Řešitelský seminář, 28. 2. 2017

Problem 1. *Evaluate*

$$\lim_{n\to\infty}\int_0^1 e^{x^n}\,dx.$$

Problem 2. *We say that a group* $(G,\cdot)$ *has the property (P), if for any automorphism* f *of* G *exist two automorphisms* g *and* h *of* G *such that* $f(x)=g(x)\cdot h(x)$*, for any* $x\in G$*. Prove that:*

1. *Any group with the property* (P) *is commutative,*
2. *Any finite abelian group of odd order has the property* (P)*,*
3. *No finite group of order* $4n+2$*,* $n\in\mathbb{N}$*, has the property* (P)*.*

Problem 3. *Let* A *and* B *be two* n *by* n *matricies with real entries such that* $AB^2=A-B$*.*

1. *Prove that* I_n+B *is a nonsingular matrix,*
2. *Prove that* $AB=BA$*.*

Problem 4. *Let* G *be a group of order* n *and let* e *be the identity element. Find all functions* $f:G\to N^*$ *such that*

1. $f(x)=1$ *iff* $x=e$*.*
2. $f(x^k)=f(x)/(f(x),k)$*, for all positive divisors* k *of* n*.*

Domácí úloha

Problem 5. *Let* G *be a group of order* $2p$*, where* p *is an odd prime. Assume that* G *has a normal subgroup of order* 2*. Prove that* G *is cyclic.*