1.1. Rovnice se separovanými proměnnými

① $2y - x^3y' = 0$

$y' = \frac{2y}{x^3}$

$\int \frac{1}{y}\,dy = 2\int \frac{1}{x^3}\,dx$ $\qquad y = 0$ je řešením

$\ln|y| = -\frac{2}{2x^2} + c$

$|y| = k\cdot e^{-x^{-2}}$

$y = c\cdot e^{-\frac{1}{x^2}} \quad c \in \mathbb{R}^+$

... obecné řešení

$\Big\}\ y = c\cdot e^{-\frac{1}{x^2}} \quad c \in \mathbb{R}$

② $(x+1)\,dy + xy\,dx = 0$

$(x+1)\,dy = -xy\,dx$

$\int \frac{1}{y}\,dy = \int -\frac{x}{x+1}\,dx$

$\int \frac{x}{x+1}\,dx = \int \frac{x+1-1}{x+1}\,dx = \int\left(1 - \frac{1}{x+1}\right)dx = x - \ln|x+1|$

$\ln|y| = -x + \ln|x+1| + c \quad c \in \mathbb{R}$ $\qquad y = 0$ NŘ

$|y| = e^{-x}\cdot|x+1|\,k \quad k \in \mathbb{R}^+ = (0,\infty)$

$y = k\cdot(x+1)\,e^{-x} \quad k \in \mathbb{R} - \{0\}$

③ $(x+1)^{-1}dx - (y-1)^{-1}dy = 0$

$\int \frac{1}{y-1}\,dy = \int \frac{1}{x+1}\,dx$ $\qquad y - 1 = 0$

$\ln|y-1| = \ln|x+1| + c \quad (c = \ln e^c)$ $\qquad y = 1$ je zahrnuto v řešení, připustíme-li $k = 0$

$y - 1 = k\cdot(x-1) \quad k \in \mathbb{R} - \{0\}$

$y = k(x-1) + 1$

$\Big\}\ y = k(x-1) + 1 \quad k \in \mathbb{R}$

④ $y - y^2 + xy' = 0$ $\qquad ⊕\ y \neq 0,\ y \neq 1$

$xy' = y^2 - y$

$⊕\ \int \frac{1}{y(y-1)}\,dx = \int\left(\frac{1}{y-1} - \frac{1}{y}\right)dy = \ln|y-1| - \ln|y|$

$\int \frac{1}{y^2-y}\,dy = \int \frac{1}{x}\,dx$

$\frac{1}{y(y-1)} = \frac{A}{y} + \frac{B}{y-1}$ $\qquad A = -1,\ B = 1$

$\ln\left|\frac{y-1}{y}\right| = \ln|x| + c$

$1 = A(y-1) + By$

$1 = Ay + By - A$

$\left|\frac{y-1}{y}\right| = k\cdot|x| \quad k \in \mathbb{R}^+$

$y - 1 = kxy \quad k \in \mathbb{R} - \{0\}$ $\qquad y = 0$ ✓, nelze zahrnout volbou k

$y(1-kx) = 1$ $\qquad y = 1$ ✓, $k = 0$

$y = \frac{1}{(1-kx)} \quad k \in \mathbb{R}^*$

$y = \frac{1}{(1-kx)} \quad k \in \mathbb{R}, \quad y = 0$

⑤ $y^2+1+xyy'=0$

$$\frac{1}{2}\int\frac{2y}{y^2+1}\,dy=\int-\frac{1}{x}\,dx$$

$$\frac{1}{2}\ln|y^2+1|=-\ln|x|+c \qquad c\in\mathbb{R}$$

$$\ln\sqrt{|y^2+1|}=\ln\frac{1}{|x|}\cdot k \qquad k\in\mathbb{R}^+$$

$$\sqrt{|y^2+1|}=\frac{k}{|x|} \qquad |^2$$

$$\underline{\underline{k=(y^2+1)\cdot x^2}}$$

$$\cos^2x+\sin^2x=1$$
$$-\sin^2x=\cos^2x-1$$
$$\cos 2x=\cos^2x-\sin^2x=\cos^2x+\cos^2x-1=2\cdot\cos^2x-1$$
$$\cos^2x=\frac{1+\cos 2x}{2}$$

⑥ $y'\cdot\cos^2x=(1+\cos^2x)\cdot\sqrt{1-y^2}$

$$\int\frac{1}{\sqrt{1-y^2}}\,dy=\int\left(\frac{1}{\cos^2x}+1\right)dx$$

$$\arcsin y=\operatorname{tg}x+x+c \qquad c\in\mathbb{R}$$

$$\underline{\underline{y=\sin(x+\operatorname{tg}x+c) \qquad c\in\mathbb{R}}}$$

$$\underline{\underline{y=\pm1}}$$

⑦ $\dfrac{x^3\,dx}{\sin y}=-\dfrac{y\,dy}{x}$

$$\int x^4\,dx=\int-y\cdot\sin y\,dy$$

$$\underline{\underline{\frac{x^5}{5}=y\cos y-\sin y+c \qquad c\in\mathbb{R}}}$$

⑧ $e^{-s}\left(1+\dfrac{ds}{dt}\right)=1$

$$1+\frac{ds}{dt}=e^s$$

$$\frac{ds}{dt}=e^s-1$$

$$\int\frac{1}{e^s-1}\,ds=\int dt$$

$$\int\frac{1}{e^s-1}\,ds=\left|\begin{array}{l}y=e^s\\ dy=e^s\,ds\\ dy=y\,ds\\ ds=\frac{1}{y}\,dy\end{array}\right|=\int\frac{1}{y(y-1)}\,dy=\ln\left|\frac{y-1}{y}\right|$$

$$\ln\left|\frac{y-1}{y}\right|=t+c$$

$$\ln|1-e^{-s}|=t+c$$

$$|1-e^{-s}|=e^{t+c}$$

$$\underline{\underline{e^{-s}=k\cdot e^t+1}} \qquad k\in\mathbb{R}-\{0\}$$

$$t=\ln\left|\frac{y-1}{y}\right|+c$$

$$\frac{y-1}{y}=\text{ATD}\ldots$$

$$y=$$

⑨ $y'\cdot\operatorname{tg}x-y^2=1-2y$

$$y'\operatorname{tg}x=1-2y+y^2$$

$$\int\frac{1}{(y-1)^2}\,dy=\int\frac{\cos x}{\sin x}\,dx$$

$$-(y-1)^{-1}=\ln|\sin x|+c \qquad c=\ln e^k$$

$$-\frac{1}{y-1}=\ln k\cdot|\sin x|$$

$$\underline{\underline{(y-1)\cdot\ln(k\cdot\sin x)=1}}$$

(10) $(y^2 + xy^2)\,y' + (x^2 - x^2 y) = 0$

$$y^2(1+x)\,y' + x^2(1-y) = 0$$

$$y^2(1+x)\,y' = x^2(y-1)$$

$$\frac{y^2}{y-1}\,dy = \frac{x^2}{x+1}\,dx$$

$y^2 : (y-1) = y + 1 + \frac{1}{y-1}$

$-(y^2 - y)$

y

$-\,(y-1)$

1

$$\int\left(y + 1 + \frac{1}{y-1}\right)dy = \int\left(x - 1 + \frac{1}{x+1}\right)dx$$

$$\frac{y^2}{2} + y + \ln|y-1| = \frac{x^2}{2} - x + \ln|x+1| + c$$

$$y = 1$$

(11) $y' = 6x + 2y + 3$

$z = 6x + 2y + 3$

$z' = 6 + 2y' + 0 \quad \Rightarrow \quad y' = \frac{1}{2}z' - 3$

$$\frac{1}{2}z' - 3 = z$$

$$\frac{1}{2}z' = z + 3$$

$$\frac{1}{z+3}\,dz = 2\,dx$$

$$\ln|z+3| = 2x + c$$

$$z + 3 = e^{2x+c} = k \cdot e^{2x}$$

$$6x + 2y + 6 = k \cdot e^{2x}$$

$$y = k \cdot e^{2x} - 3(x+1)$$

(12) $y' + 1 = x + y$

$y' = x + y - 1$

$z = x + y - 1$

$z' = 1 + y'$

$$z' - 1 = z$$

$$z' = z + 1$$

$\ln|z+1| = x$ $\qquad z = 0$ ne

$$z + 1 = e^x \cdot k$$

$x + y - 1 + 1 = e^x \cdot k \qquad k \in \mathbb{R} - \{0\}$

$$y = k \cdot e^x - x$$

(13) $y' + y = 2x + 3$

$z = -y + 2x + 3$

$z' = -y' + 2 \quad \Rightarrow \quad y' = -z' + 2$

$$y' = -y + 2x + 3$$

$$-z' + 2 = z$$

$$z' = 2 - z$$

$$-\ln|2 - z| = x$$

$2 - z = c \cdot e^{-x} \qquad c \in \mathbb{R}$

$$2 + y - 2x - 3 = c \cdot e^{-x}$$

$y = c \cdot e^{-x} + 2x + 1 \qquad c \in \mathbb{R}$

(14) $y' - y + 2x = 0$

$z = y - 2x$

$z' = y' - 2$

$y' = y - 2x$

$z' + 2 = z$

$\ln|z-2| = x$

$z - 2 = c \cdot e^{x}$

$z = c \cdot e^{x} + 2$

$y - 2x = c \cdot e^{x} + 2$

$$y = c \cdot e^{x} + 2 + 2x \qquad c \in \mathbb{R}$$

(15) $y' = x + 2y$

$z = x + 2y$

$z' = 1 + 2y'$

$\frac{z' - 1}{2} = z$

$z' = 2z + 1$

$\frac{1}{2} \ln|2z+1| = x + c$

$2z + 1 = c \cdot e^{2x}$

$2x + 4y + 1 = c \cdot e^{2x}$

$$y = c \cdot e^{2x} - \frac{1}{2}x - \frac{1}{4}$$