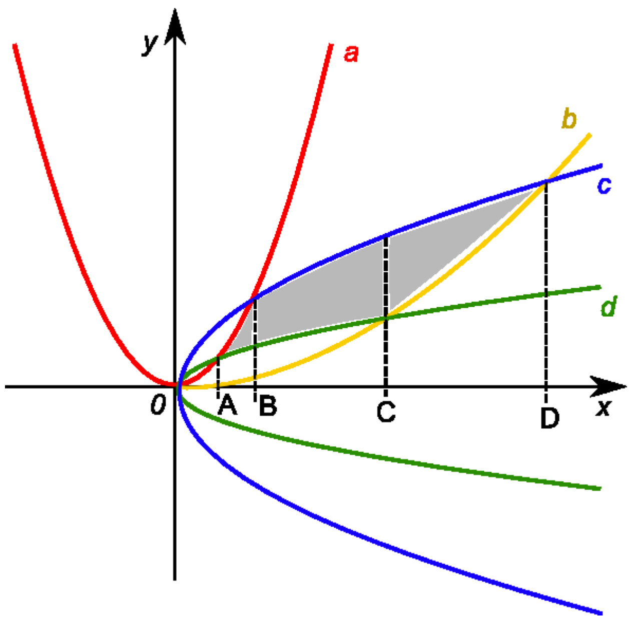
$$\iint\limits_{\Omega} \mathrm{d}x\mathrm{d}y \qquad \underbrace{\Omega}_{\blacksquare} : \quad \underbrace{x^2 = 4y}_{a}, \ \underbrace{x^2 = 8y}_{b}, \ \underbrace{y^2 = 2x}_{d}, \ \underbrace{y^2 = 4x}_{c}$$



Výpočet důležitých průsečíků křivek:

pro A z a, d:

$$\frac{x^4}{16} = 2x, \quad x^3 = 32, \quad x = \sqrt[3]{32} = 2\sqrt[3]{4}, \quad A = 2\sqrt[3]{4}$$

pro B z a, c:

$$\frac{x^4}{16} = 4x, \quad x^3 = 64, \quad x = \sqrt[3]{64} = 4, \quad B = 4$$

pro C z b, d:

$$\frac{x^4}{64} = 2x, \quad x^3 = 128, \quad x = \sqrt[3]{128} = 4\sqrt[3]{2}, \quad C = 4\sqrt[3]{2}$$

pro D z b, c:

$$\frac{x^4}{64} = 4x, \quad x^3 = 256, \quad x = \sqrt[3]{256} = 4\sqrt[3]{4}, \quad D = 4\sqrt[3]{4}$$

Platí tedy:

$$\iint\limits_{\Omega} \mathrm{d}x\mathrm{d}y = \underbrace{\int_{2\sqrt[3]{4}}^{4} \mathrm{d}x \int_{\sqrt{2x}}^{\frac{x^2}{4}} \mathrm{d}y}_{I_1} + \underbrace{\int_{4}^{4\sqrt[3]{2}} \mathrm{d}x \int_{\sqrt{2x}}^{\sqrt{4x}} \mathrm{d}y}_{I_2} + \underbrace{\int_{4\sqrt[3]{2}}^{4\sqrt[3]{4}} \mathrm{d}x \int_{\frac{x^2}{8}}^{2\sqrt{x}} \mathrm{d}y}_{I_3}$$

$$I_1 = \int_{2\sqrt[3]{4}}^{4} \mathrm{d}x \int_{\sqrt{2x}}^{\frac{x^2}{4}} \mathrm{d}y = \int_{2\sqrt[3]{4}}^{4} \frac{x^2}{4} - \sqrt{2x}\ \mathrm{d}x = \left[\frac{x^3}{12} - \frac{2(2x)^{\frac{3}{2}}}{6}\right]_{2\sqrt[3]{4}}^{4} = \left(\frac{64}{12} - \frac{8^{\frac{3}{2}}}{3}\right) - \left(\frac{8\cdot 4}{12} - \frac{2\cdot 16}{6}\right) =$$

$$= \underline{8 - \frac{16}{3}\sqrt{2}},$$

$$I_2 = \int_{4}^{4\sqrt[3]{2}} \mathrm{d}x \int_{\sqrt{2x}}^{\sqrt{4x}} \mathrm{d}y = \int_{2\sqrt[3]{4}}^{4} \sqrt{4x} - \sqrt{2x}\ \mathrm{d}x = \left[2\frac{2x^3}{3} - \frac{2(2x)^{\frac{3}{2}}}{6}\right]_{4}^{4\sqrt[3]{2}} =$$

$$= \left(\frac{4}{3}\left(4\sqrt[3]{2}\right)^{\frac{3}{2}} - \frac{1}{3}\left(8\sqrt[3]{2}\right)^{\frac{3}{2}}\right) - \left(\frac{4\cdot 8}{3} - \frac{1}{3}\cdot 8^{\frac{3}{2}}\right) = \underline{16\sqrt{2} - \frac{64}{3}}$$

$$4\sqrt[3]{2}I_3 = \int_{4\sqrt[3]{2}}^{4\sqrt[3]{4}} \mathrm{d}x \int_{\frac{x^2}{8}}^{2\sqrt{x}} \mathrm{d}y = \int_{4\sqrt[3]{2}}^{4\sqrt[3]{4}} 2\sqrt{x} - \frac{x^2}{8}\ \mathrm{d}x = \left[\frac{4x^{\frac{3}{2}}}{3} - \frac{x^3}{24}\right]_{4\sqrt[3]{2}}^{4\sqrt[3]{4}} =$$

$$= \left(\frac{4\left(4\sqrt[3]{4}\right)^{\frac{3}{2}}}{3} - \frac{\left(4\sqrt[3]{4}\right)^3}{24}\right) - \left(\frac{4\left(4\sqrt[3]{2}\right)^{\frac{3}{2}}}{3} - \frac{\left(4\sqrt[3]{2}\right)^3}{24}\right) = \underline{16 - \frac{32}{3}\sqrt{2}}$$

$$\iint\limits_{\Omega} \mathrm{d}x\mathrm{d}y = I_1 + I_2 + I_3 = 8 - \frac{16}{3}\sqrt{2} + 16\sqrt{2} - \frac{64}{3} + 16 - \frac{32}{3}\sqrt{2} = \underline{\underline{\frac{8}{3}}} \qquad ☺$$