3. 3 Plasma sheath

Why plasma sheath? Consider thermal velocity (eT/m)^1/2
It is at least 100x higher for electrons than for ions

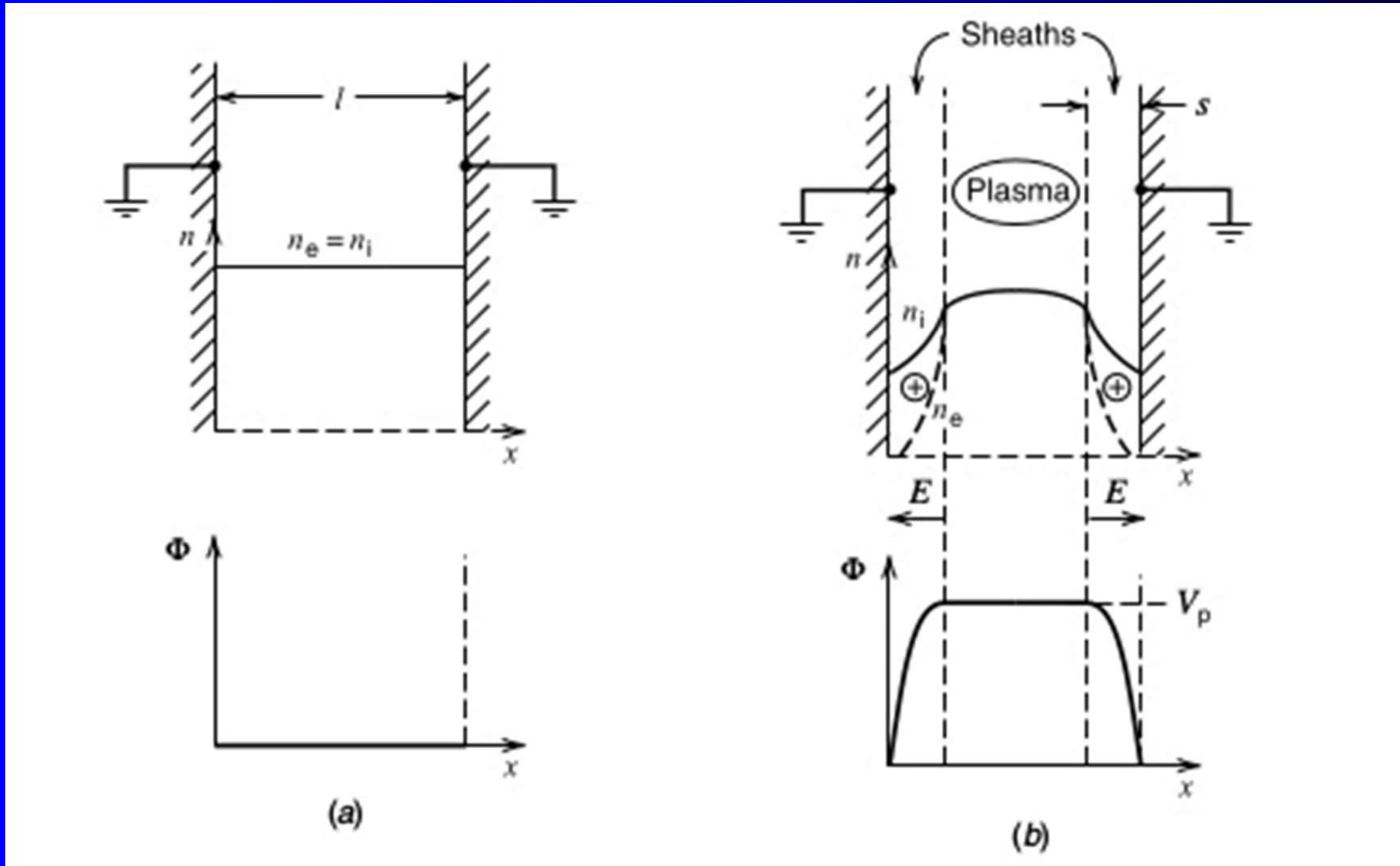
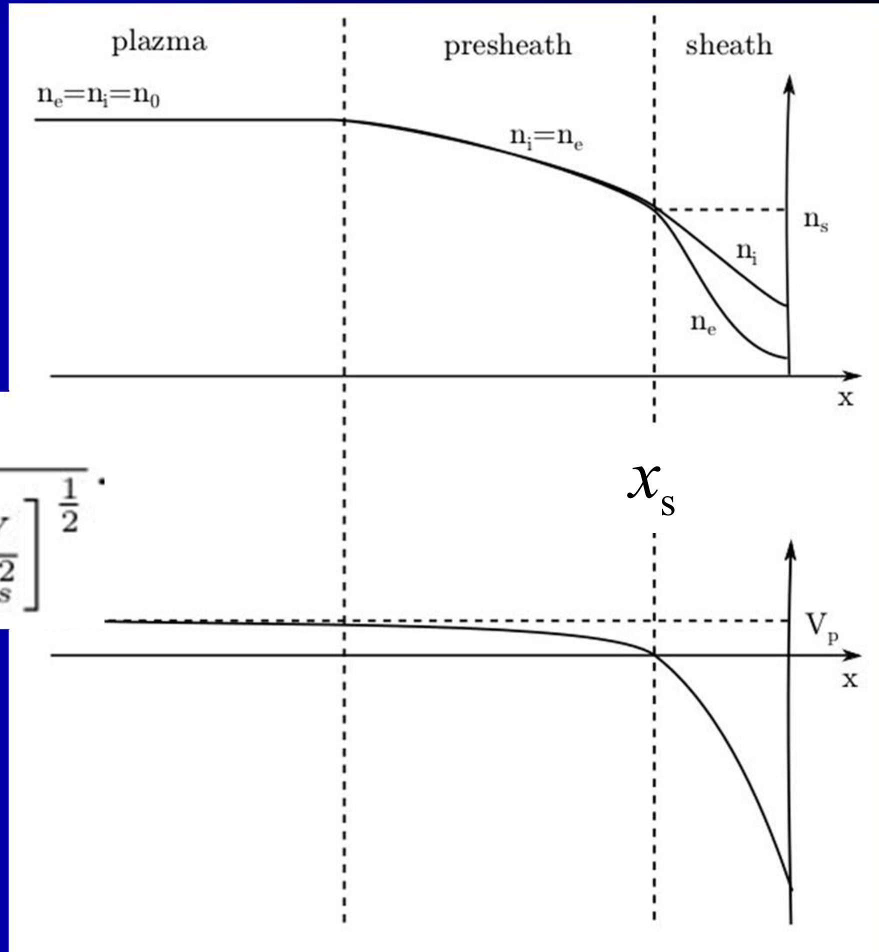


FIGURE 1.10. The formation of plasma sheaths: (*a*) initial ion and electron densities and potential; (*b*) densities, electric field, and potential after formation of the sheath.

Plasma sheath – Debye sheath

in weakly ionized plasma with $T_e \approx \text{few eV}$

at grounded or floating wall (low potential drop), collisionless $T_i \approx 0$



density of e$^-$ / ions in the sheath:

$$n_e = n_s \mathrm{e}^{\frac{eV}{k_b T_e}},\qquad n_i = \frac{n_s}{\left[1 - \frac{2eV}{M v_s^2}\right]^{\frac{1}{2}}}.$$

velocity of ions at sheath edge (Bohm velocity):

$$v_s > \left(\frac{k_b T_e}{M}\right)^{\frac{1}{2}} = u_B$$

density at the sheath edge: $n_s \approx 0.5\, n_0$

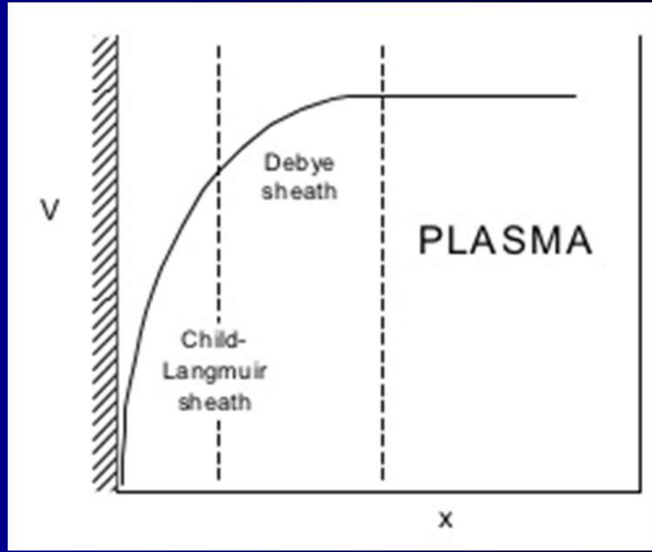
electron and ion flux has to equal at floating wall $\Gamma_i = n_s u_B$ $\quad \Gamma_e = \frac{1}{4} n_s \sqrt{\frac{8 k_b T_e}{\pi m}}\, \mathrm{e}^{\frac{eV}{k_b T_e}}$

$$V_w = \frac{k_b T_e}{2e} \ln\left(\frac{2\pi m}{M}\right)$$

calculation for Ar discharge with T_e = 2 eV, n=10^8 cm^{-3}:
floating potential approx. 5T_e =10 V,
sheath thickness approx. 5λ_D = 0.37 mm

Plasma sheath – Child-Langmuir sheath

high-voltage sheath (when a voltage is applied)



Sheath can be artificially divided into Debye sheath which contains electrons and high-voltage Child-Langmuir sheath which has ions only.

Then, current density *j*, voltage drop V_0 and sheath thickness *d* are related by the Child-Langmuir Law of Space-Charge-Limited Diodes

$$j = \frac{4}{9}\left(\frac{2e}{M}\right)^{1/2} \frac{\varepsilon_0 V_0^{3/2}}{d^2}$$

$$d = \frac{2}{3}\left(\frac{2V_0}{T_{eV}}\right)^{3/4} \lambda_D$$

following previous example with assumption V_0 = 400 V: d = 30λ_D,
total sheath thickness 35λ_D, i.e. about 1 cm

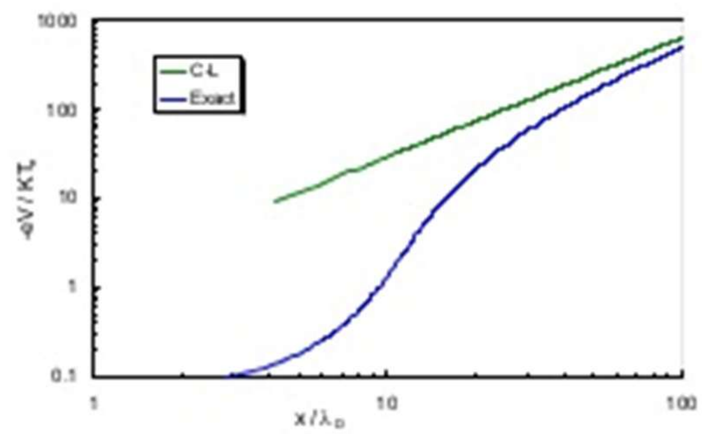


Fig. 3. An exact calculation for a plane sheath shows that C-L scaling is not followed unless the sheath is very thick (log-log scale).