Řešitelský seminář, 4. 4. 2017

Problem 1. *Prove that* $\frac{S(n)}{S(3n)}$ *is unbounded. (S(n) is the sum of the digits of positive integer* n *in its decimal representation.*

Problem 2. *Suppose that a graph* G *is the union of three trees. Is it true that* G *can be covered by two planar graphs?*

Problem 3. *Show that if* $k \leq \frac{n}{2}$ *and* $\mathcal{F}$ *is a family of* $k \times k$ *submatrices of and* $n \times n$ *matrix such that any two intersect then*

$$|\mathcal{F}| \leq \binom{n-1}{k-1}^2.$$

Problem 4. *Let* n *be a positive integer, and* $S_1, \dots S_n$ *be a collection of finite non-empty sets such that*

$$\sum_{1 \leq i < j \leq n} \frac{|S_i \cap S_j|}{|S_i||S_j|} < 1.$$

Prove that there exits pairwaise distinct elements $x_1, \dots x_n$ *such that* x_i *is a member of* S_i *for each index* i*.*

Domácí úloha

Problem 5. *Determine the integers* $k \geq 2$ *for which the sequence* $\binom{2n}{n}$, $n = 1, 2, 3, \dots$ *is eventually periodic.*