Řešitelský seminář, 7. 3. 2017

Problem 1. *Let* $f:\mathbb{R}\to\mathbb{R}$ *be continuous. Suppose that* $\mathbb{R}$ *contains a countably infinite subset* S *such that*

$$\int_p^q f(x)\,dx = 0$$

if p *and* q *are not in* S. *Prove that* f *is identicaly zero.*

Problem 2. *Let* n *be a positive integer and* $A \in \mathcal{M}_n(\mathbb{C})$, $A = (a_{pq}), 1 \le p, q \le n$ *be such that* $a_{ij} + a_{jk} + a_{ki} = 0$ *for all* $i, j, k \in \{1, 2 \dots n\}$. *Prove that* $\operatorname{rank}(A) \le 2$.

Problem 3. *Let* $(A, +, \cdot)$ *be a finite unitary and commutative ring. Denote by* d *the number of divisors of zero and by* n *the number of nilpotents. Prove*

1. *If* x *and* y *are nilpotents, then* $x + y$ *and* xy *are nilpotents,*
2. n *is a divisor of* d.

Problem 4. *Prove or give an counterexample: Every connected, locally pathwise connected set in* $\mathbb{R}^n$ *is pathwise connected.*

Domácí úloha

Problem 5. *For which numbers* $a \in (1, \infty)$ *is true that* $x^a \le a^x$ *for all* $x \in (1, \infty)$*?*