpler, cheaper, for any specimens
- integral detectors for various signals

Disadvantages:

- image not present at one instant
- resolution due to scanning

State of the art SEM: general specifications

Parameter	Value(s)	
Landing energy	20 ÷ 30000 eV	*~3 orders of magnitude*
Probe current	0.78 pA ÷ 410 nA	*~6 orders of magnitude*
Maximum/minimum field of view	3 mm / 50 nm	*~6 orders of magnitude*
Resolution	<1 nm	
Beam current control	Continuous	
Vacuum modes	Hi Vac (~10^{-4}Pa), Low Vac (10-50 Pa)	

Scanning electron microscope

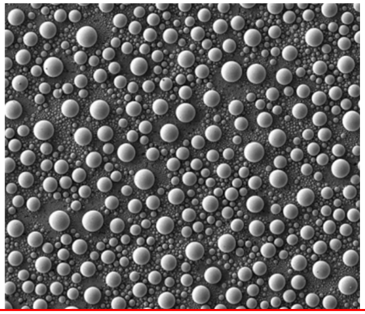
Different modes

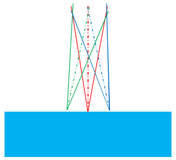
Full frame, Line, Spot, Pattern

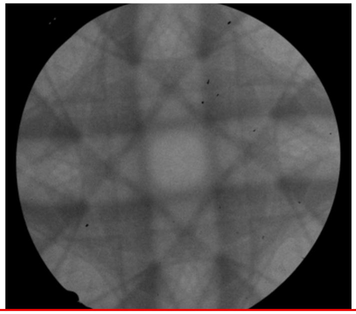
Channeling contrast

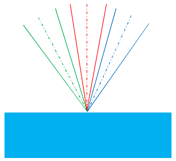
STEM

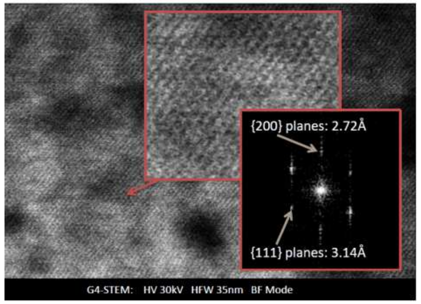
Imaging

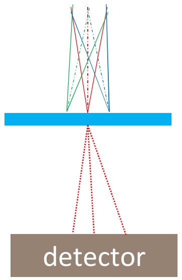






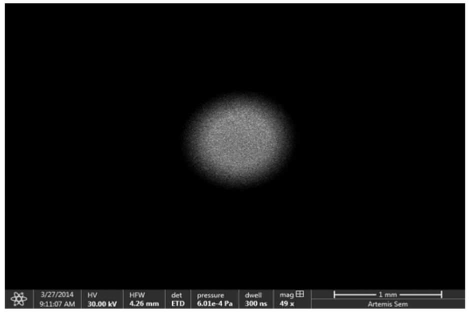


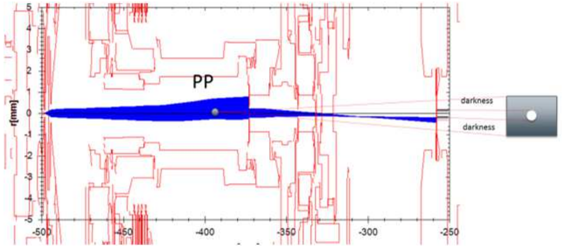




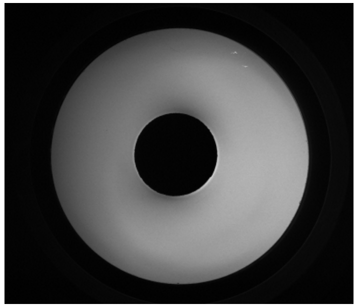
Diagnostics

Crossover

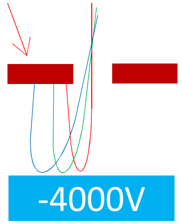




Mirror

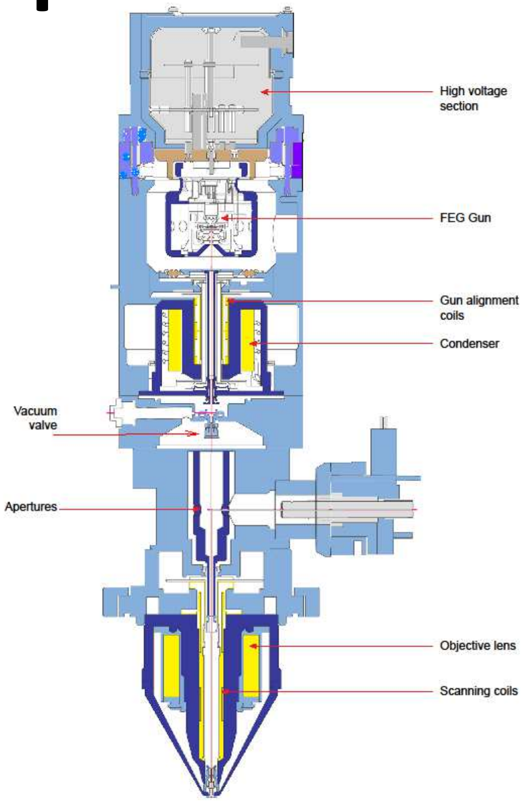


Detector +8kV



Scanning electron microscope

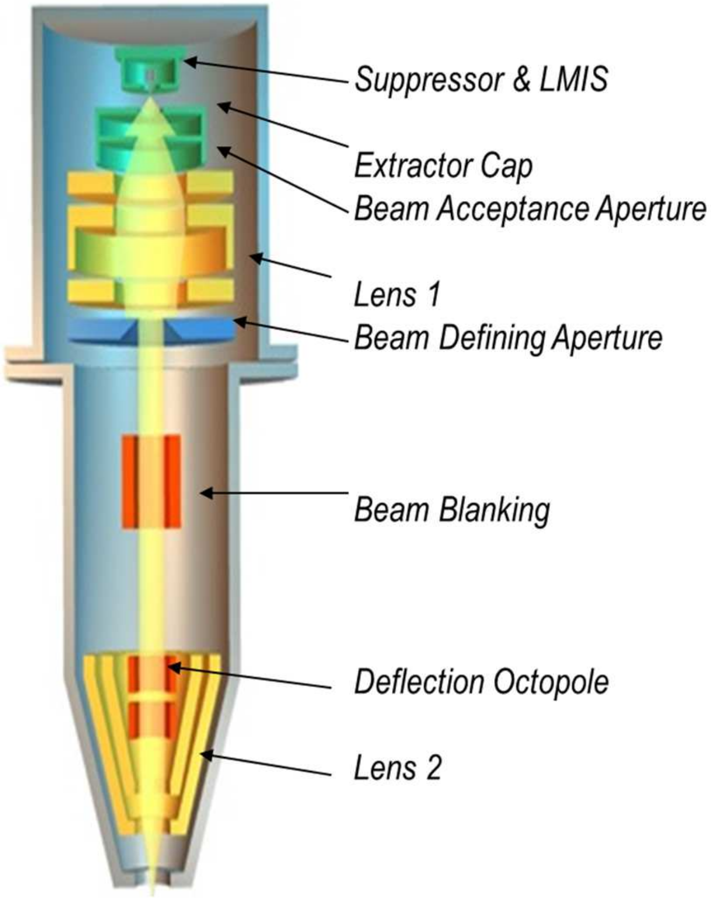
- Electron or ion source
- Lenses
- Mechanical alignment
- Alignment deflectors
- Scanning deflectors linear device
- Stigmators
- Apertures
- Magnetic shielding
- Vacuum 10^{-4} to 10^{-8} Pa
- Differential pumping
- Mechanical stiffness, vibration and acoustic resistance
- Drifts
- Detectors
- Stable sources
- Alignments
- Control model and SW



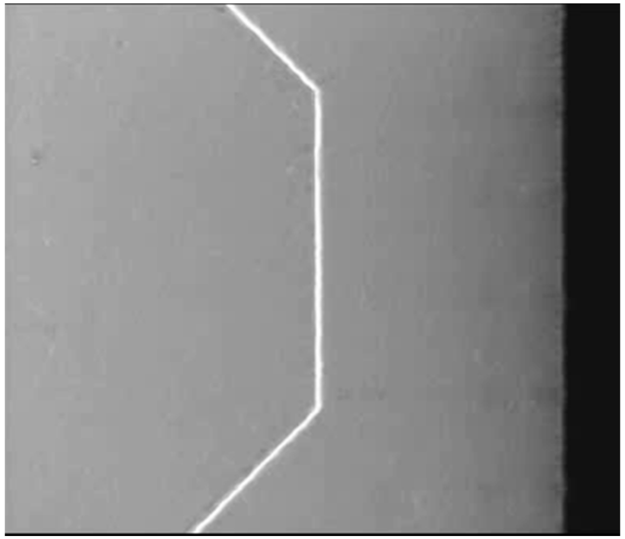
Ion Columns

- Various sources and species: Ga, In, Li, He, Ar
- Electrostatic elements
- Coulomb interactions crucial for resolution
- Sputtering of apertures
- Higher sensitivity on vacuum – beam sputtering

- Micromachining and Gas-assisted etching
- Destructive method
- Volume sputtered is independent of the scan field

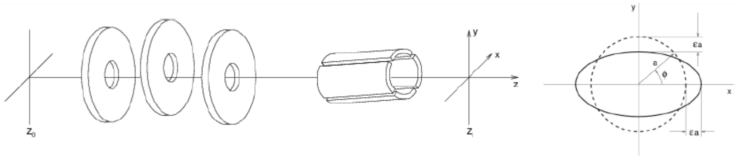


Workflow for electronic industry



1. Select the region of interest
2. Dig out the material in the vicinity
3. Form the lamella, cut on one side
4. Approach the needle
5. Weld the needle and lamella, cut the lamella
6. Transport the lamella to special holder, weld it
7. Cut the needle, repolish the lamella to ~20nm
8. Transport to TEM

Design issues



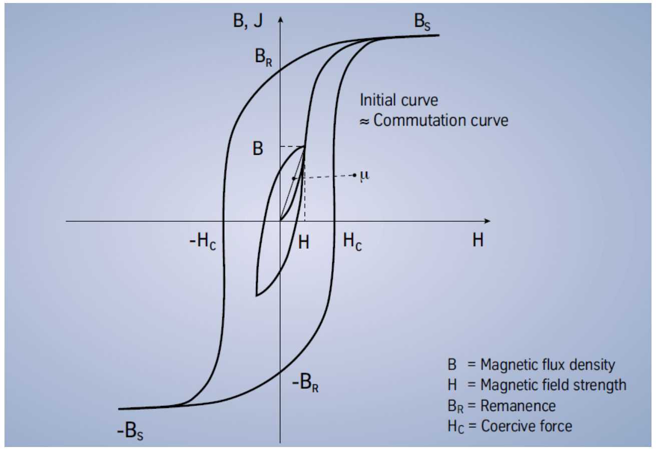
- Colinearity and circularity of the optical elements better in sub-µm region
- Assembling of heavy parts with micron precision – different systems for mechanical alignment: kinematical mounting (cone, prism & plane), deformable string of balls + V grooves
- Vacuum issues:
    - usage of a liner – thin non magnetic tube – excludes electrostatics
    - absence of a liner implies usage of vacuum sealed magnetostatic lenses
    - pressure gradient from the chamber up to the electron gun – differential pumping

Electrostatic lenses

- Electrostatic breakdown on the electrostatic lenses: vacuum 10kV/mm, surface much less – rounded shapes of the insulators: ruby balls, PEEK, Macor, ceramics

Design issues

- Saturation in the yokes of the magnetostatic lenses: design, high saturation soft magnetic materials: ARMCO iron (pure annealed Fe 2.15 T), permalloy (48% Ni and Fe)
- Remanence – degaussing rigorous but lengthy   normalization easier
- Soft magnetic material turned before and after annealing
- Mechanical strain destroys the ferromagnetic properties
- Thermal load of the magnetostatic lenses
due to the Joule heating
water cooling ~ 1000 W – vibrations, drifts



Design issues

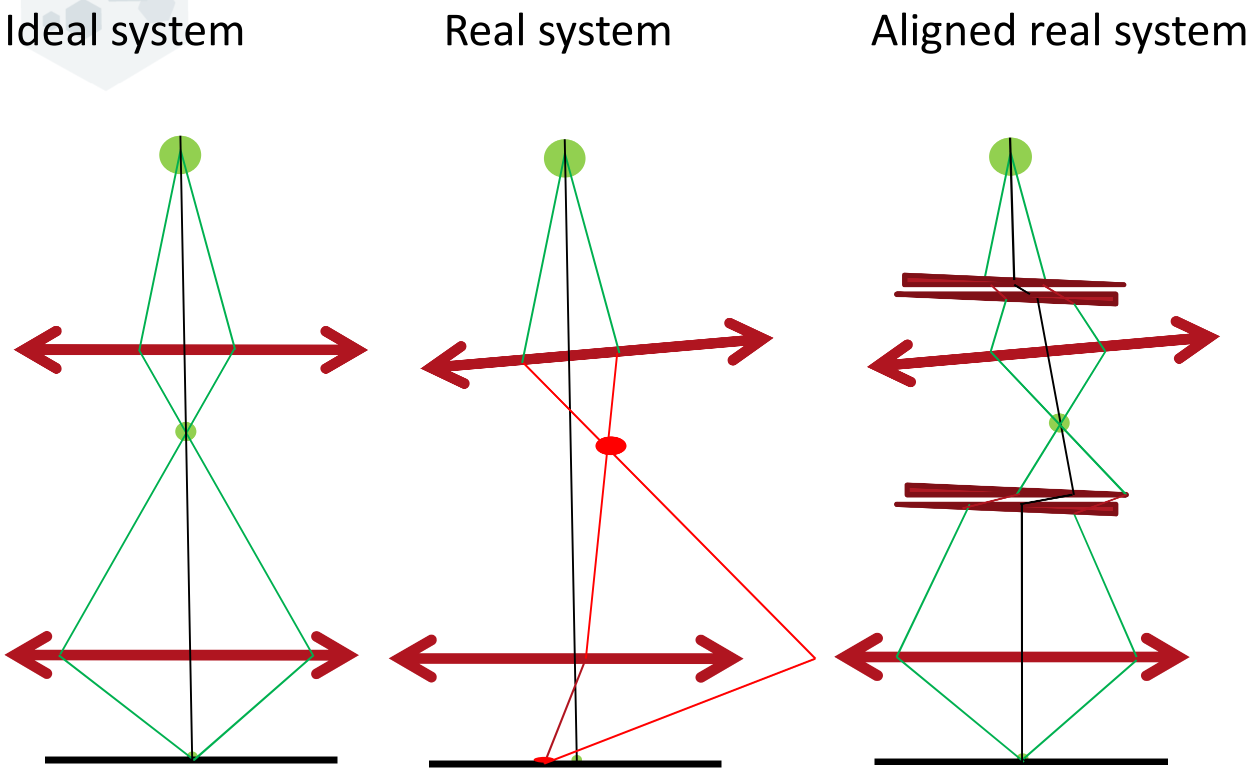
Design constraints results in the physical dimensions of the lenses
Magnification in the full field view systems obtained by ratios of object and image distance → TEM length ~ 3 m : condenser + objective + projective
Scanning probe system more length effective ~ 0.5 m: condenser + final lens

Charging of the materials
Almost all design materials are covered with thin semi or non conductive oxide layer
Solution: covering with carbon

Electrostatic versus Magnetic

	Electrostatic	Magnetostatic
Drivers	High voltage	High currents
Environment	Vacuum	Non magnetic
Column heating	no	yes
Scaling with beam energy	~ HV	~ $HV^{1/2}$
Scaling with particle mass	no	~ $m^{1/2}$
Speed	Faster	Slower
Accuracy	High	Lower (hysteresis)
Cost	More expensive	Less expensive
Application area	Ion optics Low energy electrons Fast systems (beam blanker, lithography)	High energy electrons (TEM) Low cost systems

Alignments



Pairs of deflectors are mostly used for alignments.
The position of the pivot point defines their usage.
Deflections with different pivot points can be added thanks to linearity.

Thank you for your attention

Further reading

- M. Lenc, B. Lencová: Optické prvky elektronových mikroskopů
Metody analýzy povrchů 2. Elektronová mikroskopie a difrakce.
(L. Eckertová, L. Frank ed.) Academia 1996
- B. Lencová, M. Lenc: Optika iontových svazků. Metody analýzy povrchů 3.
(L Frank, J.Král, ed.), Academia 2002
- L. Reimer, Scaning Electron Microscopy -- Physics of Image Formation and Microanalysis, Springer-Verlag 1998
- J. Orloff, ed. Handbook of Charged Particle Optics, CRC Press, 2nd edition 2008
- P. W Hawkes, E. Kasper: Principles of electron optics, vol. 1-3. Academic 1988 and 1996

Determination of the field in 3D

Finite differences method (FDM …Simion)

Advantages: easy to program

Disadvantage: not precise enough

Finite elements method (FEM … MEBS, CST, Comsol + open source…)

Advantages: covers any geometry, non-linearities, commercial packages

Disadvantage: Precise field values only on the nods, rest interpolated, evaluation of the higher order derivatives restriction to the modeled space

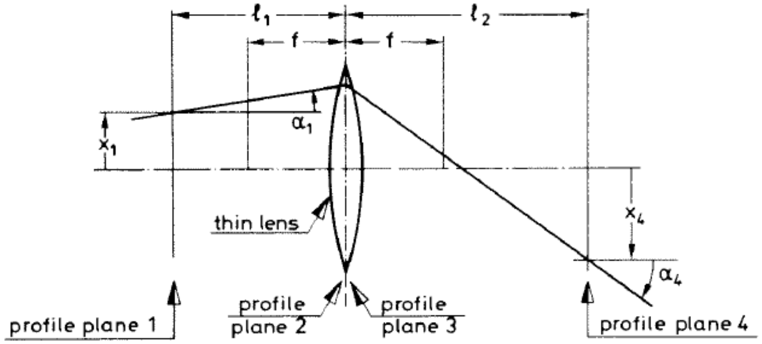
Boundary element method (BEM … Lorentz, open source)

Advantages: reduces dimensionality of the task by 1, precise field, any geometry, suitable for semi-infinite space

Disadvantages: not suitable for non-linear material properties

Transfer matrix method

Thin lens approximation



$$\begin{pmatrix} x_4 \\ \tan\alpha_4 \end{pmatrix} = \begin{pmatrix} 1 & l_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & l_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ \tan\alpha_1 \end{pmatrix}$$

Drift space

$$\begin{bmatrix} x_2 \\ x'_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & L & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x'_1 \\ 1 \end{bmatrix}$$

Lens

$$\begin{bmatrix} x_2 \\ x'_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{f} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x'_1 \\ 1 \end{bmatrix}$$

Deflector

$$\begin{bmatrix} x_2 \\ x'_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{I\,B_0\eta}{\sqrt{\hat{\Phi}}} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x'_1 \\ 1 \end{bmatrix}$$

$$f = \frac{(s+d)k}{10\,P\,(1-P)}$$

$$P = \frac{(NI)^2}{300\,\hat{\Phi}}$$