Řešitelský seminář, 14. 3. 2017

Problem 1. *Let A be a unitary and commutative ring with an odd number of elements. If n is the number of solutions of the equation $x^2 = x$, $x \in A$, and m the number of invertible elemets, show that n divides m.*

Problem 2. *Let $f : [0,1] \to \mathbb{R}$ be a continuous differentiable function, such that*

$$\int_0^1 (f'(x))^2 dx \leq 2 \int_0^1 f(x) dx.$$

Find f if $f(1) = -\frac{1}{6}$.

Problem 3. *Prove or give an counterexample: Every connected, locally pathwise connected set in $\mathbb{R}^n$ is pathwise connected.*

Domácí úloha

Problem 4. *Show that a positive constant t can satisfy $e^x > x^t$ for all $x > 0$, iff $t < e$.*